

# Computer algebra independent integration tests

Summer 2022 edition

2-Exponentials/53-2.1-u-F<sup>-c-a+b-x</sup>-<sup>n</sup>

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# Chapter 1

## Introduction

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This report gives the result of running the computer algebra independent integration test. The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [ 98 ]. This is test number [ 53 ].

## 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 ( 98 )	0.00 ( 0 )
Mathematica	100.00 ( 98 )	0.00 ( 0 )
Fricas	94.90 ( 93 )	5.10 ( 5 )
Maple	79.59 ( 78 )	20.41 ( 20 )
Maxima	65.31 ( 64 )	34.69 ( 34 )
Mupad	59.18 ( 58 )	40.82 ( 40 )
Giac	57.14 ( 56 )	42.86 ( 42 )
Sympy	42.86 ( 42 )	57.14 ( 56 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

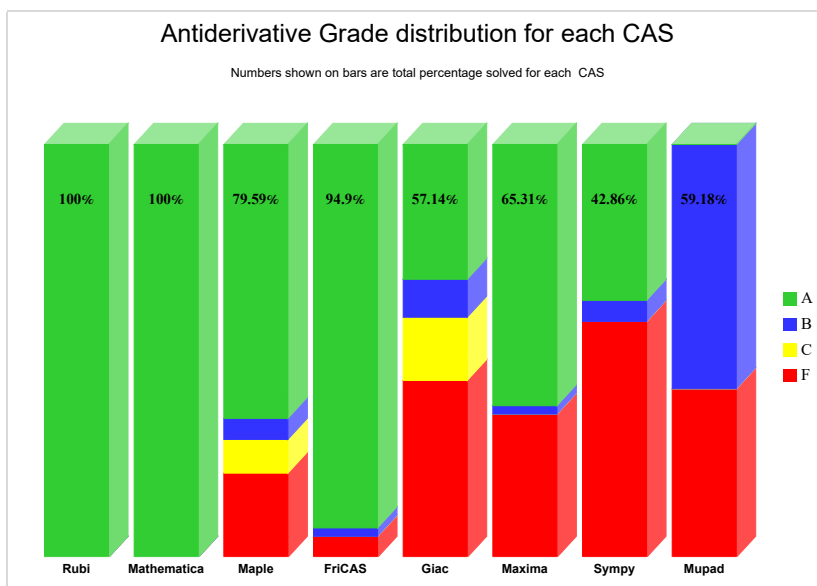
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

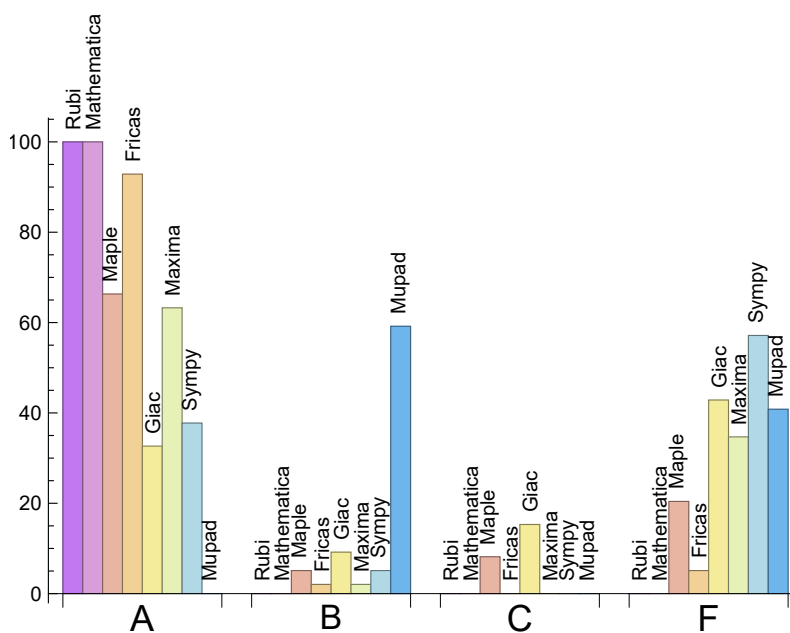
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	100.00	0.00	0.00	0.00
Fricas	92.86	2.04	0.00	5.10
Maple	66.33	5.10	8.16	20.41
Maxima	63.27	2.04	0.00	34.69
Sympy	37.76	5.10	0.00	57.14
Giac	32.65	9.18	15.31	42.86
Mupad	N/A	59.18	0.00	40.82

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	20	100.00 %	0.00 %	0.00 %
Fricas	5	100.00 %	0.00 %	0.00 %
Giac	42	88.10 %	0.00 %	11.90 %
Maxima	34	100.00 %	0.00 %	0.00 %
Sympy	56	78.57 %	8.93 %	12.50 %
Mupad	40	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS



## 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

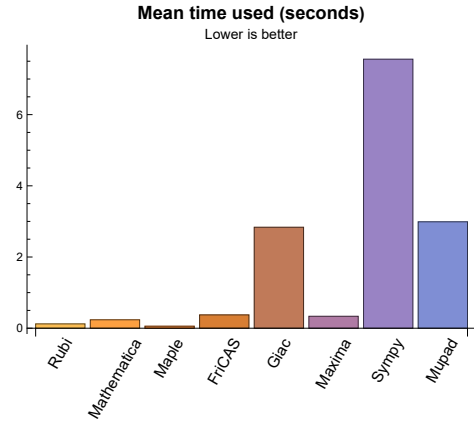
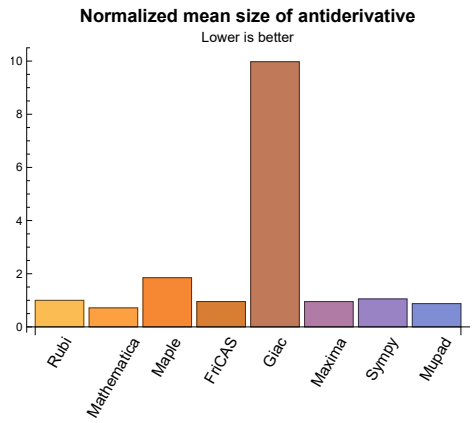
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.12	134.81	1.00	95.50	1.00
Mathematica	0.24	86.01	0.72	65.50	0.71
Maple	0.06	186.65	1.85	141.00	1.17
Maxima	0.33	114.47	0.95	62.50	0.81
Fricas	0.37	130.90	0.95	83.00	0.88
Sympy	7.56	178.00	1.05	101.50	0.84
Giac	2.84	1837.70	9.98	190.50	1.15
Mupad	2.99	114.10	0.88	80.00	0.82

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.



## 1.4 list of integrals that has no closed form antiderivative

{

## 1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima `integrate` was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

### 1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

### 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at [https://ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

### 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand,the_variable)
```

Which gives  $\sin(x)^2/2$



## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in Rubi Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntax



# Chapter 2

## detailed summary tables of results

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## 2.1 List of integrals sorted by grade for each CAS

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### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

B grade: { }

C grade: { }

F grade: { }

### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

B grade: { }

C grade: { }

F grade: { }

### 2.1.3 Maple

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 48, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 90, 91, 92, 93, 94 }

B grade: { 64, 95, 96, 97, 98 }

C grade: { 55, 83, 84, 85, 86, 87, 88, 89 }

F grade: { 1, 19, 20, 21, 22, 23, 24, 25, 26, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50 }

### 2.1.4 Maxima

A grade: { 3, 4, 5, 6, 13, 14, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 48, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

B grade: { 2, 12 }

C grade: { }

F grade: { 1, 7, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50, 78, 79, 80, 81, 82 }

### 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 23, 24, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

B grade: { 81, 82 }

C grade: { }

F grade: { 20, 21, 22, 25, 26 }

### 2.1.6 Sympy

A grade: { 4, 5, 6, 14, 27, 28, 29, 31, 32, 35, 36, 37, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 75, 76, 77, 90, 91, 92, 93, 94 }

B grade: { 2, 3, 12, 13, 74 }

C grade: { }

F grade: { 1, 7, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 30, 33, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 64, 69, 70, 71, 72, 73, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 95, 96, 97, 98 }

### 2.1.7 Giac

A grade: { 6, 27, 28, 29, 30, 31, 32, 33, 34, 43, 48, 56, 57, 58, 59, 60, 61, 62, 63, 74, 75, 76, 77, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

B grade: { 39, 40, 41, 42, 78, 79, 80, 81, 82 }

C grade: { 2, 3, 4, 5, 12, 13, 14, 51, 52, 53, 54, 65, 66, 67, 68 }

F grade: { 1, 7, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 35, 36, 37, 38, 44, 45, 46, 47, 49, 50, 55, 64, 69, 70, 71, 72, 73, 83, 84, 85, 86, 87, 88, 89 }

### 2.1.8 Mupad

A grade: { }

B grade: { 2, 3, 4, 5, 6, 12, 13, 14, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 48, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94 }

C grade: { }

F grade: { 1, 7, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50, 55, 64, 78, 79, 80, 81, 82, 95, 96, 97, 98 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column N.S. in the table below, which stands for **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$ . To help make the table fit, **Mathematica** was abbrev-

	Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
viated to MMA.	grade	A	A	A	F	F	A	F	F	F
	verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
	size	67	67	67	0	0	65	0	0	-1
	N.S.	1	1.00	1.00	0.00	0.00	0.97	0.00	0.00	-0.01
	time (sec)	N/A	0.027	0.043	0.022	0.000	0.096	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	100	260	311	217	350	7859	260
N.S.	1	1.00	0.71	1.84	2.21	1.54	2.48	55.74	1.84
time (sec)	N/A	0.080	0.091	0.097	0.291	0.395	0.107	2.678	3.681

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	78	165	208	142	231	4685	165
N.S.	1	1.00	0.71	1.50	1.89	1.29	2.10	42.59	1.50
time (sec)	N/A	0.053	0.070	0.062	0.279	0.453	0.085	3.246	3.564

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	56	91	125	83	133	2490	91
N.S.	1	1.00	0.71	1.15	1.58	1.05	1.68	31.52	1.15
time (sec)	N/A	0.035	0.063	0.057	0.301	0.433	0.066	2.984	3.507

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	34	38	62	40	60	1079	38
N.S.	1	1.00	0.71	0.79	1.29	0.83	1.25	22.48	0.79
time (sec)	N/A	0.014	0.043	0.011	0.280	0.417	0.058	2.962	3.371

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	21	21	20	21	20	21	21
N.S.	1	1.00	1.05	1.05	1.00	1.05	1.00	1.05	1.05
time (sec)	N/A	0.003	0.014	0.017	0.263	0.433	0.027	3.428	3.428

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	56	0	38	0	0	-1
N.S.	1	1.00	1.00	1.81	0.00	1.23	0.00	0.00	-0.03
time (sec)	N/A	0.023	0.053	0.070	0.000	0.373	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	99	0	77	0	0	-1
N.S.	1	1.00	0.96	1.74	0.00	1.35	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.104	0.080	0.000	0.407	0.000	0.000	0.000



Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	88	155	0	130	0	0	-1
N.S.	1	1.00	0.93	1.63	0.00	1.37	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.139	0.072	0.000	0.357	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	99	199	0	200	0	0	-1
N.S.	1	1.00	0.77	1.55	0.00	1.56	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.165	0.071	0.000	0.355	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	121	243	0	285	0	0	-1
N.S.	1	1.00	0.75	1.51	0.00	1.77	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.150	0.073	0.000	0.401	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	100	260	311	217	350	7859	260
N.S.	1	1.00	0.71	1.84	2.21	1.54	2.48	55.74	1.84
time (sec)	N/A	0.088	0.023	0.025	0.511	0.419	0.105	3.657	3.486

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	78	165	208	142	231	4685	165
N.S.	1	1.00	0.71	1.50	1.89	1.29	2.10	42.59	1.50
time (sec)	N/A	0.082	0.034	0.020	0.297	0.390	0.083	3.528	3.429

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	56	91	125	83	133	2490	91
N.S.	1	1.00	0.71	1.15	1.58	1.05	1.68	31.52	1.15
time (sec)	N/A	0.032	0.017	0.016	0.287	0.376	0.067	2.937	3.403

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	99	0	77	0	0	-1
N.S.	1	1.00	0.96	1.74	0.00	1.35	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.022	0.088	0.000	0.360	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	88	155	0	130	0	0	-1
N.S.	1	1.00	0.93	1.63	0.00	1.37	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.037	0.046	0.000	0.366	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	99	199	0	200	0	0	-1
N.S.	1	1.00	0.77	1.55	0.00	1.56	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.019	0.077	0.000	0.376	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	121	243	0	285	0	0	-1
N.S.	1	1.00	0.75	1.51	0.00	1.77	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.021	0.076	0.000	0.372	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	0	0	68	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.94	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.031	0.013	0.000	0.103	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.032	0.024	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.032	0.019	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.031	0.028	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	65	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.011	0.000	0.000	0.087	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	0	0	67	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.032	0.014	0.000	0.106	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.030	0.029	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.031	0.021	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	14	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	0.87
time (sec)	N/A	0.003	0.012	0.011	0.276	0.451	0.025	2.532	3.538

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	15	15	15	15
N.S.	1	1.00	1.00	1.07	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.003	0.013	0.011	0.271	0.470	0.025	2.293	3.442

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	13	10	13	11
N.S.	1	1.00	1.00	0.74	0.68	0.68	0.53	0.68	0.58
time (sec)	N/A	0.004	0.013	0.028	0.269	0.380	0.025	3.470	0.092

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	36	99	24	89	0	94	82
N.S.	1	1.00	0.27	0.76	0.18	0.68	0.00	0.72	0.63
time (sec)	N/A	0.113	0.064	0.027	0.320	0.373	0.000	2.926	3.433

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	36	87	24	77	37	82	72
N.S.	1	1.00	0.33	0.81	0.22	0.71	0.34	0.76	0.67
time (sec)	N/A	0.069	0.063	0.008	0.385	0.391	128.850	2.078	3.464

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	36	75	24	65	37	70	75
N.S.	1	1.00	0.42	0.88	0.28	0.76	0.44	0.82	0.88
time (sec)	N/A	0.048	0.089	0.008	0.328	0.376	8.364	3.006	3.417

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	30	66	24	51	0	58	55
N.S.	1	1.00	0.48	1.06	0.39	0.82	0.00	0.94	0.89
time (sec)	N/A	0.032	0.054	0.007	0.320	0.390	0.000	3.132	3.411

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	30	27	29	34	0	28	32
N.S.	1	1.00	0.79	0.71	0.76	0.89	0.00	0.74	0.84
time (sec)	N/A	0.017	0.059	0.019	0.318	0.395	0.000	3.095	3.497

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	38	64	24	44	34	0	42
N.S.	1	1.00	0.70	1.19	0.44	0.81	0.63	0.00	0.78
time (sec)	N/A	0.032	0.110	0.008	0.320	0.450	0.926	0.000	3.488

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	49	72	24	58	39	0	61
N.S.	1	1.00	0.64	0.94	0.31	0.75	0.51	0.00	0.79
time (sec)	N/A	0.053	0.100	0.012	0.321	0.452	9.662	0.000	3.505

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	61	84	24	74	39	0	80
N.S.	1	1.00	0.61	0.84	0.24	0.74	0.39	0.00	0.80
time (sec)	N/A	0.063	0.105	0.009	0.327	0.421	144.633	0.000	3.453

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	73	96	24	86	0	0	99
N.S.	1	1.00	0.59	0.78	0.20	0.70	0.00	0.00	0.80
time (sec)	N/A	0.078	0.116	0.009	0.325	0.385	0.000	0.000	3.525

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	72	0	0	224	0	1091	-1
N.S.	1	1.00	0.35	0.00	0.00	1.08	0.00	5.25	-0.00
time (sec)	N/A	0.205	0.146	0.007	0.000	0.361	0.000	2.595	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	72	0	0	165	0	691	-1
N.S.	1	1.00	0.42	0.00	0.00	0.95	0.00	3.99	-0.01
time (sec)	N/A	0.125	0.124	0.007	0.000	0.371	0.000	2.600	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	63	0	0	122	0	401	-1
N.S.	1	1.00	0.46	0.00	0.00	0.88	0.00	2.91	-0.01
time (sec)	N/A	0.094	0.145	0.007	0.000	0.373	0.000	3.530	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	63	0	0	91	0	198	-1
N.S.	1	1.00	0.60	0.00	0.00	0.87	0.00	1.89	-0.01
time (sec)	N/A	0.065	0.090	0.007	0.000	0.365	0.000	4.240	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	63	0	0	63	0	58	-1
N.S.	1	1.00	0.88	0.00	0.00	0.88	0.00	0.81	-0.01
time (sec)	N/A	0.037	0.092	0.008	0.000	0.417	0.000	3.046	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	75	0	0	91	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.94	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.172	0.008	0.000	0.397	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	92	0	0	139	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.284	0.007	0.000	0.389	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	118	0	0	224	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.268	0.007	0.000	0.378	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	144	0	0	309	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	1.54	0.00	0.00	-0.00
time (sec)	N/A	0.159	0.356	0.016	0.000	0.375	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	24	145	79	82	0	80	89
N.S.	1	1.00	0.16	0.96	0.52	0.54	0.00	0.53	0.59
time (sec)	N/A	0.107	0.597	0.101	0.274	0.393	0.000	2.744	3.434



Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	63	0	0	118	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	1.66	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.174	0.007	0.000	0.130	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	78	0	0	135	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	1.38	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.202	0.020	0.000	0.104	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	34	38	62	40	60	1079	38
N.S.	1	1.00	0.71	0.79	1.29	0.83	1.25	22.48	0.79
time (sec)	N/A	0.013	0.011	0.000	0.273	0.372	0.055	2.319	0.002

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	56	80	119	76	116	2486	80
N.S.	1	1.00	0.41	0.59	0.88	0.56	0.86	18.41	0.59
time (sec)	N/A	0.076	0.080	0.015	0.293	0.388	0.063	3.985	3.396

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	84	138	196	124	190	4275	138
N.S.	1	1.00	0.37	0.60	0.86	0.54	0.83	18.67	0.60
time (sec)	N/A	0.136	0.127	0.018	0.298	0.370	0.086	3.198	3.542

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	117	212	293	184	284	7421	212
N.S.	1	1.00	0.34	0.61	0.84	0.53	0.82	21.32	0.61
time (sec)	N/A	0.238	0.438	0.025	0.299	0.398	0.106	2.368	3.531

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	61	334	123	126	99	0	-1
N.S.	1	1.00	0.53	2.88	1.06	1.09	0.85	0.00	-0.01
time (sec)	N/A	0.127	0.100	0.088	0.103	0.150	11.988	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	121	432	196	121	236	202	175
N.S.	1	1.00	0.30	1.09	0.49	0.30	0.59	0.51	0.44
time (sec)	N/A	0.360	0.226	0.074	0.289	0.440	0.096	2.331	3.535

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	130	291	164	102	196	163	126
N.S.	1	1.00	0.41	0.92	0.52	0.32	0.62	0.51	0.40
time (sec)	N/A	0.292	0.189	0.060	0.439	0.378	0.097	2.277	3.523

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	96	173	132	78	148	123	117
N.S.	1	1.00	0.52	0.94	0.72	0.42	0.80	0.67	0.64
time (sec)	N/A	0.173	0.152	0.065	0.373	0.353	0.079	2.963	3.430

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	41	77	103	57	104	87	66
N.S.	1	1.00	0.51	0.96	1.29	0.71	1.30	1.09	0.82
time (sec)	N/A	0.047	0.146	0.059	0.301	0.412	0.065	3.560	0.109

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	52	113	69	50	70	95	69
N.S.	1	1.00	0.51	1.11	0.68	0.49	0.69	0.93	0.68
time (sec)	N/A	0.121	0.151	0.074	0.340	0.371	4.831	3.715	3.558

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	54	92	61	56	99	92	72
N.S.	1	1.00	0.57	0.98	0.65	0.60	1.05	0.98	0.77
time (sec)	N/A	0.113	0.159	0.065	0.409	0.401	1.937	3.659	3.635

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	68	112	64	70	56	125	100
N.S.	1	1.00	0.52	0.86	0.49	0.54	0.43	0.96	0.77
time (sec)	N/A	0.153	0.147	0.071	0.497	0.446	1.750	3.792	3.636

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	81	167	63	83	53	183	142
N.S.	1	1.00	0.41	0.84	0.32	0.42	0.27	0.92	0.72
time (sec)	N/A	0.204	0.183	0.074	0.507	0.361	1.789	3.297	3.565

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	86	433	123	161	0	0	-1
N.S.	1	1.00	0.62	3.12	0.88	1.16	0.00	0.00	-0.01
time (sec)	N/A	0.211	0.237	0.101	0.183	0.106	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	159	250	330	229	323	9945	249
N.S.	1	1.00	0.38	0.60	0.80	0.55	0.78	24.02	0.60
time (sec)	N/A	0.476	0.267	0.061	0.417	0.377	0.109	3.935	3.646

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	121	197	264	179	260	7057	196
N.S.	1	1.00	0.37	0.60	0.80	0.55	0.79	21.52	0.60
time (sec)	N/A	0.372	0.237	0.058	0.415	0.549	0.101	3.863	3.559

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	91	144	198	133	199	4688	143
N.S.	1	1.00	0.38	0.60	0.82	0.55	0.82	19.37	0.59
time (sec)	N/A	0.247	0.204	0.059	0.433	0.390	0.080	2.191	3.519

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	58	93	135	86	134	2743	92
N.S.	1	1.00	0.68	1.09	1.59	1.01	1.58	32.27	1.08
time (sec)	N/A	0.082	0.167	0.054	0.440	0.373	0.067	3.038	3.443

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	54	118	87	75	0	0	80
N.S.	1	1.00	0.56	1.23	0.91	0.78	0.00	0.00	0.83
time (sec)	N/A	0.171	0.226	0.080	0.501	0.366	0.000	0.000	3.584

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	58	135	68	82	0	0	89
N.S.	1	1.00	0.68	1.59	0.80	0.96	0.00	0.00	1.05
time (sec)	N/A	0.192	0.225	0.084	0.350	0.368	0.000	0.000	3.588

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	76	204	74	88	0	0	133
N.S.	1	1.00	0.56	1.50	0.54	0.65	0.00	0.00	0.98
time (sec)	N/A	0.241	0.213	0.091	0.360	0.373	0.000	0.000	3.601

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	116	290	85	136	0	0	202
N.S.	1	1.00	0.53	1.34	0.39	0.63	0.00	0.00	0.93
time (sec)	N/A	0.314	0.251	0.096	0.453	0.347	0.000	0.000	3.660

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	156	382	93	185	0	0	258
N.S.	1	1.00	0.49	1.19	0.29	0.58	0.00	0.00	0.80
time (sec)	N/A	0.397	0.297	0.100	0.408	0.503	0.000	0.000	3.679

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	754	754	458	1240	894	544	1445	1096	803
N.S.	1	1.00	0.61	1.64	1.19	0.72	1.92	1.45	1.06
time (sec)	N/A	0.649	1.882	0.085	0.338	0.487	0.345	2.078	3.819

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	320	694	599	354	899	674	537
N.S.	1	1.00	0.65	1.40	1.21	0.72	1.82	1.36	1.08
time (sec)	N/A	0.451	1.979	0.075	0.348	0.362	0.213	1.844	3.663

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	191	322	344	197	447	331	264
N.S.	1	1.00	0.70	1.19	1.27	0.73	1.65	1.22	0.97
time (sec)	N/A	0.241	0.650	0.066	0.345	0.364	0.149	1.945	0.178

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	50	99	149	83	158	132	120
N.S.	1	1.00	0.49	0.97	1.46	0.81	1.55	1.29	1.18
time (sec)	N/A	0.069	0.231	0.056	0.299	0.392	0.076	3.313	0.110

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	175	489	0	235	0	546	-1
N.S.	1	1.00	0.63	1.77	0.00	0.85	0.00	1.97	-0.00
time (sec)	N/A	0.244	0.702	0.065	0.000	0.347	0.000	2.649	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	163	406	0	353	0	2861	-1
N.S.	1	1.00	0.63	1.57	0.00	1.37	0.00	11.09	-0.00
time (sec)	N/A	0.270	1.092	0.066	0.000	0.383	0.000	3.177	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	267	418	0	550	0	1995	-1
N.S.	1	1.00	0.91	1.42	0.00	1.87	0.00	6.79	-0.00
time (sec)	N/A	0.294	1.807	0.076	0.000	0.404	0.000	2.771	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	389	511	0	793	0	3178	-1
N.S.	1	1.00	0.98	1.29	0.00	2.00	0.00	8.03	-0.00
time (sec)	N/A	0.365	2.227	0.085	0.000	0.392	0.000	1.773	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	669	596	0	1084	0	16988	-1
N.S.	1	1.00	1.20	1.07	0.00	1.95	0.00	30.50	-0.00
time (sec)	N/A	0.484	2.283	0.073	0.000	0.377	0.000	2.854	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	192	44	34	0	0	25
N.S.	1	1.00	1.00	8.00	1.83	1.42	0.00	0.00	1.04
time (sec)	N/A	0.103	0.187	0.196	0.352	0.410	0.000	0.000	3.589

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	198	44	26	0	0	23
N.S.	1	1.00	1.05	9.00	2.00	1.18	0.00	0.00	1.05
time (sec)	N/A	0.093	0.111	0.122	0.365	0.440	0.000	0.000	3.499

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	198	44	26	0	0	23
N.S.	1	1.00	1.05	9.00	2.00	1.18	0.00	0.00	1.05
time (sec)	N/A	0.064	0.093	0.121	0.346	0.445	0.000	0.000	3.392

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	21	186	40	24	0	0	21
N.S.	1	1.00	1.05	9.30	2.00	1.20	0.00	0.00	1.05
time (sec)	N/A	0.034	0.088	0.133	0.358	0.498	0.000	0.000	3.479

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	180	38	23	0	0	20
N.S.	1	1.00	1.00	9.47	2.00	1.21	0.00	0.00	1.05
time (sec)	N/A	0.092	0.025	0.115	0.358	0.485	0.000	0.000	3.516

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	136	41	26	0	0	23
N.S.	1	1.00	1.05	6.18	1.86	1.18	0.00	0.00	1.05
time (sec)	N/A	0.097	0.113	0.130	0.348	0.457	0.000	0.000	3.571



Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	136	41	26	0	0	23
N.S.	1	1.00	1.05	6.18	1.86	1.18	0.00	0.00	1.05
time (sec)	N/A	0.095	0.114	0.128	0.357	0.386	0.000	0.000	3.583

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	45	43	60	43	51	43	45
N.S.	1	1.00	0.49	0.47	0.66	0.47	0.56	0.47	0.49
time (sec)	N/A	0.097	0.046	0.036	0.298	0.415	0.046	1.764	0.130

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	37	35	48	35	42	35	37
N.S.	1	1.00	0.51	0.49	0.67	0.49	0.58	0.49	0.51
time (sec)	N/A	0.073	0.040	0.012	0.275	0.493	0.043	1.860	0.055

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	29	27	36	27	34	27	29
N.S.	1	1.00	0.55	0.51	0.68	0.51	0.64	0.51	0.55
time (sec)	N/A	0.046	0.037	0.010	0.277	0.364	0.040	1.878	0.058

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	21	19	24	19	26	19	18
N.S.	1	1.00	0.62	0.56	0.71	0.56	0.76	0.56	0.53
time (sec)	N/A	0.019	0.027	0.010	0.272	0.342	0.037	1.992	0.045

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	14	14	14	14	14	13
N.S.	1	1.00	1.00	0.88	0.88	0.88	0.88	0.88	0.81
time (sec)	N/A	0.005	0.023	0.012	0.271	0.384	0.028	1.885	3.388

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	57	10	10	0	10	-1
N.S.	1	1.00	1.00	2.11	0.37	0.37	0.00	0.37	-0.04
time (sec)	N/A	0.038	0.031	0.015	0.315	0.380	0.000	2.428	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	47	116	13	29	0	29	-1
N.S.	1	1.00	0.98	2.42	0.27	0.60	0.00	0.60	-0.02
time (sec)	N/A	0.059	0.042	0.015	0.320	0.380	0.000	2.180	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	56	155	15	38	0	46	-1
N.S.	1	1.00	0.79	2.18	0.21	0.54	0.00	0.65	-0.01
time (sec)	N/A	0.082	0.056	0.018	0.309	0.362	0.000	2.926	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	64	189	15	46	0	63	-1
N.S.	1	1.00	0.70	2.05	0.16	0.50	0.00	0.68	-0.01
time (sec)	N/A	0.107	0.066	0.019	0.324	0.390	0.000	2.290	0.000

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [18] had the largest ratio of [61]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	17	0.059
2	A	5	2	1.00	17	0.118
3	A	4	2	1.00	17	0.118
4	A	3	2	1.00	17	0.118
5	A	2	2	1.00	15	0.133
6	A	1	1	1.00	9	0.111
7	A	1	1	1.00	17	0.059
8	A	2	2	1.00	17	0.118
9	A	3	2	1.00	17	0.118
10	A	4	2	1.00	17	0.118
11	A	5	2	1.00	17	0.118
12	A	6	3	1.00	48	0.062
13	A	5	3	1.00	37	0.081
14	A	4	3	1.00	26	0.115
15	A	3	3	1.00	28	0.107
16	A	4	3	1.00	39	0.077
17	A	5	3	1.00	50	0.060
18	A	6	3	1.00	61	0.049
19	A	2	2	1.00	19	0.105
20	A	2	2	1.00	50	0.040
21	A	2	2	1.00	39	0.051
22	A	2	2	1.00	28	0.071
23	A	1	1	1.00	17	0.059
24	A	1	1	1.00	19	0.053
25	A	2	2	1.00	30	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	2	2	1.00	41	0.049
27	A	1	1	1.00	7	0.143
28	A	1	1	1.00	7	0.143
29	A	1	1	1.00	7	0.143
30	A	6	3	1.00	13	0.231
31	A	5	3	1.00	13	0.231
32	A	4	3	1.00	13	0.231
33	A	3	3	1.00	13	0.231
34	A	2	2	1.00	13	0.154
35	A	3	3	1.00	13	0.231
36	A	4	3	1.00	13	0.231
37	A	5	3	1.00	13	0.231
38	A	6	3	1.00	13	0.231
39	A	6	3	1.00	19	0.158
40	A	5	3	1.00	19	0.158
41	A	4	3	1.00	19	0.158
42	A	3	3	1.00	19	0.158
43	A	2	2	1.00	19	0.105
44	A	3	3	1.00	19	0.158
45	A	4	3	1.00	19	0.158
46	A	5	3	1.00	19	0.158
47	A	6	3	1.00	19	0.158
48	A	9	3	1.00	12	0.250
49	A	1	1	1.00	19	0.053
50	A	2	2	1.00	21	0.095
51	A	2	2	1.00	15	0.133
52	A	8	3	1.00	20	0.150
53	A	12	3	1.00	25	0.120
54	A	17	3	1.00	30	0.100
55	A	6	2	1.00	21	0.095
56	A	24	3	1.00	21	0.143
57	A	20	3	1.00	21	0.143
58	A	11	3	1.00	19	0.158
59	A	4	2	1.00	18	0.111
60	A	9	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	8	5	1.00	21	0.238
62	A	9	4	1.00	21	0.190
63	A	12	3	1.00	21	0.143
64	A	5	2	1.00	22	0.091
65	A	17	3	1.00	22	0.136
66	A	14	3	1.00	22	0.136
67	A	11	3	1.00	20	0.150
68	A	4	3	1.00	19	0.158
69	A	6	4	1.00	22	0.182
70	A	6	4	1.00	22	0.182
71	A	8	3	1.00	22	0.136
72	A	11	3	1.00	22	0.136
73	A	14	3	1.00	22	0.136
74	A	28	3	1.00	25	0.120
75	A	20	3	1.00	25	0.120
76	A	13	3	1.00	23	0.130
77	A	5	2	1.00	18	0.111
78	A	13	4	1.00	25	0.160
79	A	11	5	1.00	25	0.200
80	A	11	5	1.00	25	0.200
81	A	13	4	1.00	25	0.160
82	A	17	3	1.00	25	0.120
83	A	1	1	1.00	39	0.026
84	A	1	1	1.00	38	0.026
85	A	1	1	1.00	36	0.028
86	A	1	1	1.00	35	0.029
87	A	1	1	1.00	35	0.029
88	A	1	1	1.00	38	0.026
89	A	1	1	1.00	38	0.026
90	A	5	2	1.00	15	0.133
91	A	4	2	1.00	15	0.133
92	A	3	2	1.00	15	0.133
93	A	2	2	1.00	13	0.154
94	A	1	1	1.00	11	0.091
95	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	3	1.00	15	0.200
97	A	4	3	1.00	15	0.200
98	A	5	3	1.00	15	0.200

# Chapter 3

## Listing of integrals

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3.19	$\int F^{c(a+bx)}((d+ex)^n)^m dx$	123
3.20	$\int F^{c(a+bx)}(d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4)^m dx$	126
3.21	$\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^m dx$	129
3.22	$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2)^m dx$	132
3.23	$\int F^{c(a+bx)}(d+ex)^m dx$	135
3.24	$\int F^{c(a+bx)}(d+ex)^{-m} dx$	138
3.25	$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2)^{-m} dx$	141

3.26	$\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^{-m} dx$	144
3.27	$\int F^{2+5x} dx$	147
3.28	$\int F^{a+bx} dx$	150
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3.30	$\int F^{a+bx} x^{7/2} dx$	156
3.31	$\int F^{a+bx} x^{5/2} dx$	160
3.32	$\int F^{a+bx} x^{3/2} dx$	164
3.33	$\int F^{a+bx} \sqrt{x} dx$	168
3.34	$\int \frac{F^{a+bx}}{\sqrt{x}} dx$	171
3.35	$\int \frac{F^{a+bx}}{x^{3/2}} dx$	174
3.36	$\int \frac{F^{a+bx}}{x^{5/2}} dx$	177
3.37	$\int \frac{F^{a+bx}}{x^{7/2}} dx$	180
3.38	$\int \frac{F^{a+bx}}{x^{9/2}} dx$	184
3.39	$\int F^{c(a+bx)}(d + ex)^{7/2} dx$	188
3.40	$\int F^{c(a+bx)}(d + ex)^{5/2} dx$	192
3.41	$\int F^{c(a+bx)}(d + ex)^{3/2} dx$	196
3.42	$\int F^{c(a+bx)} \sqrt{d + ex} dx$	200
3.43	$\int \frac{F^{c(a+bx)}}{\sqrt{d + ex}} dx$	204
3.44	$\int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx$	207
3.45	$\int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx$	211
3.46	$\int \frac{F^{c(a+bx)}}{(d+ex)^{7/2}} dx$	215
3.47	$\int \frac{F^{c(a+bx)}}{(d+ex)^{9/2}} dx$	219
3.48	$\int e^{-bx} x^{13/2} dx$	223
3.49	$\int F^{c(a+bx)}(d + ex)^{4/3} dx$	230
3.50	$\int (F^{c(a+bx)})^n (d + ex)^{4/3} dx$	233
3.51	$\int F^{c(a+bx)}(d + ex) dx$	236
3.52	$\int F^{c(a+bx)}(d + ex + fx^2) dx$	240
3.53	$\int F^{c(a+bx)}(d + ex + fx^2 + gx^3) dx$	245
3.54	$\int F^{c(a+bx)}(d + ex + fx^2 + gx^3 + hx^4) dx$	251
3.55	$\int e^{-a-bx} x^m (a + bx)^3 dx$	257
3.56	$\int e^{-a-bx} x^3 (a + bx)^3 dx$	261
3.57	$\int e^{-a-bx} x^2 (a + bx)^3 dx$	265
3.58	$\int e^{-a-bx} x (a + bx)^3 dx$	269
3.59	$\int e^{-a-bx} (a + bx)^3 dx$	273
3.60	$\int \frac{e^{-a-bx} (a+bx)^3}{x} dx$	276
3.61	$\int \frac{e^{-a-bx} (a+bx)^3}{x^2} dx$	280
3.62	$\int \frac{e^{-a-bx} (a+bx)^3}{x^3} dx$	284
3.63	$\int \frac{e^{-a-bx} (a+bx)^3}{x^4} dx$	288
3.64	$\int F^{a+b(c+dx)} x^m (e + fx)^2 dx$	292
3.65	$\int F^{a+b(c+dx)} x^3 (e + fx)^2 dx$	296



3.66	$\int F^{a+b(c+dx)} x^2 (e+fx)^2 dx$	302
3.67	$\int F^{a+b(c+dx)} x (e+fx)^2 dx$	308
3.68	$\int F^{a+b(c+dx)} (e+fx)^2 dx$	314
3.69	$\int \frac{F^{a+b(c+dx)} (e+fx)^2}{x} dx$	319
3.70	$\int \frac{F^{a+b(c+dx)} (e+fx)^2}{x^2} dx$	323
3.71	$\int \frac{F^{a+b(c+dx)} (e+fx)^2}{x^3} dx$	327
3.72	$\int \frac{F^{a+b(c+dx)} (e+fx)^2}{x^4} dx$	331
3.73	$\int \frac{F^{a+b(c+dx)} (e+fx)^2}{x^5} dx$	335
3.74	$\int e^{-a-bx} (a+bx)^4 (c+dx)^3 dx$	339
3.75	$\int e^{-a-bx} (a+bx)^4 (c+dx)^2 dx$	346
3.76	$\int e^{-a-bx} (a+bx)^4 (c+dx) dx$	351
3.77	$\int e^{-a-bx} (a+bx)^4 dx$	356
3.78	$\int \frac{e^{-a-bx} (a+bx)^4}{c+dx} dx$	360
3.79	$\int \frac{e^{-a-bx} (a+bx)^4}{(c+dx)^2} dx$	365
3.80	$\int \frac{e^{-a-bx} (a+bx)^4}{(c+dx)^3} dx$	371
3.81	$\int \frac{e^{-a-bx} (a+bx)^4}{(c+dx)^4} dx$	376
3.82	$\int \frac{e^{-a-bx} (a+bx)^4}{(c+dx)^5} dx$	383
3.83	$\int F^{c(a+bx)} x^m \log^n(dx) (e+en+e(1+m+bcx \log(F)) \log(dx)) dx$	390
3.84	$\int F^{c(a+bx)} x^2 \log^n(dx) (e+en+e(3+bcx \log(F)) \log(dx)) dx$	393
3.85	$\int F^{c(a+bx)} x \log^n(dx) (e+en+e(2+bcx \log(F)) \log(dx)) dx$	396
3.86	$\int F^{c(a+bx)} \log^n(dx) (e+en+e(1+bcx \log(F)) \log(dx)) dx$	399
3.87	$\int \frac{F^{c(a+bx)} \log^n(dx) (e+en+bcex \log(F) \log(dx))}{x} dx$	402
3.88	$\int \frac{F^{c(a+bx)} \log^n(dx) (e+en+e(-1+bcx \log(F)) \log(dx))}{x^2} dx$	405
3.89	$\int \frac{F^{c(a+bx)} \log^n(dx) (e+en+e(-2+bcx \log(F)) \log(dx))}{x^3} dx$	408
3.90	$\int \sqrt{e^{a+bx}} x^4 dx$	411
3.91	$\int \sqrt{e^{a+bx}} x^3 dx$	415
3.92	$\int \sqrt{e^{a+bx}} x^2 dx$	418
3.93	$\int \sqrt{e^{a+bx}} x dx$	421
3.94	$\int \sqrt{e^{a+bx}} dx$	424
3.95	$\int \frac{\sqrt{e^{a+bx}}}{x} dx$	427
3.96	$\int \frac{\sqrt{e^{a+bx}}}{x^2} dx$	430
3.97	$\int \frac{\sqrt{e^{a+bx}}}{x^3} dx$	433
3.98	$\int \frac{\sqrt{e^{a+bx}}}{x^4} dx$	437

### 3.1 $\int F^{c(a+bx)}(d+ex)^m dx$

Optimal. Leaf size=67

$$\frac{F^{c\left(a-\frac{bd}{e}\right)}(d+ex)^m \Gamma\left(1+m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-m}}{bc \log(F)}$$

[Out]  $F^{c*(a-b*d/e)}*(e*x+d)^m*\text{GAMMA}(1+m, -b*c*(e*x+d)*\ln(F)/e)/b/c/\ln(F)/((-b*c*(e*x+d)*\ln(F)/e)^m)$

Rubi [A]

time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {2212}

$$\frac{(d+ex)^m F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-m} \text{Gamma}\left(m+1, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c*(a+b*x)}*(d+e*x)^m, x]$

[Out]  $(F^{c*(a-(b*d)/e)}*(d+e*x)^m*\text{Gamma}[1+m, -((b*c*(d+e*x)*\text{Log}[F])/e)])/(b*c*\text{Log}[F]*(-((b*c*(d+e*x)*\text{Log}[F])/e))^m)$

Rule 2212

```
Int[(F_)^((g_.)*((e_.)+(f_.)*(x_)))*((c_.)+(d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e-c*(f/d))))*((c+d*x)^FracPart[m]/(d*(-f)*g*(Log[F]/d))
)^(IntPart[m]+1)*((-f)*g*Log[F]*((c+d*x)/d)^FracPart[m])*Gamma[m+1,
((-f)*g*(Log[F]/d))*(c+d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rubi steps

$$\int F^{c(a+bx)}(d+ex)^m dx = \frac{F^{c\left(a-\frac{bd}{e}\right)}(d+ex)^m \Gamma\left(1+m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-m}}{bc \log(F)}$$

Mathematica [A]

time = 0.04, size = 67, normalized size = 1.00

$$\frac{F^{c\left(a-\frac{bd}{e}\right)}(d+ex)^m \Gamma\left(1+m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-m}}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d + e\*x)^m,x]

[Out] (F^(c\*(a - (b\*d)/e))\*(d + e\*x)^m\*Gamma[1 + m, -((b\*c\*(d + e\*x)\*Log[F])/e)]) / (b\*c\*Log[F]\*(-(b\*c\*(d + e\*x)\*Log[F])/e))^m

**Maple** [F]

time = 0.02, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}(ex + d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*(e\*x+d)^m,x)

[Out] int(F^(c\*(b\*x+a))\*(e\*x+d)^m,x)

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^m,x, algorithm="maxima")

[Out] integrate((x\*e + d)^m\*F^((b\*x + a)\*c), x)

**Fricas** [A]

time = 0.10, size = 65, normalized size = 0.97

$$\frac{e^{(-(me \log(-bce^{(-1)} \log(F)) + (bcd - ace) \log(F))e^{(-1)})} \Gamma(m + 1, -(bcxe + bcd)e^{(-1)} \log(F))}{bc \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^m,x, algorithm="fricas")

[Out] e^(-(m\*e\*log(-b\*c\*e^(-1)\*log(F)) + (b\*c\*d - a\*c\*e)\*log(F))\*e^(-1))\*gamma(m + 1, -(b\*c\*x\*e + b\*c\*d)\*e^(-1)\*log(F))/(b\*c\*log(F))

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}(d + ex)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(e\*x+d)\*\*m,x)

[Out] Integral(F\*\*(c\*(a + b\*x))\*(d + e\*x)\*\*m, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^m,x, algorithm="giac")

[Out] integrate((x\*e + d)^m\*F^((b\*x + a)\*c), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int F^{c(a+bx)} (d+ex)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))\*(d + e\*x)^m,x)

[Out] int(F^(c\*(a + b\*x))\*(d + e\*x)^m, x)

### 3.2 $\int F^{c(a+bx)}(d+ex)^4 dx$

**Optimal.** Leaf size=141

$$\frac{24e^4 F^{c(a+bx)}}{b^5 c^5 \log^5(F)} - \frac{24e^3 F^{c(a+bx)}(d+ex)}{b^4 c^4 \log^4(F)} + \frac{12e^2 F^{c(a+bx)}(d+ex)^2}{b^3 c^3 \log^3(F)} - \frac{4e F^{c(a+bx)}(d+ex)^3}{b^2 c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^4}{bc \log(F)}$$

[Out]  $24e^4 F^{c(bx+a)}/b^5/c^5/\ln(F)^5 - 24e^3 F^{c(bx+a)}(ex+d)/b^4/c^4/\ln(F)^4 + 12e^2 F^{c(bx+a)}(ex+d)^2/b^3/c^3/\ln(F)^3 - 4e F^{c(bx+a)}(ex+d)^3/b^2/c^2/\ln(F)^2 + F^{c(bx+a)}(ex+d)^4/b/c/\ln(F)$

**Rubi** [A]

time = 0.08, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2207, 2225}

$$\frac{24e^4 F^{c(a+bx)}}{b^5 c^5 \log^5(F)} - \frac{24e^3(d+ex)F^{c(a+bx)}}{b^4 c^4 \log^4(F)} + \frac{12e^2(d+ex)^2 F^{c(a+bx)}}{b^3 c^3 \log^3(F)} - \frac{4e(d+ex)^3 F^{c(a+bx)}}{b^2 c^2 \log^2(F)} + \frac{(d+ex)^4 F^{c(a+bx)}}{bc \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c(a+bx)}(d+ex)^4, x]$

[Out]  $(24e^4 F^{c(a+bx)})/(b^5 c^5 \text{Log}[F]^5) - (24e^3 F^{c(a+bx)}(d+ex))/(b^4 c^4 \text{Log}[F]^4) + (12e^2 F^{c(a+bx)}(d+ex)^2)/(b^3 c^3 \text{Log}[F]^3) - (4e F^{c(a+bx)}(d+ex)^3)/(b^2 c^2 \text{Log}[F]^2) + (F^{c(a+bx)}(d+ex)^4)/(b c \text{Log}[F])$

Rule 2207

$\text{Int}[(b_.)(F_)^{((g_.)((e_.) + (f_.)(x_)))}^{(n_.)((c_.) + (d_.)(x_))}^{(m_.)}, x\_Symbol] :> \text{Simp}[(c + dx)^m ((b F^{(g(e + fx))})^n / (f g^n \text{Log}[F])), x] - \text{Dist}[d(m/(f g^n \text{Log}[F])), \text{Int}[(c + dx)^{(m-1)} (b F^{(g(e + fx))})^n, x], x] /; \text{FreeQ}\{F, b, c, d, e, f, g, n\}, x\} \&\& \text{GtQ}[m, 0] \&\& \text{IntegerQ}[2m] \&\& !\text{TrueQ}[\$UseGamma]$

Rule 2225

$\text{Int}[(F_)^{((c_.)((a_.) + (b_.)(x_)))}^{(n_.)}, x\_Symbol] :> \text{Simp}[F^{c(a+bx)}]^{(n)} / (b c^n \text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x\}$

Rubi steps

$$\begin{aligned}
\int F^{c(a+bx)}(d+ex)^4 dx &= \frac{F^{c(a+bx)}(d+ex)^4}{bc \log(F)} - \frac{(4e) \int F^{c(a+bx)}(d+ex)^3 dx}{bc \log(F)} \\
&= -\frac{4eF^{c(a+bx)}(d+ex)^3}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^4}{bc \log(F)} + \frac{(12e^2) \int F^{c(a+bx)}(d+ex)^2 dx}{b^2c^2 \log^2(F)} \\
&= \frac{12e^2 F^{c(a+bx)}(d+ex)^2}{b^3c^3 \log^3(F)} - \frac{4eF^{c(a+bx)}(d+ex)^3}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^4}{bc \log(F)} - \frac{(24e^3) \int F^{c(a+bx)}(d+ex) dx}{b^3c^3 \log^3(F)} \\
&= -\frac{24e^3 F^{c(a+bx)}(d+ex)}{b^4c^4 \log^4(F)} + \frac{12e^2 F^{c(a+bx)}(d+ex)^2}{b^3c^3 \log^3(F)} - \frac{4eF^{c(a+bx)}(d+ex)^3}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^4}{bc \log(F)} \\
&= \frac{24e^4 F^{c(a+bx)}}{b^5c^5 \log^5(F)} - \frac{24e^3 F^{c(a+bx)}(d+ex)}{b^4c^4 \log^4(F)} + \frac{12e^2 F^{c(a+bx)}(d+ex)^2}{b^3c^3 \log^3(F)} - \frac{4eF^{c(a+bx)}(d+ex)^3}{b^2c^2 \log^2(F)}
\end{aligned}$$

**Mathematica [A]**

time = 0.09, size = 100, normalized size = 0.71

$$\frac{F^{c(a+bx)}(24e^4 - 24bce^3(d+ex) \log(F) + 12b^2c^2e^2(d+ex)^2 \log^2(F) - 4b^3c^3e(d+ex)^3 \log^3(F) + b^4c^4(d+ex)^4 \log^4(F))}{b^5c^5 \log^5(F)}$$

Antiderivative was successfully verified.

**[In]** Integrate[F^(c\*(a + b\*x))\*(d + e\*x)^4,x]

**[Out]** (F^(c\*(a + b\*x))\*(24\*e^4 - 24\*b\*c\*e^3\*(d + e\*x)\*Log[F] + 12\*b^2\*c^2\*e^2\*(d + e\*x)^2\*Log[F]^2 - 4\*b^3\*c^3\*e\*(d + e\*x)^3\*Log[F]^3 + b^4\*c^4\*(d + e\*x)^4\*Log[F]^4))/(b^5\*c^5\*Log[F]^5)

**Maple [A]**

time = 0.10, size = 260, normalized size = 1.84

method	result
gospers	$\frac{(e^4 x^4 c^4 b^4 \ln(F)^4 + 4 \ln(F)^4 b^4 c^4 d e^3 x^3 + 6 \ln(F)^4 b^4 c^4 d^2 e^2 x^2 + 4 \ln(F)^4 b^4 c^4 d^3 e x + \ln(F)^4 b^4 c^4 d^4 - 4 \ln(F)^3 b^3 c^3 e^4 x^3 - 12 \ln(F)^3 b^3 c^3 d e^3 x^2 + 4 \ln(F)^3 b^3 c^3 d^2 e^2 x + 4 \ln(F)^3 b^3 c^3 d^3 e - 4 \ln(F)^3 b^3 c^3 d^4 - 4 \ln(F)^2 b^2 c^2 e^3 x^3 + 12 \ln(F)^2 b^2 c^2 d e^2 x^2 - 24 \ln(F)^2 b^2 c^2 d^2 e x + 24 \ln(F)^2 b^2 c^2 d^3 - 24 \ln(F) b c^3 e^4 x^3 + 12 \ln(F) b c^3 d e^3 x^2 - 12 \ln(F) b c^3 d^2 e^2 x + 12 \ln(F) b c^3 d^3 e - 12 \ln(F) b c^3 d^4 + 120 b^4 c^4 e^4 x^4 - 120 b^4 c^4 d e^3 x^3 + 60 b^4 c^4 d^2 e^2 x^2 - 120 b^4 c^4 d^3 e x + 120 b^4 c^4 d^4) e^{c(bx+a) \ln(F)}}{c^5 b^5 \ln(F)^5} + \frac{e^4 x^4 e^{c(bx+a) \ln(F)}}{cb \ln(F)} + \frac{4e(c^3 b^3 \ln(F)^3)}{c^4 b^4}$
risch	$\frac{(e^4 x^4 c^4 b^4 \ln(F)^4 + 4 \ln(F)^4 b^4 c^4 d e^3 x^3 + 6 \ln(F)^4 b^4 c^4 d^2 e^2 x^2 + 4 \ln(F)^4 b^4 c^4 d^3 e x + \ln(F)^4 b^4 c^4 d^4 - 4 \ln(F)^3 b^3 c^3 e^4 x^3 - 12 \ln(F)^3 b^3 c^3 d e^3 x^2 + 4 \ln(F)^3 b^3 c^3 d^2 e^2 x + 4 \ln(F)^3 b^3 c^3 d^3 e - 4 \ln(F)^3 b^3 c^3 d^4 - 4 \ln(F)^2 b^2 c^2 e^3 x^3 + 12 \ln(F)^2 b^2 c^2 d e^2 x^2 - 24 \ln(F)^2 b^2 c^2 d^2 e x + 24 \ln(F)^2 b^2 c^2 d^3 - 24 \ln(F) b c^3 e^4 x^3 + 12 \ln(F) b c^3 d e^3 x^2 - 12 \ln(F) b c^3 d^2 e^2 x + 12 \ln(F) b c^3 d^3 e - 12 \ln(F) b c^3 d^4 + 120 b^4 c^4 e^4 x^4 - 120 b^4 c^4 d e^3 x^3 + 60 b^4 c^4 d^2 e^2 x^2 - 120 b^4 c^4 d^3 e x + 120 b^4 c^4 d^4) e^{bcx \ln(F)}}{c^5 b^5 \ln(F)^5} + \frac{4F^{ca} e^3 d}{c^4 b^4} \left( 6 - \frac{(-4b^3 c^3 x^3 \ln(F)^3 + 12b^2 c^3 d e^3 x^2 - 12 \ln(F) b c^3 d^2 e^2 x + 12 \ln(F) b c^3 d^3 e - 12 \ln(F) b c^3 d^4 + 120 b^4 c^4 e^4 x^4 - 120 b^4 c^4 d e^3 x^3 + 60 b^4 c^4 d^2 e^2 x^2 - 120 b^4 c^4 d^3 e x + 120 b^4 c^4 d^4)}{c^5 b^5 \ln(F)^5} \right)$
norman	$\frac{(\ln(F)^4 b^4 c^4 d^4 - 4 \ln(F)^3 b^3 c^3 d^3 e + 12 c^2 b^2 \ln(F)^2 d^2 e^2 - 24 d e^3 c b \ln(F) + 24 e^4) e^{c(bx+a) \ln(F)}}{c^5 b^5 \ln(F)^5} + \frac{e^4 x^4 e^{c(bx+a) \ln(F)}}{cb \ln(F)} + \frac{4e(c^3 b^3 \ln(F)^3)}{c^4 b^4}$
meijerg	$-\frac{F^{ca} e^4 \left( 24 - \frac{(5b^4 c^4 x^4 \ln(F)^4 - 20b^3 c^3 x^3 \ln(F)^3 + 60b^2 c^2 x^2 \ln(F)^2 - 120bcx \ln(F) + 120) e^{bcx \ln(F)}}{5} \right)}{c^5 b^5 \ln(F)^5} + \frac{4F^{ca} e^3 d}{c^4 b^4} \left( 6 - \frac{(-4b^3 c^3 x^3 \ln(F)^3 + 12b^2 c^3 d e^3 x^2 - 12 \ln(F) b c^3 d^2 e^2 x + 12 \ln(F) b c^3 d^3 e - 12 \ln(F) b c^3 d^4 + 120 b^4 c^4 e^4 x^4 - 120 b^4 c^4 d e^3 x^3 + 60 b^4 c^4 d^2 e^2 x^2 - 120 b^4 c^4 d^3 e x + 120 b^4 c^4 d^4)}{c^5 b^5 \ln(F)^5} \right)$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(F^(c\*(b\*x+a))\*(e\*x+d)^4,x,method=\_RETURNVERBOSE)

[Out]  $(e^{4*x^4*c^4*b^4*\ln(F)^4+4*\ln(F)^4*b^4*c^4*d*e^3*x^3+6*\ln(F)^4*b^4*c^4*d^2*e^2*x^2+4*\ln(F)^4*b^4*c^4*d^3*e*x+\ln(F)^4*b^4*c^4*d^4-4*\ln(F)^3*b^3*c^3*e^4*x^3-12*\ln(F)^3*b^3*c^3*d*e^3*x^2-12*\ln(F)^3*b^3*c^3*d^2*e^2*x-4*\ln(F)^3*b^3*c^3*d^3*e+12*\ln(F)^2*b^2*c^2*e^4*x^2+24*\ln(F)^2*b^2*c^2*d*e^3*x+12*c^2*b^2*2*\ln(F)^2*d^2*e^2-24*\ln(F)*b*c*e^4*x-24*d*e^3*c*b*\ln(F)+24*e^4)*F^{(c*(b*x+a))}/c^5/b^5/\ln(F)^5$

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs.  $2(143) = 286$ .

time = 0.29, size = 311, normalized size = 2.21

$$\frac{F^{bc+4d} - 4(F^{bc}\log(F) - F^{bc})d^{bc}\log(F^{b+1})}{bc\log(F)^5} + \frac{6(F^{bc}d^2\log(F)^2 - 2F^{bc}\log(F) + 2F^{bc})d^{bc}\log(F^{b+1})}{b^2c^2\log(F)^5} + \frac{4(F^{bc}d^3\log(F)^3 - 3F^{bc}d^2\log(F)^2 + 6F^{bc}\log(F) - 6F^{bc})d^{bc}\log(F^{b+1})}{b^3c^3\log(F)^5} + \frac{(F^{bc}d^4\log(F)^4 - 4F^{bc}d^3\log(F)^3 + 12F^{bc}d^2\log(F)^2 - 24F^{bc}\log(F) + 24F^{bc})d^{bc}\log(F^{b+1})}{b^4c^4\log(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e*x+d)^4,x, algorithm="maxima")`

[Out]  $F^{(b*c*x + a*c)}*d^4/(b*c*\log(F)) + 4*(F^{(a*c)}*b*c*x*\log(F) - F^{(a*c)})*d^3*e^{(b*c*x*\log(F) + 1)}/(b^2*c^2*\log(F)^2) + 6*(F^{(a*c)}*b^2*c^2*x^2*\log(F)^2 - 2*F^{(a*c)}*b*c*x*\log(F) + 2*F^{(a*c)})*d^2*e^{(b*c*x*\log(F) + 2)}/(b^3*c^3*\log(F)^3) + 4*(F^{(a*c)}*b^3*c^3*x^3*\log(F)^3 - 3*F^{(a*c)}*b^2*c^2*x^2*\log(F)^2 + 6*F^{(a*c)}*b*c*x*\log(F) - 6*F^{(a*c)})*d*e^{(b*c*x*\log(F) + 3)}/(b^4*c^4*\log(F)^4) + (F^{(a*c)}*b^4*c^4*x^4*\log(F)^4 - 4*F^{(a*c)}*b^3*c^3*x^3*\log(F)^3 + 12*F^{(a*c)}*b^2*c^2*x^2*\log(F)^2 - 24*F^{(a*c)}*b*c*x*\log(F) + 24*F^{(a*c)})*e^{(b*c*x*\log(F) + 4)}/(b^5*c^5*\log(F)^5)$

**Fricas** [A]

time = 0.39, size = 217, normalized size = 1.54

$$\frac{((b^4c^4x^4e^4 + 4b^4c^4d^2x^2e^2 + 6b^4c^4d^2x^2e^2 + 4b^4c^4d^3xe + b^4c^4d^4)\log(F)^4 - 4(b^2c^2x^3e^4 + 3b^2c^2dx^2e^3 + 3b^2c^2d^2xe^2 + b^2c^2d^3e)\log(F)^3 + 12(b^2c^2x^2e^4 + 2b^2c^2dxe^3 + b^2c^2d^2e^2)\log(F)^2 - 24(bcxe^4 + bde^3)\log(F) + 24e^4)F^{bc+4d}}{b^5c^5\log(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e*x+d)^4,x, algorithm="fricas")`

[Out]  $((b^4*c^4*4*x^4*e^4 + 4*b^4*c^4*d*x^3*e^3 + 6*b^4*c^4*d^2*x^2*e^2 + 4*b^4*c^4*d^3*x*e + b^4*c^4*d^4)*\log(F)^4 - 4*(b^3*c^3*x^3*e^4 + 3*b^3*c^3*d*x^2*e^3 + 3*b^3*c^3*d^2*x*e^2 + b^3*c^3*d^3*e)*\log(F)^3 + 12*(b^2*c^2*x^2*e^4 + 2*b^2*c^2*d*x*e^3 + b^2*c^2*d^2*e^2)*\log(F)^2 - 24*(b*c*x*e^4 + b*c*d*e^3)*\log(F) + 24*e^4)*F^{(b*c*x + a*c)}/(b^5*c^5*\log(F)^5)$

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs.  $2(139) = 278$ .

time = 0.11, size = 350, normalized size = 2.48

$$\begin{cases} \frac{F^{c(bx+a)}(b^4c^4d^4\log(F)^4 + 4b^4c^4d^3c\log(F)^3 + 4b^4c^4d^2c^2\log(F)^2 + 4b^4c^4d^2c^2\log(F)^2 + b^4c^4c^2\log(F)^2 - 4b^4c^4d^3c\log(F)^3 - 12b^4c^4d^2c^2\log(F)^2 - 12b^4c^4d^2c^2\log(F)^2 - 4b^4c^4c^2\log(F)^2 + 24b^4c^4d^2c\log(F)^2 + 24b^4c^4d^2c\log(F)^2 + 24b^4c^4d^2c\log(F)^2 - 24bc^4a\log(F) + 24c^4)}{b^5c^5\log(F)^5} & \text{for } b^5c^5\log(F)^5 \neq 0 \\ d^4x + 2d^3cx^2 + 2d^2c^2x^3 + dc^3x^4 + \frac{c^4}{c^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(e*x+d)**4,x)
```

```
[Out] Piecewise((F**(c*(a + b*x))*(b**4*c**4*d**4*log(F)**4 + 4*b**4*c**4*d**3*e*
x*log(F)**4 + 6*b**4*c**4*d**2*e**2*x**2*log(F)**4 + 4*b**4*c**4*d*e**3*x**
3*log(F)**4 + b**4*c**4*e**4*x**4*log(F)**4 - 4*b**3*c**3*d**3*e*log(F)**3
- 12*b**3*c**3*d**2*e**2*x*log(F)**3 - 12*b**3*c**3*d*e**3*x**2*log(F)**3 -
4*b**3*c**3*e**4*x**3*log(F)**3 + 12*b**2*c**2*d**2*e**2*log(F)**2 + 24*b*
**2*c**2*d*e**3*x*log(F)**2 + 12*b**2*c**2*e**4*x**2*log(F)**2 - 24*b*c*d*e*
**3*log(F) - 24*b*c*e**4*x*log(F) + 24*e**4)/(b**5*c**5*log(F)**5), Ne(b**5*
c**5*log(F)**5, 0)), (d**4*x + 2*d**3*e*x**2 + 2*d**2*e**2*x**3 + d*e**3*x*
**4 + e**4*x**5/5, True))
```

**Giac [C]** Result contains complex when optimal does not.

time = 2.68, size = 7859, normalized size = 55.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -(((pi^3*b^4*c^4*x^4*log(abs(F))*sgn(F) - pi*b^4*c^4*x^4*log(abs(F)))^3*sg
n(F) - pi^3*b^4*c^4*x^4*log(abs(F)) + pi*b^4*c^4*x^4*log(abs(F))^3 - pi^3*b
^3*c^3*x^3*sgn(F) + 3*pi*b^3*c^3*x^3*log(abs(F))^2*sgn(F) + pi^3*b^3*c^3*x^
3 - 3*pi*b^3*c^3*x^3*log(abs(F))^2 - 6*pi*b^2*c^2*x^2*log(abs(F))*sgn(F) +
6*pi*b^2*c^2*x^2*log(abs(F)) + 6*pi*b*c*x*sgn(F) - 6*pi*b*c*x*(pi^5*b^5*c^
5*sgn(F) - 10*pi^3*b^5*c^5*log(abs(F))^2*sgn(F) + 5*pi*b^5*c^5*log(abs(F))^
4*sgn(F) - pi^5*b^5*c^5 + 10*pi^3*b^5*c^5*log(abs(F))^2 - 5*pi*b^5*c^5*log(
abs(F))^4)/((pi^5*b^5*c^5*sgn(F) - 10*pi^3*b^5*c^5*log(abs(F))^2*sgn(F) + 5
*pi*b^5*c^5*log(abs(F))^4*sgn(F) - pi^5*b^5*c^5 + 10*pi^3*b^5*c^5*log(abs(F
))^2 - 5*pi*b^5*c^5*log(abs(F))^4)^2 + (5*pi^4*b^5*c^5*log(abs(F))*sgn(F) -
10*pi^2*b^5*c^5*log(abs(F))^3*sgn(F) - 5*pi^4*b^5*c^5*log(abs(F)) + 10*pi^
2*b^5*c^5*log(abs(F))^3 - 2*b^5*c^5*log(abs(F))^5)^2) - (pi^4*b^4*c^4*x^4*sg
n(F) - 6*pi^2*b^4*c^4*x^4*log(abs(F))^2*sgn(F) - pi^4*b^4*c^4*x^4 + 6*pi^2
*b^4*c^4*x^4*log(abs(F))^2 - 2*b^4*c^4*x^4*log(abs(F))^4 + 12*pi^2*b^3*c^3*
x^3*log(abs(F))*sgn(F) - 12*pi^2*b^3*c^3*x^3*log(abs(F)) + 8*b^3*c^3*x^3*lo
g(abs(F))^3 - 12*pi^2*b^2*c^2*x^2*sgn(F) + 12*pi^2*b^2*c^2*x^2 - 24*b^2*c^2
*x^2*log(abs(F))^2 + 48*b*c*x*log(abs(F)) - 48)*(5*pi^4*b^5*c^5*log(abs(F))
*sgn(F) - 10*pi^2*b^5*c^5*log(abs(F))^3*sgn(F) - 5*pi^4*b^5*c^5*log(abs(F))
+ 10*pi^2*b^5*c^5*log(abs(F))^3 - 2*b^5*c^5*log(abs(F))^5)/((pi^5*b^5*c^5*
sgn(F) - 10*pi^3*b^5*c^5*log(abs(F))^2*sgn(F) + 5*pi*b^5*c^5*log(abs(F))^4*
sgn(F) - pi^5*b^5*c^5 + 10*pi^3*b^5*c^5*log(abs(F))^2 - 5*pi*b^5*c^5*log(ab
s(F))^4)^2 + (5*pi^4*b^5*c^5*log(abs(F))*sgn(F) - 10*pi^2*b^5*c^5*log(abs(F
))^3*sgn(F) - 5*pi^4*b^5*c^5*log(abs(F)) + 10*pi^2*b^5*c^5*log(abs(F))^3 -
2*b^5*c^5*log(abs(F))^5)^2))*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*
pi*a*c*sgn(F) + 1/2*pi*a*c) - ((pi^4*b^4*c^4*x^4*sgn(F) - 6*pi^2*b^4*c^4*x^
4*log(abs(F))^2*sgn(F) - pi^4*b^4*c^4*x^4 + 6*pi^2*b^4*c^4*x^4*log(abs(F))^
```



$$\begin{aligned}
& 2 - 2*b^4*c^4*x^4*\log(\text{abs}(F))^4 + 12*\pi^2*b^3*c^3*x^3*\log(\text{abs}(F))*\text{sgn}(F) - \\
& 12*\pi^2*b^3*c^3*x^3*\log(\text{abs}(F)) + 8*b^3*c^3*x^3*\log(\text{abs}(F))^3 - 12*\pi^2*b^2 \\
& *c^2*x^2*\text{sgn}(F) + 12*\pi^2*b^2*c^2*x^2 - 24*b^2*c^2*x^2*\log(\text{abs}(F))^2 + 48*b \\
& *c*x*\log(\text{abs}(F)) - 48*(\pi^5*b^5*c^5*\text{sgn}(F) - 10*\pi^3*b^5*c^5*\log(\text{abs}(F))^2 \\
& *\text{sgn}(F) + 5*\pi*b^5*c^5*\log(\text{abs}(F))^4*\text{sgn}(F) - \pi^5*b^5*c^5 + 10*\pi^3*b^5*c^ \\
& 5*\log(\text{abs}(F))^2 - 5*\pi*b^5*c^5*\log(\text{abs}(F))^4)/((\pi^5*b^5*c^5*\text{sgn}(F) - 10*\pi \\
& ^3*b^5*c^5*\log(\text{abs}(F))^2*\text{sgn}(F) + 5*\pi*b^5*c^5*\log(\text{abs}(F))^4*\text{sgn}(F) - \pi^5* \\
& b^5*c^5 + 10*\pi^3*b^5*c^5*\log(\text{abs}(F))^2 - 5*\pi*b^5*c^5*\log(\text{abs}(F))^4)^2 + ( \\
& 5*\pi^4*b^5*c^5*\log(\text{abs}(F))*\text{sgn}(F) - 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3*\text{sgn}(F) - \\
& 5*\pi^4*b^5*c^5*\log(\text{abs}(F)) + 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3 - 2*b^5*c^5*\log( \\
& \text{abs}(F))^5)^2) + 4*(\pi^3*b^4*c^4*x^4*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^4*c^4*x^4*\log \\
& (\text{abs}(F))^3*\text{sgn}(F) - \pi^3*b^4*c^4*x^4*\log(\text{abs}(F)) + \pi*b^4*c^4*x^4*\log(\text{abs}(F) \\
& ))^3 - \pi^3*b^3*c^3*x^3*\text{sgn}(F) + 3*\pi*b^3*c^3*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) + \pi \\
& ^3*b^3*c^3*x^3 - 3*\pi*b^3*c^3*x^3*\log(\text{abs}(F))^2 - 6*\pi*b^2*c^2*x^2*\log(\text{abs}( \\
& F))*\text{sgn}(F) + 6*\pi*b^2*c^2*x^2*\log(\text{abs}(F)) + 6*\pi*b*c*x*\text{sgn}(F) - 6*\pi*b*c*x) \\
& *(5*\pi^4*b^5*c^5*\log(\text{abs}(F))*\text{sgn}(F) - 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3*\text{sgn}(F) \\
& - 5*\pi^4*b^5*c^5*\log(\text{abs}(F)) + 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3 - 2*b^5*c^5*\log \\
& (\text{abs}(F))^5)/((\pi^5*b^5*c^5*\text{sgn}(F) - 10*\pi^3*b^5*c^5*\log(\text{abs}(F))^2*\text{sgn}(F) + \\
& 5*\pi*b^5*c^5*\log(\text{abs}(F))^4*\text{sgn}(F) - \pi^5*b^5*c^5 + 10*\pi^3*b^5*c^5*\log(\text{abs} \\
& (F))^2 - 5*\pi*b^5*c^5*\log(\text{abs}(F))^4)^2 + (5*\pi^4*b^5*c^5*\log(\text{abs}(F))*\text{sgn}(F) \\
& - 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3*\text{sgn}(F) - 5*\pi^4*b^5*c^5*\log(\text{abs}(F)) + 10*\pi \\
& i^2*b^5*c^5*\log(\text{abs}(F))^3 - 2*b^5*c^5*\log(\text{abs}(F))^5)^2))*\sin(-1/2*\pi*b*c*x* \\
& \text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c))*e^{(b*c*x*\log(\text{abs}(F) \\
& )) + a*c*\log(\text{abs}(F)) + 4) - 8*I*((I*\pi^4*b^4*c^4*x^4*\text{sgn}(F) - 4*\pi^3*b^4*c^ \\
& 4*x^4*\log(\text{abs}(F))*\text{sgn}(F) - 6*I*\pi^2*b^4*c^4*x^4*\log(\text{abs}(F))^2*\text{sgn}(F) + 4*\pi \\
& *b^4*c^4*x^4*\log(\text{abs}(F))^3*\text{sgn}(F) - I*\pi^4*b^4*c^4*x^4 + 4*\pi^3*b^4*c^4*x^4 \\
& *\log(\text{abs}(F)) + 6*I*\pi^2*b^4*c^4*x^4*\log(\text{abs}(F))^2 - 4*\pi*b^4*c^4*x^4*\log(\text{ab} \\
& s(F))^3 - 2*I*b^4*c^4*x^4*\log(\text{abs}(F))^4 + 4*\pi^3*b^3*c^3*x^3*\text{sgn}(F) + 12*I* \\
& \pi^2*b^3*c^3*x^3*\log(\text{abs}(F))*\text{sgn}(F) - 12*\pi*b^3*c^3*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) \\
& ) - 4*\pi^3*b^3*c^3*x^3 - 12*I*\pi^2*b^3*c^3*x^3*\log(\text{abs}(F)) + 12*\pi*b^3*c^3* \\
& x^3*\log(\text{abs}(F))^2 + 8*I*b^3*c^3*x^3*\log(\text{abs}(F))^3 - 12*I*\pi^2*b^2*c^2*x^2*s \\
& \text{gn}(F) + 24*\pi*b^2*c^2*x^2*\log(\text{abs}(F))*\text{sgn}(F) + 12*I*\pi^2*b^2*c^2*x^2 - 24*\pi \\
& i*b^2*c^2*x^2*\log(\text{abs}(F)) - 24*I*b^2*c^2*x^2*\log(\text{abs}(F))^2 - 24*\pi*b*c*x*\text{sg} \\
& n(F) + 24*\pi*b*c*x + 48*I*b*c*x*\log(\text{abs}(F)) - 48*I)*e^{(1/2*I*\pi*b*c*x*\text{sgn}(F) \\
& ) - 1/2*I*\pi*b*c*x + 1/2*I*\pi*a*c*\text{sgn}(F) - 1/2*I*\pi*a*c)/(16*I*\pi^5*b^5*c^5 \\
& *\text{sgn}(F) - 80*\pi^4*b^5*c^5*\log(\text{abs}(F))*\text{sgn}(F) - 160*I*\pi^3*b^5*c^5*\log(\text{abs}(F) \\
& ))^2*\text{sgn}(F) + 160*\pi^2*b^5*c^5*\log(\text{abs}(F))^3*\text{sgn}(F) + 80*I*\pi*b^5*c^5*\log(\text{a} \\
& bs(F))^4*\text{sgn}(F) - 16*I*\pi^5*b^5*c^5 + 80*\pi^4*b^5*c^5*\log(\text{abs}(F)) + 160*I*\pi \\
& i^3*b^5*c^5*\log(\text{abs}(F))^2 - 160*\pi^2*b^5*c^5*\log(\text{abs}(F))^3 - 80*I*\pi*b^5*c^ \\
& 5*\log(\text{abs}(F))^4 + 32*b^5*c^5*\log(\text{abs}(F))^5) - (I*\pi^4*b^4*c^4*x^4*\text{sgn}(F) + \\
& 4*\pi^3*b^4*c^4*x^4*\log(\text{abs}(F))*\text{sgn}(F) - 6*I*\pi^...
\end{aligned}$$

**Mupad [B]**

time = 3.68, size = 260, normalized size = 1.84

$$\frac{P^{(4,4)}(b^4 c^4 \ln(F)^4 + 4b^4 c^4 d^2 \ln(F)^3 + 6b^4 c^4 d^2 x^2 \ln(F)^3 + 4b^4 c^4 d^2 x^2 \ln(F)^3 + b^4 c^4 e^2 \ln(F)^3 - 4b^4 c^4 d^2 \ln(F)^2 - 12b^4 c^4 d^2 x^2 \ln(F)^2 - 12b^4 c^4 d^2 x^2 \ln(F)^2 - 4b^4 c^4 e^2 \ln(F)^2 + 12b^4 c^4 d^2 \ln(F)^2 + 24b^4 c^4 d^2 x^2 \ln(F)^2 + 12b^4 c^4 d^2 x^2 \ln(F)^2 - 24bcd^2 \ln(F) - 24bcd^2 x \ln(F) + 24c^4}{4b^4 c^4 \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(F^{(c*(a + b*x))}*(d + e*x)^4, x)$

[Out]  $(F^{(a*c + b*c*x)}*(24*e^4 + b^4*c^4*d^4*\log(F)^4 - 24*b*c*e^4*x*\log(F) - 4*b^3*c^3*d^3*e*\log(F)^3 + 12*b^2*c^2*d^2*e^2*\log(F)^2 + 12*b^2*c^2*e^4*x^2*\log(F)^2 - 4*b^3*c^3*e^4*x^3*\log(F)^3 + b^4*c^4*e^4*x^4*\log(F)^4 - 24*b*c*d*e^3*\log(F) + 6*b^4*c^4*d^2*e^2*x^2*\log(F)^4 + 24*b^2*c^2*d*e^3*x*\log(F)^2 + 4*b^4*c^4*d^3*e*x*\log(F)^4 - 12*b^3*c^3*d^2*e^2*x*\log(F)^3 - 12*b^3*c^3*d*e^3*x^2*\log(F)^3 + 4*b^4*c^4*d*e^3*x^3*\log(F)^4))/(b^5*c^5*\log(F)^5)$

### 3.3 $\int F^{c(a+bx)}(d+ex)^3 dx$

**Optimal.** Leaf size=110

$$-\frac{6e^3 F^{c(a+bx)}}{b^4 c^4 \log^4(F)} + \frac{6e^2 F^{c(a+bx)}(d+ex)}{b^3 c^3 \log^3(F)} - \frac{3e F^{c(a+bx)}(d+ex)^2}{b^2 c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^3}{bc \log(F)}$$

[Out]  $-6e^3 F^{c(a+bx)}/b^4/c^4/\ln(F)^4 + 6e^2 F^{c(a+bx)}(d+ex)/b^3/c^3/\ln(F)^3 - 3e F^{c(a+bx)}(d+ex)^2/b^2/c^2/\ln(F)^2 + F^{c(a+bx)}(d+ex)^3/bc/\ln(F)$

**Rubi [A]**

time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2207, 2225}

$$-\frac{6e^3 F^{c(a+bx)}}{b^4 c^4 \log^4(F)} + \frac{6e^2 (d+ex) F^{c(a+bx)}}{b^3 c^3 \log^3(F)} - \frac{3e (d+ex)^2 F^{c(a+bx)}}{b^2 c^2 \log^2(F)} + \frac{(d+ex)^3 F^{c(a+bx)}}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))\*(d + e\*x)^3, x]

[Out]  $(-6e^3 F^{c(a+bx)})/(b^4 c^4 \text{Log}[F]^4) + (6e^2 F^{c(a+bx)}(d+ex))/(b^3 c^3 \text{Log}[F]^3) - (3e F^{c(a+bx)}(d+ex)^2)/(b^2 c^2 \text{Log}[F]^2) + (F^{c(a+bx)}(d+ex)^3)/(bc \text{Log}[F])$

Rule 2207

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(c + d\*x)^m\*((b\*F^(g\*(e + f\*x)))^n/(f\*g\*n\*Log[F])), x] - Dist[d\*(m/(f\*g\*n\*Log[F])), Int[(c + d\*x)^(m-1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

Rule 2225

Int[(F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned}
\int F^{c(a+bx)}(d+ex)^3 dx &= \frac{F^{c(a+bx)}(d+ex)^3}{bc \log(F)} - \frac{(3e) \int F^{c(a+bx)}(d+ex)^2 dx}{bc \log(F)} \\
&= -\frac{3eF^{c(a+bx)}(d+ex)^2}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^3}{bc \log(F)} + \frac{(6e^2) \int F^{c(a+bx)}(d+ex) dx}{b^2c^2 \log^2(F)} \\
&= \frac{6e^2 F^{c(a+bx)}(d+ex)}{b^3c^3 \log^3(F)} - \frac{3eF^{c(a+bx)}(d+ex)^2}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^3}{bc \log(F)} - \frac{(6e^3) \int F^{c(a+bx)} dx}{b^3c^3 \log^3(F)} \\
&= -\frac{6e^3 F^{c(a+bx)}}{b^4c^4 \log^4(F)} + \frac{6e^2 F^{c(a+bx)}(d+ex)}{b^3c^3 \log^3(F)} - \frac{3eF^{c(a+bx)}(d+ex)^2}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^3}{bc \log(F)}
\end{aligned}$$

**Mathematica [A]**

time = 0.07, size = 78, normalized size = 0.71

$$\frac{F^{c(a+bx)}(-6e^3 + 6bce^2(d+ex) \log(F) - 3b^2c^2e(d+ex)^2 \log^2(F) + b^3c^3(d+ex)^3 \log^3(F))}{b^4c^4 \log^4(F)}$$

Antiderivative was successfully verified.

`[In] Integrate[F^(c*(a + b*x))*(d + e*x)^3,x]`

```
[Out] (F^(c*(a + b*x))*(-6*e^3 + 6*b*c*e^2*(d + e*x)*Log[F] - 3*b^2*c^2*e*(d + e*x)^2*Log[F]^2 + b^3*c^3*(d + e*x)^3*Log[F]^3))/(b^4*c^4*Log[F]^4)
```

**Maple [A]**

time = 0.06, size = 165, normalized size = 1.50

method	result
gospers	$\frac{(e^3x^3c^3b^3 \ln(F)^3 + 3 \ln(F)^3b^3c^3d^2e^2x^2 + 3 \ln(F)^3b^3c^3d^2ex + c^3b^3 \ln(F)^3d^3 - 3 \ln(F)^2b^2c^2e^3x^2 - 6 \ln(F)^2b^2c^2d^2ex - 3 \ln(F)^2b^2c^2d^2e + 6c^4b^4 \ln(F)^4)}{c^4b^4 \ln(F)^4}$
risch	$\frac{(e^3x^3c^3b^3 \ln(F)^3 + 3 \ln(F)^3b^3c^3d^2e^2x^2 + 3 \ln(F)^3b^3c^3d^2ex + c^3b^3 \ln(F)^3d^3 - 3 \ln(F)^2b^2c^2e^3x^2 - 6 \ln(F)^2b^2c^2d^2ex - 3 \ln(F)^2b^2c^2d^2e + 6c^4b^4 \ln(F)^4)}{c^4b^4 \ln(F)^4}$
norman	$\frac{(c^3b^3 \ln(F)^3d^3 - 3 \ln(F)^2b^2c^2d^2e + 6 \ln(F)bcd^2e^2 - 6e^3)e^{c(bx+a) \ln(F)}}{c^4b^4 \ln(F)^4} + \frac{e^3x^3e^{c(bx+a) \ln(F)}}{cb \ln(F)} + \frac{3e(\ln(F)^2b^2c^2d^2 - 2 \ln(F)bced + 2e^2)}{c^3b^3 \ln(F)^3}$
meijerg	$\frac{F^{ca}e^3 \left( 6 - \frac{(-4b^3c^3x^3 \ln(F)^3 + 12b^2c^2x^2 \ln(F)^2 - 24bcx \ln(F) + 24)e^{bcx \ln(F)}}{4} \right)}{c^4b^4 \ln(F)^4} - \frac{3F^{ca}e^2d \left( 2 - \frac{(3b^2c^2x^2 \ln(F)^2 - 6bcx \ln(F) + 6)e^{bcx \ln(F)}}{3} \right)}{c^3b^3 \ln(F)^3}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(F^(c*(b*x+a))*(e*x+d)^3,x,method=_RETURNVERBOSE)`

```
[Out] (e^3*x^3*c^3*b^3*ln(F)^3+3*ln(F)^3*b^3*c^3*d^2*e^2*x^2+3*ln(F)^3*b^3*c^3*d^2*ex+c^3*b^3*ln(F)^3*d^3-3*ln(F)^2*b^2*c^2*e^3*x^2-6*ln(F)^2*b^2*c^2*d^2*ex+6*c^4*b^4*ln(F)^4)/(b^4*c^4*ln(F)^4)
```

$-3*\ln(F)^2*b^2*c^2*d^2*e+6*\ln(F)*b*c*e^3*x+6*\ln(F)*b*c*d*e^2-6*e^3)*F^(c*(b*x+a))/c^4/b^4/\ln(F)^4$

**Maxima [A]**

time = 0.28, size = 208, normalized size = 1.89

$$\frac{F^{bcx+ac}d^3}{bc\log(F)} + \frac{3(F^{ac}bcx\log(F) - F^{ac})d^2e^{(bcx\log(F)+1)}}{b^2c^2\log(F)^2} + \frac{3(F^{ac}b^2c^2x^2\log(F)^2 - 2F^{ac}bcx\log(F) + 2F^{ac})de^{(bcx\log(F)+2)}}{b^3c^3\log(F)^3} + \frac{(F^{ac}b^3c^3x^3\log(F)^3 - 3F^{ac}b^2c^2x^2\log(F)^2 + 6F^{ac}bcx\log(F) - 6F^{ac})e^{(bcx\log(F)+3)}}{b^4c^4\log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^3,x, algorithm="maxima")

[Out]  $F^{(b*c*x + a*c)}*d^3/(b*c*\log(F)) + 3*(F^{(a*c)}*b*c*x*\log(F) - F^{(a*c)})*d^2*e^{(b*c*x*\log(F) + 1)}/(b^2*c^2*\log(F)^2) + 3*(F^{(a*c)}*b^2*c^2*x^2*\log(F)^2 - 2*F^{(a*c)}*b*c*x*\log(F) + 2*F^{(a*c)})*d*e^{(b*c*x*\log(F) + 2)}/(b^3*c^3*\log(F)^3) + (F^{(a*c)}*b^3*c^3*x^3*\log(F)^3 - 3*F^{(a*c)}*b^2*c^2*x^2*\log(F)^2 + 6*F^{(a*c)}*b*c*x*\log(F) - 6*F^{(a*c)})*e^{(b*c*x*\log(F) + 3)}/(b^4*c^4*\log(F)^4)$

**Fricas [A]**

time = 0.45, size = 142, normalized size = 1.29

$$\frac{((b^3c^3x^3e^3 + 3b^3c^3dx^2e^2 + 3b^3c^3d^2xe + b^3c^3d^3)\log(F)^3 - 3(b^2c^2x^2e^3 + 2b^2c^2dxe^2 + b^2c^2d^2e)\log(F)^2 + 6(bcxe^3 + bcde^2)\log(F) - 6e^3)F^{bcx+ac}}{b^4c^4\log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^3,x, algorithm="fricas")

[Out]  $((b^3*c^3*x^3*e^3 + 3*b^3*c^3*d*x^2*e^2 + 3*b^3*c^3*d^2*x*e + b^3*c^3*d^3)*\log(F)^3 - 3*(b^2*c^2*x^2*e^3 + 2*b^2*c^2*d*x*e^2 + b^2*c^2*d^2*e)*\log(F)^2 + 6*(b*c*x*e^3 + b*c*d*e^2)*\log(F) - 6*e^3)*F^{(b*c*x + a*c)}/(b^4*c^4*\log(F)^4)$

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(107) = 214.

time = 0.09, size = 231, normalized size = 2.10

$$\begin{cases} \frac{F^{c(a+bx)}(b^3c^3d^3\log(F)^3 + 3b^3c^3d^2cx\log(F)^3 + 3b^3c^3d^2e^2x^2\log(F)^3 + b^3c^3e^3x^3\log(F)^3 - 3b^2c^2d^2e\log(F)^2 - 6b^2c^2d^2e^2x\log(F)^2 - 3b^2c^2e^3x^2\log(F)^2 + 6bcde^2\log(F) + 6bcc^3x\log(F) - 6e^3)}{b^4c^4\log(F)^4} & \text{for } b^4c^4\log(F)^4 \neq 0 \\ d^3x + \frac{3d^2cx^2}{2} + de^2x^3 + \frac{e^3x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(e\*x+d)\*\*3,x)

[Out] Piecewise((F\*\*(c\*(a + b\*x))\*(b\*\*3\*c\*\*3\*d\*\*3\*log(F)\*\*3 + 3\*b\*\*3\*c\*\*3\*d\*\*2\*e\*x\*log(F)\*\*3 + 3\*b\*\*3\*c\*\*3\*d\*e\*\*2\*x\*\*2\*log(F)\*\*3 + b\*\*3\*c\*\*3\*e\*\*3\*x\*\*3\*log(F))\*\*3 - 3\*b\*\*2\*c\*\*2\*d\*\*2\*e\*log(F)\*\*2 - 6\*b\*\*2\*c\*\*2\*d\*e\*\*2\*x\*log(F)\*\*2 - 3\*b\*\*2\*c\*\*2\*e\*\*3\*x\*\*2\*log(F)\*\*2 + 6\*b\*c\*d\*e\*\*2\*log(F) + 6\*b\*c\*e\*\*3\*x\*log(F) - 6\*e\*\*3)/(b\*\*4\*c\*\*4\*log(F)\*\*4), Ne(b\*\*4\*c\*\*4\*log(F)\*\*4, 0)), (d\*\*3\*x + 3\*d\*\*2\*e\*x\*\*2/2 + d\*e\*\*2\*x\*\*3 + e\*\*3\*x\*\*4/4, True))



$$\begin{aligned}
& I\pi^2 b^3 c^3 x^3 \log(\text{abs}(F)) + 3\pi b^3 c^3 x^3 \log(\text{abs}(F))^2 + 2I b^3 c^3 x^3 \log(\text{abs}(F))^3 - 3I\pi^2 b^2 c^2 x^2 \text{sgn}(F) + 6\pi b^2 c^2 x^2 \log(\text{abs}(F)) \text{sgn}(F) \\
& + 3I\pi^2 b^2 c^2 x^2 - 6\pi b^2 c^2 x^2 \log(\text{abs}(F)) - 6I b^2 c^2 x^2 \log(\text{abs}(F))^2 - 6\pi b^2 c^2 x^2 \text{sgn}(F) + 6\pi b^2 c^2 x^2 \log(\text{abs}(F)) \text{sgn}(F) \\
& + 12I b^2 c^2 x^2 \log(\text{abs}(F)) - 12I \text{e}^{(1/2 I \pi b^2 c^2 x^2 \text{sgn}(F) - 1/2 I \pi b^2 c^2 x^2 + 1/2 I \pi a^2 c^2 \text{sgn}(F) - 1/2 I \pi a^2 c^2)} \\
& / (\pi^4 b^4 c^4 \text{sgn}(F) + 4I \pi^3 b^4 c^4 \log(\text{abs}(F)) \text{sgn}(F) - 6\pi^2 b^4 c^4 \log(\text{abs}(F))^2 \text{sgn}(F) - 4I \pi b^4 c^4 \log(\text{abs}(F))^3 \text{sgn}(F) \\
& - \pi^4 b^4 c^4 - 4I \pi^3 b^4 c^4 \log(\text{abs}(F)) + 6\pi^2 b^4 c^4 \log(\text{abs}(F))^2 + 4I \pi b^4 c^4 \log(\text{abs}(F))^3 - 2b^4 c^4 \log(\text{abs}(F))^4) \\
& + (\pi^3 b^3 c^3 x^3 \text{sgn}(F) - 3I \pi^2 b^3 c^3 x^3 \log(\text{abs}(F)) \text{sgn}(F) - 3\pi b^3 c^3 x^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 c^3 x^3 + 3I \pi^2 b^3 c^3 x^3 \log(\text{abs}(F)) \\
& + 3\pi b^3 c^3 x^3 \log(\text{abs}(F))^2 - 2I b^3 c^3 x^3 \log(\text{abs}(F))^3 + 3I \pi^2 b^2 c^2 x^2 \text{sgn}(F) + 6\pi b^2 c^2 x^2 \log(\text{abs}(F)) \text{sgn}(F) - 3I \pi^2 b^2 c^2 x^2 \\
& - 6\pi b^2 c^2 x^2 \log(\text{abs}(F)) + 6I b^2 c^2 x^2 \log(\text{abs}(F))^2 - 6\pi b^2 c^2 x^2 \text{sgn}(F) + 6\pi b^2 c^2 x^2 \log(\text{abs}(F)) \text{sgn}(F) \\
& - 12I b^2 c^2 x^2 \log(\text{abs}(F)) + 12I \text{e}^{(-1/2 I \pi b^2 c^2 x^2 \text{sgn}(F) + 1/2 I \pi b^2 c^2 x^2 - 1/2 I \pi a^2 c^2 \text{sgn}(F) + 1/2 I \pi a^2 c^2)} \\
& / (\pi^4 b^4 c^4 \text{sgn}(F) - 4I \pi^3 b^4 c^4 \log(\text{abs}(F)) \text{sgn}(F) - 6\pi^2 b^4 c^4 \log(\text{abs}(F))^2 \text{sgn}(F) + 4I \pi b^4 c^4 \log(\text{abs}(F))^3 \text{sgn}(F) \\
& - \pi^4 b^4 c^4 + 4I \pi^3 b^4 c^4 \log(\text{abs}(F)) + 6\pi^2 b^4 c^4 \log(\text{abs}(F))^2 - 4I \pi b^4 c^4 \log(\text{abs}(F))^3 - 2b^4 c^4 \log(\text{abs}(F))^4) \\
& \text{e}^{(b^2 c^2 x^2 \log(\text{abs}(F)) + a^2 c^2 \log(\text{abs}(F)) + 3)} + 3 \left( (\pi^2 b^2 c^2 d x^2 \text{sgn}(F) - \pi^2 b^2 c^2 d x^2 + 2b^2 c^2 d x^2 \log(\text{abs}(F))^2 - 4b^2 c^2 d x^2 \log(\text{abs}(F)) \text{sgn}(F) + 4d) \right. \\
& \left. (3\pi^2 b^3 c^3 \log(\text{abs}(F)) \text{sgn}(F) - 3\pi^2 b^3 c^3 \log(\text{abs}(F)) + 2b^3 c^3 \log(\text{abs}(F))^3) / ((\pi^3 b^3 c^3 \text{sgn}(F) - 3\pi b^3 c^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 c^3 + 3\pi b^3 c^3 \log(\text{abs}(F))^2) \right. \\
& \left. + (3\pi^2 b^3 c^3 \log(\text{abs}(F)) \text{sgn}(F) - 3\pi^2 b^3 c^3 \log(\text{abs}(F)) + 2b^3 c^3 \log(\text{abs}(F))^3) \right) - 2(\pi^3 b^3 c^3 \text{sgn}(F) - 3\pi b^3 c^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 c^3 + 3\pi b^3 c^3 \log(\text{abs}(F))^2) \\
& \left( \pi b^2 c^2 d x^2 \log(\text{abs}(F)) \text{sgn}(F) - \pi b^2 c^2 d x^2 \log(\text{abs}(F)) - \pi b^2 c^2 d x^2 \text{sgn}(F) + \pi b^2 c^2 d x^2 \right) / ((\pi^3 b^3 c^3 \text{sgn}(F) - 3\pi b^3 c^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 c^3 + 3\pi b^3 c^3 \log(\text{abs}(F))^2) \text{sgn}(F) - \pi^3 b^3 c^3 + 3\pi b^3 c^3 \log(\text{abs}(F))^2)
\end{aligned}$$

**Mupad [B]**

time = 3.56, size = 165, normalized size = 1.50

$$\frac{F^{a+bcx} (b^3 c^3 d^3 \ln(F)^3 + 3b^3 c^3 d^2 e x \ln(F)^3 + 3b^3 c^3 d e^2 x^2 \ln(F)^3 + b^3 c^3 e^3 x^3 \ln(F)^3 - 3b^2 c^2 d^2 e \ln(F)^2 - 6b^2 c^2 d e^2 x \ln(F)^2 - 3b^2 c^2 e^3 x^2 \ln(F)^2 + 6b c d e^2 \ln(F) + 6b c e^3 x \ln(F) - 6e^3)}{b^4 c^4 \ln(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(F^{c(a + b*x)} * (d + e*x)^3, x)$

[Out]  $(F^{(a*c + b*c*x)} * (b^3 c^3 d^3 \log(F)^3 - 6e^3 + 6*b*c*e^3*x*\log(F) - 3*b^2*c^2*d^2*e*\log(F)^2 - 3*b^2*c^2*e^3*x^2*\log(F)^2 + b^3*c^3*e^3*x^3*\log(F)^3 + 6*b*c*d*e^2*\log(F) - 6*b^2*c^2*d*e^2*x*\log(F)^2 + 3*b^3*c^3*d^2*e*x*\log(F)^3 + 3*b^3*c^3*d*e^2*x^2*\log(F)^3)) / (b^4*c^4*\log(F)^4)$

### 3.4 $\int F^{c(a+bx)}(d+ex)^2 dx$

**Optimal.** Leaf size=79

$$\frac{2e^2 F^{c(a+bx)}}{b^3 c^3 \log^3(F)} - \frac{2e F^{c(a+bx)}(d+ex)}{b^2 c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^2}{bc \log(F)}$$

[Out]  $2e^2 F^{c(a+bx)}/b^3/c^3/\ln(F)^3 - 2e F^{c(a+bx)}(d+ex)/b^2/c^2/\ln(F)^2 + F^{c(a+bx)}(d+ex)^2/b/c/\ln(F)$

**Rubi [A]**

time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2207, 2225}

$$\frac{2e^2 F^{c(a+bx)}}{b^3 c^3 \log^3(F)} - \frac{2e(d+ex)F^{c(a+bx)}}{b^2 c^2 \log^2(F)} + \frac{(d+ex)^2 F^{c(a+bx)}}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))\*(d + e\*x)^2, x]

[Out]  $(2e^2 F^{c(a+bx)})/(b^3 c^3 \text{Log}[F]^3) - (2e F^{c(a+bx)}(d+ex))/(b^2 c^2 \text{Log}[F]^2) + (F^{c(a+bx)}(d+ex)^2)/(bc \text{Log}[F])$

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int F^{c(a+bx)}(d+ex)^2 dx &= \frac{F^{c(a+bx)}(d+ex)^2}{bc \log(F)} - \frac{(2e) \int F^{c(a+bx)}(d+ex) dx}{bc \log(F)} \\ &= -\frac{2e F^{c(a+bx)}(d+ex)}{b^2 c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^2}{bc \log(F)} + \frac{(2e^2) \int F^{c(a+bx)} dx}{b^2 c^2 \log^2(F)} \\ &= \frac{2e^2 F^{c(a+bx)}}{b^3 c^3 \log^3(F)} - \frac{2e F^{c(a+bx)}(d+ex)}{b^2 c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^2}{bc \log(F)} \end{aligned}$$



**Mathematica [A]**

time = 0.06, size = 56, normalized size = 0.71

$$\frac{F^{c(a+bx)}(2e^2 - 2bce(d+ex)\log(F) + b^2c^2(d+ex)^2\log^2(F))}{b^3c^3\log^3(F)}$$

Antiderivative was successfully verified.

**[In]** Integrate[F^(c\*(a + b\*x))\*(d + e\*x)^2,x]**[Out]** (F^(c\*(a + b\*x))\*(2\*e^2 - 2\*b\*c\*e\*(d + e\*x)\*Log[F] + b^2\*c^2\*(d + e\*x)^2\*Log[F]^2))/(b^3\*c^3\*Log[F]^3)**Maple [A]**

time = 0.06, size = 91, normalized size = 1.15

method	result	size
gospers	$\frac{(e^2x^2c^2b^2\ln(F)^2+2\ln(F)^2b^2c^2dex+\ln(F)^2b^2c^2d^2-2\ln(F)bc e^2x-2\ln(F)bc ed+2e^2)F^{c(bx+a)}}{c^3b^3\ln(F)^3}$	91
risch	$\frac{(e^2x^2c^2b^2\ln(F)^2+2\ln(F)^2b^2c^2dex+\ln(F)^2b^2c^2d^2-2\ln(F)bc e^2x-2\ln(F)bc ed+2e^2)F^{c(bx+a)}}{c^3b^3\ln(F)^3}$	91
norman	$\frac{(\ln(F)^2b^2c^2d^2-2\ln(F)bc ed+2e^2)e^{c(bx+a)\ln(F)}}{c^3b^3\ln(F)^3} + \frac{e^2x^2e^{c(bx+a)\ln(F)}}{cb\ln(F)} + \frac{2e(\ln(F)bcd-e)x e^{c(bx+a)\ln(F)}}{c^2b^2\ln(F)^2}$	111
meijerg	$-\frac{F^{ca}e^2\left(2-\frac{(3b^2c^2x^2\ln(F)^2-6bcx\ln(F)+6)e^{bcx\ln(F)}}{3}\right)}{c^3b^3\ln(F)^3} + \frac{2F^{ca}ed\left(1-\frac{(-2bcx\ln(F)+2)e^{bcx\ln(F)}}{2}\right)}{c^2b^2\ln(F)^2} - \frac{F^{ca}d^2(1-e^{bcx\ln(F)})}{cb\ln(F)}$	121

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(F^(c\*(b\*x+a))\*(e\*x+d)^2,x,method=\_RETURNVERBOSE)**[Out]** (e^2\*x^2\*c^2\*b^2\*ln(F)^2+2\*ln(F)^2\*b^2\*c^2\*d\*e\*x+ln(F)^2\*b^2\*c^2\*d^2-2\*ln(F)\*b\*c\*e^2\*x-2\*ln(F)\*b\*c\*e\*d+2\*e^2)\*F^(c\*(b\*x+a))/c^3/b^3/ln(F)^3**Maxima [A]**

time = 0.30, size = 125, normalized size = 1.58

$$\frac{F^{bcx+ac}d^2}{bc\log(F)} + \frac{2(F^{ac}bcx\log(F) - F^{ac})de^{(bcx\log(F)+1)}}{b^2c^2\log(F)^2} + \frac{(F^{ac}b^2c^2x^2\log(F)^2 - 2F^{ac}bcx\log(F) + 2F^{ac})e^{(bcx\log(F)+2)}}{b^3c^3\log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(F^(c\*(b\*x+a))\*(e\*x+d)^2,x, algorithm="maxima")**[Out]** F^(b\*c\*x + a\*c)\*d^2/(b\*c\*log(F)) + 2\*(F^(a\*c)\*b\*c\*x\*log(F) - F^(a\*c))\*d\*e^(b\*c\*x\*log(F) + 1)/(b^2\*c^2\*log(F)^2) + (F^(a\*c)\*b^2\*c^2\*x^2\*log(F)^2 - 2\*F^(a\*c)\*b\*c\*x\*log(F) + 2\*F^(a\*c))\*e^(b\*c\*x\*log(F) + 2)/(b^3\*c^3\*log(F)^3)

**Fricas** [A]

time = 0.43, size = 83, normalized size = 1.05

$$\frac{((b^2c^2x^2e^2 + 2b^2c^2dxe + b^2c^2d^2)\log(F)^2 - 2(bcxe^2 + bcde)\log(F) + 2e^2)F^{bcx+ac}}{b^3c^3\log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^2,x, algorithm="fricas")

[Out] ((b^2\*c^2\*x^2\*e^2 + 2\*b^2\*c^2\*d\*x\*e + b^2\*c^2\*d^2)\*log(F)^2 - 2\*(b\*c\*x\*e^2 + b\*c\*d\*e)\*log(F) + 2\*e^2)\*F^(b\*c\*x + a\*c)/(b^3\*c^3\*log(F)^3)

**Sympy** [A]

time = 0.07, size = 133, normalized size = 1.68

$$\begin{cases} \frac{F^{c(a+bx)}(b^2c^2d^2\log(F)^2 + 2b^2c^2dex\log(F)^2 + b^2c^2e^2x^2\log(F)^2 - 2bcde\log(F) - 2bce^2x\log(F) + 2e^2)}{b^3c^3\log(F)^3} & \text{for } b^3c^3\log(F)^3 \neq 0 \\ d^2x + dex^2 + \frac{e^2x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(e\*x+d)\*\*2,x)

[Out] Piecewise(((F\*\*(c\*(a + b\*x))\*(b\*\*2\*c\*\*2\*d\*\*2\*log(F)\*\*2 + 2\*b\*\*2\*c\*\*2\*d\*e\*x\*log(F)\*\*2 + b\*\*2\*c\*\*2\*e\*\*2\*x\*\*2\*log(F)\*\*2 - 2\*b\*c\*d\*e\*log(F) - 2\*b\*c\*e\*\*2\*x\*log(F) + 2\*e\*\*2)/(b\*\*3\*c\*\*3\*log(F)\*\*3), Ne(b\*\*3\*c\*\*3\*log(F)\*\*3, 0)), (d\*\*2\*x + d\*e\*x\*\*2 + e\*\*2\*x\*\*3/3, True))

**Giac** [C] Result contains complex when optimal does not.

time = 2.98, size = 2490, normalized size = 31.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^2,x, algorithm="giac")

[Out] (((3\*pi^2\*b^3\*c^3\*log(abs(F))\*sgn(F) - 3\*pi^2\*b^3\*c^3\*log(abs(F)) + 2\*b^3\*c^3\*log(abs(F))^3)\*(pi^2\*b^2\*c^2\*x^2\*sgn(F) - pi^2\*b^2\*c^2\*x^2 + 2\*b^2\*c^2\*x^2\*log(abs(F))^2 - 4\*b\*c\*x\*log(abs(F)) + 4)/((pi^3\*b^3\*c^3\*sgn(F) - 3\*pi\*b^3\*c^3\*log(abs(F))^2\*sgn(F) - pi^3\*b^3\*c^3 + 3\*pi\*b^3\*c^3\*log(abs(F))^2)^2 + (3\*pi^2\*b^3\*c^3\*log(abs(F))\*sgn(F) - 3\*pi^2\*b^3\*c^3\*log(abs(F)) + 2\*b^3\*c^3\*log(abs(F))^3)^2) - 2\*(pi^3\*b^3\*c^3\*sgn(F) - 3\*pi\*b^3\*c^3\*log(abs(F))^2\*sgn(F) - pi^3\*b^3\*c^3 + 3\*pi\*b^3\*c^3\*log(abs(F))^2)\*(pi\*b^2\*c^2\*x^2\*log(abs(F))\*sgn(F) - pi\*b^2\*c^2\*x^2\*log(abs(F)) - pi\*b\*c\*x\*sgn(F) + pi\*b\*c\*x)/((pi^3\*b^3\*c^3\*sgn(F) - 3\*pi\*b^3\*c^3\*log(abs(F))^2\*sgn(F) - pi^3\*b^3\*c^3 + 3\*pi\*b^3\*c^3\*log(abs(F))^2)^2 + (3\*pi^2\*b^3\*c^3\*log(abs(F))\*sgn(F) - 3\*pi^2\*b^3\*c^3\*log(abs(F)) + 2\*b^3\*c^3\*log(abs(F))^3)^2)

$$\begin{aligned}
& c^3 \log(\text{abs}(F)) + 2*b^3*c^3 \log(\text{abs}(F))^3)^2) * \cos(-1/2*\pi*b*c*x*\text{sgn}(F) + 1 \\
& /2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c) + ((\pi^3*b^3*c^3*\text{sgn}(F) - 3*\pi \\
& i*b^3*c^3 \log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*b^3*c^3 + 3*\pi*b^3*c^3 \log(\text{abs}(F))^2) \\
& *(\pi^2*b^2*c^2*x^2*\text{sgn}(F) - \pi^2*b^2*c^2*x^2 + 2*b^2*c^2*x^2 \log(\text{abs}(F))^2 \\
& - 4*b*c*x \log(\text{abs}(F)) + 4)/((\pi^3*b^3*c^3*\text{sgn}(F) - 3*\pi*b^3*c^3 \log(\text{abs}(F)) \\
& ^2*\text{sgn}(F) - \pi^3*b^3*c^3 + 3*\pi*b^3*c^3 \log(\text{abs}(F))^2)^2 + (3*\pi^2*b^3*c^3 \\
& \log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*b^3*c^3 \log(\text{abs}(F)) + 2*b^3*c^3 \log(\text{abs}(F))^3)^2 \\
& + 2*(3*\pi^2*b^3*c^3 \log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*b^3*c^3 \log(\text{abs}(F)) + 2* \\
& b^3*c^3 \log(\text{abs}(F))^3)*(\pi*b^2*c^2*x^2 \log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*x^2 \\
& \log(\text{abs}(F)) - \pi*b*c*x*\text{sgn}(F) + \pi*b*c*x)/((\pi^3*b^3*c^3*\text{sgn}(F) - 3*\pi*b^3*c^3 \\
& c^3 \log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*b^3*c^3 + 3*\pi*b^3*c^3 \log(\text{abs}(F))^2)^2 + ( \\
& 3*\pi^2*b^3*c^3 \log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*b^3*c^3 \log(\text{abs}(F)) + 2*b^3*c^3 \\
& \log(\text{abs}(F))^3)^2) * \sin(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn} \\
& (F) + 1/2*\pi*a*c)) * e^{(b*c*x \log(\text{abs}(F)) + a*c \log(\text{abs}(F)) + 2) - 2*I*((-I*\pi \\
& i^2*b^2*c^2*x^2*\text{sgn}(F) + 2*\pi*b^2*c^2*x^2 \log(\text{abs}(F))*\text{sgn}(F) + I*\pi^2*b^2*c \\
& ^2*x^2 - 2*\pi*b^2*c^2*x^2 \log(\text{abs}(F)) - 2*I*b^2*c^2*x^2 \log(\text{abs}(F))^2 - 2*\pi \\
& i*b*c*x*\text{sgn}(F) + 2*\pi*b*c*x + 4*I*b*c*x \log(\text{abs}(F)) - 4*I)*e^{(1/2*I*\pi*b*c* \\
& x*\text{sgn}(F) - 1/2*I*\pi*b*c*x + 1/2*I*\pi*a*c*\text{sgn}(F) - 1/2*I*\pi*a*c)/( -4*I*\pi^3* \\
& b^3*c^3*\text{sgn}(F) + 12*\pi^2*b^3*c^3 \log(\text{abs}(F))*\text{sgn}(F) + 12*I*\pi*b^3*c^3 \log(a \\
& bs(F))^2*\text{sgn}(F) + 4*I*\pi^3*b^3*c^3 - 12*\pi^2*b^3*c^3 \log(\text{abs}(F)) - 12*I*\pi* \\
& b^3*c^3 \log(\text{abs}(F))^2 + 8*b^3*c^3 \log(\text{abs}(F))^3) - (-I*\pi^2*b^2*c^2*x^2*\text{sgn} \\
& (F) - 2*\pi*b^2*c^2*x^2 \log(\text{abs}(F))*\text{sgn}(F) + I*\pi^2*b^2*c^2*x^2 + 2*\pi*b^2*c \\
& ^2*x^2 \log(\text{abs}(F)) - 2*I*b^2*c^2*x^2 \log(\text{abs}(F))^2 + 2*\pi*b*c*x*\text{sgn}(F) - 2* \\
& \pi*b*c*x + 4*I*b*c*x \log(\text{abs}(F)) - 4*I)*e^{(-1/2*I*\pi*b*c*x*\text{sgn}(F) + 1/2*I*\pi \\
& i*b*c*x - 1/2*I*\pi*a*c*\text{sgn}(F) + 1/2*I*\pi*a*c)/(4*I*\pi^3*b^3*c^3*\text{sgn}(F) + 12 \\
& *\pi^2*b^3*c^3 \log(\text{abs}(F))*\text{sgn}(F) - 12*I*\pi*b^3*c^3 \log(\text{abs}(F))^2*\text{sgn}(F) - 4 \\
& *I*\pi^3*b^3*c^3 - 12*\pi^2*b^3*c^3 \log(\text{abs}(F)) + 12*I*\pi*b^3*c^3 \log(\text{abs}(F)) \\
& ^2 + 8*b^3*c^3 \log(\text{abs}(F))^3)} * e^{(b*c*x \log(\text{abs}(F)) + a*c \log(\text{abs}(F)) + 2) \\
& + 2*(2*((\pi*b^2*c^2 \log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2 \log(\text{abs}(F))))*(\pi*b*c*d* \\
& x*\text{sgn}(F) - \pi*b*c*d*x)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2 \log \\
& (\text{abs}(F))^2)^2 + 4*(\pi*b^2*c^2 \log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2 \log(\text{abs}(F)))^2 \\
& + (\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2 \log(\text{abs}(F))^2)*(b*c*d* \\
& x \log(\text{abs}(F)) - d)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2 \log(\text{abs} \\
& (F))^2)^2 + 4*(\pi*b^2*c^2 \log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2 \log(\text{abs}(F)))^2)* \\
& \cos(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c) + \\
& ((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2 \log(\text{abs}(F))^2)*( \pi*b*c*d* \\
& x*\text{sgn}(F) - \pi*b*c*d*x)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2 \log \\
& (\text{abs}(F))^2)^2 + 4*(\pi*b^2*c^2 \log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2 \log(\text{abs}(F)))^2 \\
& - 4*(\pi*b^2*c^2 \log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2 \log(\text{abs}(F)))*(b*c*d*x \log \\
& (\text{abs}(F)) - d)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2 \log(\text{abs}(F)) \\
& ^2)^2 + 4*(\pi*b^2*c^2 \log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2 \log(\text{abs}(F)))^2) * \sin( \\
& -1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c)) * e^{(b \\
& *c*x \log(\text{abs}(F)) + a*c \log(\text{abs}(F)) + 1) - I*((\pi*b*c*d*x*\text{sgn}(F) - \pi*b*c*d* \\
& x - 2*I*b*c*d*x \log(\text{abs}(F)) + 2*I*d)*e^{(1/2*I*\pi*b*c*x*\text{sgn}(F) - 1/2*I*\pi*b* \\
& c*x + 1/2*I*\pi*a*c*\text{sgn}(F) - 1/2*I*\pi*a*c)/(\pi^2*b^2*c^2*\text{sgn}(F) + 2*I*\pi*b^2
\end{aligned}$$

```

*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 - 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^2
*c^2*log(abs(F))^2) + (pi*b*c*d*x*sgn(F) - pi*b*c*d*x + 2*I*b*c*d*x*log(abs
(F)) - 2*I*d)*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn
(F) + 1/2*I*pi*a*c)/(pi^2*b^2*c^2*sgn(F) - 2*I*pi*b^2*c^2*log(abs(F))*sgn(F
) - pi^2*b^2*c^2 + 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^2*c^2*log(abs(F))^2)*e
^(b*c*x*log(abs(F)) + a*c*log(abs(F)) + 1) + 2*(2*b*c*d^2*cos(-1/2*pi*b*c*x
*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F))/(4*b^2
*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b*c)
*d^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a
*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(b*c*x*log(ab
s(F)) + a*c*log(abs(F))) + I*(I*d^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c
*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*...

```

**Mupad [B]**

time = 3.51, size = 91, normalized size = 1.15

$$\frac{F^{a+bcx} (b^2 c^2 d^2 \ln(F)^2 + 2b^2 c^2 d e x \ln(F)^2 + b^2 c^2 e^2 x^2 \ln(F)^2 - 2bcde \ln(F) - 2bce^2 x \ln(F) + 2e^2)}{b^3 c^3 \ln(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))\*(d + e\*x)^2,x)

[Out] (F^(a\*c + b\*c\*x)\*(2\*e^2 + b^2\*c^2\*d^2\*log(F)^2 - 2\*b\*c\*e^2\*x\*log(F) + b^2\*c^2\*e^2\*x^2\*log(F)^2 - 2\*b\*c\*d\*e\*log(F) + 2\*b^2\*c^2\*d\*e\*x\*log(F)^2))/(b^3\*c^3\*log(F)^3)

### 3.5 $\int F^{c(a+bx)}(d+ex) dx$

Optimal. Leaf size=48

$$-\frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)}{bc \log(F)}$$

[Out]  $-eF^{c(b*x+a)}/b^2/c^2/\ln(F)^2+F^{c(b*x+a)}*(e*x+d)/b/c/\ln(F)$

Rubi [A]

time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2207, 2225}

$$\frac{(d+ex)F^{c(a+bx)}}{bc \log(F)} - \frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c(a+bx)}(d+ex), x]$

[Out]  $-((eF^{c(a+bx)})/(b^2*c^2*\text{Log}[F]^2)) + (F^{c(a+bx)}*(d+ex))/(b*c*\text{Log}[F])$

Rule 2207

$\text{Int}[(b_*)*(F_*)^{((g_*)*((e_*) + (f_*)*(x_)))}^{(n_*)}*((c_*) + (d_*)*(x_))^{(m_*)}, x\_Symbol] :> \text{Simp}[(c + d*x)^m*((bF^{g*(e+f*x)})^n/(f*g*n*\text{Log}[F])), x] - \text{Dist}[d*(m/(f*g*n*\text{Log}[F])), \text{Int}[(c + d*x)^{(m-1)}*(bF^{g*(e+f*x)})^n, x], x] /; \text{FreeQ}\{F, b, c, d, e, f, g, n\}, x\} \&\& \text{GtQ}[m, 0] \&\& \text{IntegerQ}[2*m] \&\& !\text{TrueQ}[\$UseGamma]$

Rule 2225

$\text{Int}[(F_*)^{((c_*)*((a_*) + (b_*)*(x_)))}^{(n_*)}, x\_Symbol] :> \text{Simp}[F^{c(a+bx)}(d+ex)^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

Rubi steps

$$\begin{aligned} \int F^{c(a+bx)}(d+ex) dx &= \frac{F^{c(a+bx)}(d+ex)}{bc \log(F)} - \frac{e \int F^{c(a+bx)} dx}{bc \log(F)} \\ &= -\frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)}{bc \log(F)} \end{aligned}$$

**Mathematica [A]**

time = 0.04, size = 34, normalized size = 0.71

$$\frac{F^{c(a+bx)}(-e + bc(d + ex) \log(F))}{b^2 c^2 \log^2(F)}$$

Antiderivative was successfully verified.

`[In] Integrate[F^(c*(a + b*x))*(d + e*x), x]``[Out] (F^(c*(a + b*x))*(-e + b*c*(d + e*x)*Log[F]))/(b^2*c^2*Log[F]^2)`**Maple [A]**

time = 0.01, size = 38, normalized size = 0.79

method	result	size
gospers	$\frac{(\ln(F)bcex + \ln(F)bcd - e)F^{c(bx+a)}}{c^2 b^2 \ln(F)^2}$	38
risch	$\frac{(\ln(F)bcex + \ln(F)bcd - e)F^{c(bx+a)}}{c^2 b^2 \ln(F)^2}$	38
norman	$\frac{(\ln(F)bcd - e)e^{c(bx+a)\ln(F)}}{c^2 b^2 \ln(F)^2} + \frac{ex e^{c(bx+a)\ln(F)}}{cb \ln(F)}$	56
meijerg	$\frac{F^{ca} e \left( 1 - \frac{(-2bcx \ln(F) + 2)e^{bcx \ln(F)}}{2} \right)}{c^2 b^2 \ln(F)^2} - \frac{F^{ca} d (1 - e^{bcx \ln(F)})}{cb \ln(F)}$	68

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(F^(c*(b*x+a))*(e*x+d), x, method=_RETURNVERBOSE)``[Out] (ln(F)*b*c*e*x + ln(F)*b*c*d - e)*F^(c*(b*x+a))/c^2/b^2/ln(F)^2`**Maxima [A]**

time = 0.28, size = 62, normalized size = 1.29

$$\frac{F^{bcx+ac}d}{bc \log(F)} + \frac{(F^{ac}bcx \log(F) - F^{ac})e^{(bcx \log(F)+1)}}{b^2 c^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(c*(b*x+a))*(e*x+d), x, algorithm="maxima")``[Out] F^(b*c*x + a*c)*d/(b*c*log(F)) + (F^(a*c)*b*c*x*log(F) - F^(a*c))*e^(b*c*x*log(F) + 1)/(b^2*c^2*log(F)^2)`**Fricas [A]**

time = 0.42, size = 40, normalized size = 0.83

$$\frac{((bcxe + bcd) \log(F) - e)F^{bcx+ac}}{b^2 c^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x+d),x, algorithm="fricas")
```

```
[Out] ((b*c*x*e + b*c*d)*log(F) - e)*F^(b*c*x + a*c)/(b^2*c^2*log(F)^2)
```

**Sympy** [A]

time = 0.06, size = 60, normalized size = 1.25

$$\begin{cases} \frac{F^{c(a+bx)}(bcd \log(F) + bcex \log(F) - e)}{b^2 c^2 \log(F)^2} & \text{for } b^2 c^2 \log(F)^2 \neq 0 \\ dx + \frac{ex^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(e*x+d),x)
```

```
[Out] Piecewise((F**(c*(a + b*x))*(b*c*d*log(F) + b*c*e*x*log(F) - e)/(b**2*c**2*log(F)**2), Ne(b**2*c**2*log(F)**2, 0)), (d*x + e*x**2/2, True))
```

**Giac** [C] Result contains complex when optimal does not.

time = 2.96, size = 1079, normalized size = 22.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x+d),x, algorithm="giac")
```

```
[Out] (2*((pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))*(pi*b*c*x*sgn(F) - pi*b*c*x)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2) + (pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)*(b*c*x*log(abs(F)) - 1)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2))*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c) + ((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)*(pi*b*c*x*sgn(F) - pi*b*c*x)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2) - 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))*(b*c*x*log(abs(F)) - 1)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2))*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)) + 1) - 1/2*I*((pi*b*c*x*sgn(F) - pi*b*c*x - 2*I*b*c*x*log(abs(F)) + 2*I)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(pi^2*b^2*c^2*sgn(F) + 2*I*pi*b^2*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 - 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^2*c^2*log(abs(F))^2) + (pi*b*c*x*sgn(F) - pi*b*c*x + 2*I*b*c*x*log(abs(F)) - 2*I)*e^(-1/2*I*pi*b
```

```

*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(pi^2*b^
2*c^2*sgn(F) - 2*I*pi*b^2*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 + 2*I*pi*b^
2*c^2*log(abs(F)) + 2*b^2*c^2*log(abs(F))^2))*e^(b*c*x*log(abs(F)) + a*c*lo
g(abs(F)) + 1) + 2*(2*b*c*d*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*p
i*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*s
gn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b*c)*d*sin(-1/2*pi*b*c*x*sgn(F) +
1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (
pi*b*c*sgn(F) - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*
d*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*p
i*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F))) - I*d*e^(-1/2*I*pi*
b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*
b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(
abs(F)))

```

**Mupad [B]**

time = 3.37, size = 38, normalized size = 0.79

$$\frac{F^{ac+bcx} (bcd \ln(F) - e + bcex \ln(F))}{b^2 c^2 \ln(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))\*(d + e\*x),x)

[Out] (F^(a\*c + b\*c\*x)\*(b\*c\*d\*log(F) - e + b\*c\*e\*x\*log(F)))/(b^2\*c^2\*log(F)^2)



### 3.6 $\int F^{c(a+bx)} dx$

Optimal. Leaf size=20

$$\frac{F^{c(a+bx)}}{bc \log(F)}$$

[Out]  $F^{(c*(b*x+a))/b/c/\ln(F)}$

Rubi [A]

time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2225}

$$\frac{F^{c(a+bx)}}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x)),x]

[Out] F^(c\*(a + b\*x))/(b\*c\*Log[F])

Rule 2225

Int[((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(n\_), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\int F^{c(a+bx)} dx = \frac{F^{c(a+bx)}}{bc \log(F)}$$

Mathematica [A]

time = 0.01, size = 21, normalized size = 1.05

$$\frac{F^{ac+bcx}}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x)),x]

[Out] F^(a\*c + b\*c\*x)/(b\*c\*Log[F])

Maple [A]

time = 0.02, size = 21, normalized size = 1.05

method	result	size
gospers	$\frac{F^{c(bx+a)}}{bc \ln(F)}$	21
derivativedivides	$\frac{F^{c(bx+a)}}{bc \ln(F)}$	21
default	$\frac{F^{c(bx+a)}}{bc \ln(F)}$	21
risch	$\frac{F^{c(bx+a)}}{bc \ln(F)}$	21
norman	$\frac{e^{c(bx+a) \ln(F)}}{cb \ln(F)}$	22
meijerg	$-\frac{F^{ca} (1 - e^{bcx \ln(F)})}{cb \ln(F)}$	29

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a)),x,method=_RETURNVERBOSE)`

[Out]  $F^{c(bx+a)}/b/c/\ln(F)$

**Maxima** [A]

time = 0.26, size = 20, normalized size = 1.00

$$\frac{F^{(bx+a)c}}{bc \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a)),x, algorithm="maxima")`

[Out]  $F^{((b*x + a)*c)}/(b*c*\log(F))$

**Fricas** [A]

time = 0.43, size = 21, normalized size = 1.05

$$\frac{F^{bcx+ac}}{bc \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a)),x, algorithm="fricas")`

[Out]  $F^{(b*c*x + a*c)}/(b*c*\log(F))$

**Sympy** [A]

time = 0.03, size = 20, normalized size = 1.00

$$\begin{cases} \frac{F^{c(a+bx)}}{bc \log(F)} & \text{for } bc \log(F) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a)),x)

[Out] Piecewise((F\*\*(c\*(a + b\*x))/(b\*c\*log(F)), Ne(b\*c\*log(F), 0)), (x, True))

**Giac** [A]

time = 3.43, size = 21, normalized size = 1.05

$$\frac{F^{bcx+ac}}{bc \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a)),x, algorithm="giac")

[Out] F^(b\*c\*x + a\*c)/(b\*c\*log(F))

**Mupad** [B]

time = 3.43, size = 21, normalized size = 1.05

$$\frac{F^{ac+bcx}}{bc \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x)),x)

[Out] F^(a\*c + b\*c\*x)/(b\*c\*log(F))

$$3.7 \quad \int \frac{F^{c(a+bx)}}{d+ex} dx$$

Optimal. Leaf size=31

$$\frac{F^{c\left(a-\frac{bd}{e}\right)} \operatorname{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{e}$$

[Out]  $F^{c(a-b*d/e)} * \operatorname{Ei}(b*c*(e*x+d)*\ln(F)/e) / e$

Rubi [A]

time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {2209}

$$\frac{F^{c\left(a-\frac{bd}{e}\right)} \operatorname{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{e}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{c(a+b*x)} / (d+e*x), x]$

[Out]  $(F^{c(a-(b*d)/e)} * \operatorname{ExpIntegralEi}[(b*c*(d+e*x)*\operatorname{Log}[F])/e]) / e$

Rule 2209

$\operatorname{Int}[(F_)^{c((g_.) * ((e_.) + (f_.) * (x_)))} / ((c_.) + (d_.) * (x_)), x\_Symbol] \rightarrow \operatorname{Simp}[(F^{c(g*(e - c*(f/d)))/d} * \operatorname{ExpIntegralEi}[f*g*(c + d*x)*(Log[F]/d)], x] / ; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\amp; \operatorname{!TrueQ}\{ \$UseGamma \}$

Rubi steps

$$\int \frac{F^{c(a+bx)}}{d+ex} dx = \frac{F^{c\left(a-\frac{bd}{e}\right)} \operatorname{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{e}$$

Mathematica [A]

time = 0.05, size = 31, normalized size = 1.00

$$\frac{F^{c\left(a-\frac{bd}{e}\right)} \operatorname{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{e}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}[F^{c(a+b*x)} / (d+e*x), x]$

[Out]  $(F^{c(a - (b*d)/e)}) * \text{ExpIntegralEi}[(b*c*(d + e*x)*\text{Log}[F])/e]/e$

**Maple [A]**

time = 0.07, size = 56, normalized size = 1.81

method	result	size
risch	$-\frac{F^{\frac{c(ae-bd)}{e}} \text{expIntegral}\left(1, -bcx \ln(F) - ca \ln(F) - \frac{-\ln(F)ace + \ln(F)bcd}{e}\right)}{e}$	56

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a))/(e*x+d),x,method=_RETURNVERBOSE)`

[Out]  $-1/e * F^{c(a*e-b*d)/e} * \text{Ei}\left(1, -b*c*x*\ln(F) - c*a*\ln(F) - (-\ln(F)*a*c*e + \ln(F)*b*c*d)/e\right)$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))/(e*x+d),x, algorithm="maxima")`

[Out] `integrate(F^((b*x + a)*c)/(x*e + d), x)`

**Fricas [A]**

time = 0.37, size = 38, normalized size = 1.23

$$\frac{\text{Ei}((bcxe + bcd)e^{(-1)} \log(F)) e^{(-1)}}{F^{(bcd-ace)e^{(-1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))/(e*x+d),x, algorithm="fricas")`

[Out]  $\text{Ei}((b*c*x*e + b*c*d)*e^{(-1)}*\log(F))*e^{(-1)}/F^{((b*c*d - a*c*e)*e^{(-1)})}$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{c(a+bx)}}{d+ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(c*(b*x+a))/(e*x+d),x)`

[Out] `Integral(F**(c*(a + b*x))/(d + e*x), x)`

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d),x, algorithm="giac")

[Out] integrate(F^((b\*x + a)\*c)/(x\*e + d), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{F^{c(a+bx)}}{d+ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))/(d + e\*x),x)

[Out] int(F^(c\*(a + b\*x))/(d + e\*x), x)

### 3.8 $\int \frac{F^{c(a+bx)}}{(d+ex)^2} dx$

**Optimal.** Leaf size=57

$$-\frac{F^{c(a+bx)}}{e(d+ex)} + \frac{bcF^{c\left(a-\frac{bd}{e}\right)} \operatorname{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log(F)}{e^2}$$

[Out]  $-F^{c(bx+a)}/e/(ex+d)+b*c*F^{c(a-b*d/e)}*Ei(b*c*(ex+d)*ln(F)/e)*ln(F)/e^2$

**Rubi [A]**

time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2208, 2209}

$$\frac{bc \log(F) F^{c\left(a-\frac{bd}{e}\right)} \operatorname{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{e^2} - \frac{F^{c(a+bx)}}{e(d+ex)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{c(a+bx)}/(d+ex)^2, x]$

[Out]  $-(F^{c(a+bx)}/(e*(d+ex))) + (b*c*F^{c(a-(b*d)/e)}*ExpIntegralEi[(b*c*(d+ex)*Log[F])/e]*Log[F])/e^2$

Rule 2208

$\operatorname{Int}[(b_*)*(F_*)^{((g_*)*((e_*) + (f_*)*(x_)))})^{(n_*)}*((c_*) + (d_*)*(x_))^{(m_*)}, x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m+1)}*((b*F^{g*(e+f*x)})^n/(d*(m+1))), x] - \operatorname{Dist}[f*g*n*(\operatorname{Log}[F]/(d*(m+1))), \operatorname{Int}[(c + d*x)^{(m+1)}*(b*F^{g*(e+f*x)})^n, x], x] /; \operatorname{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{IntegerQ}[2*m] \&\& \operatorname{!TrueQ}[\$UseGamma]$

Rule 2209

$\operatorname{Int}[(F_*)^{((g_*)*((e_*) + (f_*)*(x_)))}/((c_*) + (d_*)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[(F^{g*(e-c*(f/d))}/d)*ExpIntegralEi[f*g*(c+d*x)*(\operatorname{Log}[F]/d)], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& \operatorname{!TrueQ}[\$UseGamma]$

Rubi steps

$$\int \frac{F^{c(a+bx)}}{(d+ex)^2} dx = -\frac{F^{c(a+bx)}}{e(d+ex)} + \frac{(bc \log(F)) \int \frac{F^{c(a+bx)}}{d+ex} dx}{e}$$

$$= -\frac{F^{c(a+bx)}}{e(d+ex)} + \frac{bcF^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log(F)}{e^2}$$

**Mathematica [A]**

time = 0.10, size = 55, normalized size = 0.96

$$\frac{F^{ac} \left( -\frac{eF^{bcx}}{d+ex} + bcF^{-\frac{bcd}{e}} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log(F) \right)}{e^2}$$

Antiderivative was successfully verified.

`[In] Integrate[F^(c*(a + b*x))/(d + e*x)^2, x]``[Out] (F^(a*c)*(-(eF^(b*c*x))/(d + e*x)) + (b*c*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F])/F^((b*c*d)/e))/e^2`**Maple [A]**

time = 0.08, size = 99, normalized size = 1.74

method	result	size
risch	$-\frac{cb \ln(F) F^{bcx} F^{ca}}{e^2 \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)} - \frac{cb \ln(F) F^{\frac{c(ae-bd)}{e}} \text{expIntegral}\left(1, -bcx \ln(F) - ca \ln(F) - \frac{-\ln(F) ace + \ln(F) bcd}{e}\right)}{e^2}$	99

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(F^(c*(b*x+a))/(e*x+d)^2, x, method=_RETURNVERBOSE)``[Out] -c*b*ln(F)/e^2*F^(b*c*x)*F^(c*a)/(b*c*x*ln(F)+1/e*ln(F)*b*c*d)-c*b*ln(F)/e^2*F^(c*(a*e-b*d)/e)*Ei(1, -b*c*x*ln(F)-c*a*ln(F)-(-ln(F)*a*c*e+ln(F)*b*c*d)/e)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(c*(b*x+a))/(e*x+d)^2, x, algorithm="maxima")``[Out] integrate(F^((b*x + a)*c)/(x*e + d)^2, x)`



**Fricas** [A]

time = 0.41, size = 77, normalized size = 1.35

$$\frac{F^{bcx+ac} e^{-\frac{(bcxe+bcd)\text{Ei}((bcxe+bcd)e^{(-1)} \log(F)) \log(F)}{F^{(bcd-ace)e^{(-1)}}}}}{xe^3 + de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^2,x, algorithm="fricas")

[Out] -(F^(b\*c\*x + a\*c)\*e - (b\*c\*x\*e + b\*c\*d)\*Ei((b\*c\*x\*e + b\*c\*d)\*e^(-1)\*log(F)) \*log(F)/F^((b\*c\*d - a\*c\*e)\*e^(-1)))/(x\*e^3 + d\*e^2)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{c(a+bx)}}{(d+ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))/(e\*x+d)\*\*2,x)

[Out] Integral(F\*\*(c\*(a + b\*x))/(d + e\*x)\*\*2, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^2,x, algorithm="giac")

[Out] integrate(F^((b\*x + a)\*c)/(x\*e + d)^2, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{F^{c(a+bx)}}{(d+ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))/(d + e\*x)^2,x)

[Out] int(F^(c\*(a + b\*x))/(d + e\*x)^2, x)

### 3.9 $\int \frac{F^{c(a+bx)}}{(d+ex)^3} dx$

**Optimal.** Leaf size=95

$$-\frac{F^{c(a+bx)}}{2e(d+ex)^2} - \frac{bcF^{c(a+bx)} \log(F)}{2e^2(d+ex)} + \frac{b^2c^2F^{c(a-\frac{bd}{e})} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log^2(F)}{2e^3}$$

[Out]  $-1/2 * F^{(c*(b*x+a))} / e / (e*x+d)^2 - 1/2 * b*c * F^{(c*(b*x+a))} * \ln(F) / e^2 / (e*x+d) + 1/2 * b^2 * c^2 * F^{(c*(a-b*d/e))} * \text{Ei}(b*c*(e*x+d) * \ln(F) / e) * \ln(F)^2 / e^3$

**Rubi [A]**

time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2208, 2209}

$$\frac{b^2c^2 \log^2(F) F^{c(a-\frac{bd}{e})} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{2e^3} - \frac{bc \log(F) F^{c(a+bx)}}{2e^2(d+ex)} - \frac{F^{c(a+bx)}}{2e(d+ex)^2}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))/(d + e\*x)^3, x]

[Out]  $-1/2 * F^{(c*(a + b*x))} / (e*(d + e*x)^2) - (b*c * F^{(c*(a + b*x))} * \text{Log}[F]) / (2 * e^2 * (d + e*x)) + (b^2 * c^2 * F^{(c*(a - (b*d)/e)}) * \text{ExpIntegralEi}[(b*c*(d + e*x) * \text{Log}[F]) / e] * \text{Log}[F]^2) / (2 * e^3)$

Rule 2208

```
Int[((b._)*(F_)^((g._)*((e._) + (f._)*(x_))))^(n._)*((c._) + (d._)*(x_))^(m
_), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b * F^(g*(e + f*x)))^n / (d*(m + 1)))
, x] - Dist[f*g*n*(Log[F] / (d*(m + 1))), Int[(c + d*x)^(m + 1)*(b * F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g._)*((e._) + (f._)*(x_))) / ((c._) + (d._)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - c*(f/d))) / d) * ExpIntegralEi[f*g*(c + d*x) * (Log[F] / d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rubi steps

$$\begin{aligned}
\int \frac{F^{c(a+bx)}}{(d+ex)^3} dx &= -\frac{F^{c(a+bx)}}{2e(d+ex)^2} + \frac{(bc \log(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^2} dx}{2e} \\
&= -\frac{F^{c(a+bx)}}{2e(d+ex)^2} - \frac{bcF^{c(a+bx)} \log(F)}{2e^2(d+ex)} + \frac{(b^2c^2 \log^2(F)) \int \frac{F^{c(a+bx)}}{d+ex} dx}{2e^2} \\
&= -\frac{F^{c(a+bx)}}{2e(d+ex)^2} - \frac{bcF^{c(a+bx)} \log(F)}{2e^2(d+ex)} + \frac{b^2c^2 F^{c(a-\frac{bd}{e})} \text{Ei}\left(\frac{bc(d+ex) \log(F)}{e}\right) \log^2(F)}{2e^3}
\end{aligned}$$

**Mathematica [A]**

time = 0.14, size = 88, normalized size = 0.93

$$\frac{F^{c(a-\frac{bd}{e})} \left( b^2c^2(d+ex)^2 \text{Ei}\left(\frac{bc(d+ex) \log(F)}{e}\right) \log^2(F) - eF^{\frac{bc(d+ex)}{e}} (e + bc(d+ex) \log(F)) \right)}{2e^3(d+ex)^2}$$

Antiderivative was successfully verified.

`[In] Integrate[F^(c*(a + b*x))/(d + e*x)^3,x]`

```
[Out] (F^(c*(a - (b*d)/e))*(b^2*c^2*(d + e*x)^2*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F]^2 - eF^((b*c*(d + e*x))/e)*(e + b*c*(d + e*x)*Log[F]))/(2*e^3*(d + e*x)^2)
```

**Maple [A]**

time = 0.07, size = 155, normalized size = 1.63

method	result
risch	$ -\frac{c^2b^2 \ln(F)^2 F^{bcx} F^{ca}}{2e^3 \left( bcx \ln(F) + \frac{\ln(F)bcd}{e} \right)^2} - \frac{c^2b^2 \ln(F)^2 F^{bcx} F^{ca}}{2e^3 \left( bcx \ln(F) + \frac{\ln(F)bcd}{e} \right)} - \frac{c^2b^2 \ln(F)^2 F^{\frac{c(ae-bd)}{e}} \text{expIntegral}\left(1, -bcx \ln(F) - ca \ln(F) - \frac{-\ln(F)}{e}\right)}{2e^3} $

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(F^(c*(b*x+a))/(e*x+d)^3,x,method=_RETURNVERBOSE)`

```
[Out] -1/2*c^2*b^2*ln(F)^2/e^3*F^(b*c*x)*F^(c*a)/(b*c*x*ln(F)+1/e*ln(F)*b*c*d)^2-1/2*c^2*b^2*ln(F)^2/e^3*F^(b*c*x)*F^(c*a)/(b*c*x*ln(F)+1/e*ln(F)*b*c*d)-1/2*c^2*b^2*ln(F)^2/e^3*F^(c*(a*e-b*d)/e)*Ei(1,-b*c*x*ln(F)-c*a*ln(F)-(-ln(F)*a*c*e+ln(F)*b*c*d)/e)
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^3,x, algorithm="maxima")

[Out] integrate(F^((b\*x + a)\*c)/(x\*e + d)^3, x)

**Fricas** [A]

time = 0.36, size = 130, normalized size = 1.37

$$\frac{(b^2c^2x^2e^2+2b^2c^2dxe+b^2c^2d^2)\text{Ei}((bcxe+bcd)e^{(-1)}\log(F))\log(F)^2}{F^{(bcd-ace)e^{(-1)}}} - ((bcxe^2 + bcde)\log(F) + e^2)F^{bcx+ac}$$


---


$$2(x^2e^5 + 2dxe^4 + d^2e^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^3,x, algorithm="fricas")

[Out] 1/2\*((b^2\*c^2\*x^2\*e^2 + 2\*b^2\*c^2\*d\*x\*e + b^2\*c^2\*d^2)\*Ei((b\*c\*x\*e + b\*c\*d)\*e^(-1)\*log(F))\*log(F)^2/F^((b\*c\*d - a\*c\*e)\*e^(-1)) - ((b\*c\*x\*e^2 + b\*c\*d\*e)\*log(F) + e^2)\*F^(b\*c\*x + a\*c))/(x^2\*e^5 + 2\*d\*x\*e^4 + d^2\*e^3)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{c(a+bx)}}{(d+ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))/(e\*x+d)\*\*3,x)

[Out] Integral(F\*\*(c\*(a + b\*x))/(d + e\*x)\*\*3, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^3,x, algorithm="giac")

[Out] integrate(F^((b\*x + a)\*c)/(x\*e + d)^3, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c(a+bx)}}{(d+ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))/(d + e\*x)^3,x)

[Out] int(F^(c\*(a + b\*x))/(d + e\*x)^3, x)

### 3.10 $\int \frac{F^{c(a+bx)}}{(d+ex)^4} dx$

**Optimal.** Leaf size=128

$$-\frac{F^{c(a+bx)}}{3e(d+ex)^3} - \frac{bcF^{c(a+bx)} \log(F)}{6e^2(d+ex)^2} - \frac{b^2c^2F^{c(a+bx)} \log^2(F)}{6e^3(d+ex)} + \frac{b^3c^3F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log^3(F)}{6e^4}$$

[Out]  $-1/3 * F^{(c*(b*x+a))} / e / (e*x+d)^3 - 1/6 * b*c * F^{(c*(b*x+a))} * \ln(F) / e^2 / (e*x+d)^2 - 1/6 * b^2 * c^2 * F^{(c*(b*x+a))} * \ln(F)^2 / e^3 / (e*x+d) + 1/6 * b^3 * c^3 * F^{(c*(a-b*d/e))} * \text{Ei}(b*c*(e*x+d)*\ln(F)/e) * \ln(F)^3 / e^4$

**Rubi [A]**

time = 0.07, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2208, 2209}

$$\frac{b^3c^3 \log^3(F) F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{6e^4} - \frac{b^2c^2 \log^2(F) F^{c(a+bx)}}{6e^3(d+ex)} - \frac{bc \log(F) F^{c(a+bx)}}{6e^2(d+ex)^2} - \frac{F^{c(a+bx)}}{3e(d+ex)^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{(c*(a + b*x))} / (d + e*x)^4, x]$

[Out]  $-1/3 * F^{(c*(a + b*x))} / (e*(d + e*x)^3) - (b*c * F^{(c*(a + b*x))} * \text{Log}[F]) / (6 * e^2 * (d + e*x)^2) - (b^2 * c^2 * F^{(c*(a + b*x))} * \text{Log}[F]^2) / (6 * e^3 * (d + e*x)) + (b^3 * c^3 * F^{(c*(a - (b*d)/e)}) * \text{ExpIntegralEi}[(b*c*(d + e*x)*\text{Log}[F])/e] * \text{Log}[F]^3) / (6 * e^4)$

Rule 2208

$\text{Int}[(b_*) * (F_*)^{((g_*) * ((e_*) + (f_*) * (x_*)))^n) * ((c_*) + (d_*) * (x_*))^{(m_*)}, x\_Symbol] :> \text{Simp}[(c + d*x)^{(m+1)} * ((b * F^{(g*(e + f*x))})^n / (d * (m+1))) , x] - \text{Dist}[f * g * n * (\text{Log}[F] / (d * (m+1))) , \text{Int}[(c + d*x)^{(m+1)} * (b * F^{(g*(e + f*x))})^n, x], x] /; \text{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \text{LtQ}[m, -1] \&\& \text{IntegerQ}[2*m] \&\& !\text{TrueQ}[\$UseGamma]$

Rule 2209

$\text{Int}[(F_*)^{((g_*) * ((e_*) + (f_*) * (x_*))) / ((c_*) + (d_*) * (x_*))}, x\_Symbol] :> \text{Simp}[(F^{(g*(e - c*(f/d))}) / d) * \text{ExpIntegralEi}[f * g * (c + d*x) * (\text{Log}[F] / d)], x] /; \text{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\text{TrueQ}[\$UseGamma]$

Rubi steps

$$\begin{aligned}
\int \frac{F^{c(a+bx)}}{(d+ex)^4} dx &= -\frac{F^{c(a+bx)}}{3e(d+ex)^3} + \frac{(bc \log(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^3} dx}{3e} \\
&= -\frac{F^{c(a+bx)}}{3e(d+ex)^3} - \frac{bcF^{c(a+bx)} \log(F)}{6e^2(d+ex)^2} + \frac{(b^2c^2 \log^2(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^2} dx}{6e^2} \\
&= -\frac{F^{c(a+bx)}}{3e(d+ex)^3} - \frac{bcF^{c(a+bx)} \log(F)}{6e^2(d+ex)^2} - \frac{b^2c^2F^{c(a+bx)} \log^2(F)}{6e^3(d+ex)} + \frac{(b^3c^3 \log^3(F)) \int \frac{F^{c(a+bx)}}{d+ex} dx}{6e^3} \\
&= -\frac{F^{c(a+bx)}}{3e(d+ex)^3} - \frac{bcF^{c(a+bx)} \log(F)}{6e^2(d+ex)^2} - \frac{b^2c^2F^{c(a+bx)} \log^2(F)}{6e^3(d+ex)} + \frac{b^3c^3F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex) \log(F)}{e}\right)}{6e^4}
\end{aligned}$$

**Mathematica [A]**

time = 0.17, size = 99, normalized size = 0.77

$$\frac{F^{ac} \left( b^3 c^3 F^{-\frac{bcd}{e}} \text{Ei}\left(\frac{bc(d+ex) \log(F)}{e}\right) \log^3(F) - \frac{eF^{bcx} (2e^2 + bce(d+ex) \log(F) + b^2c^2(d+ex)^2 \log^2(F))}{(d+ex)^3} \right)}{6e^4}$$

Antiderivative was successfully verified.

`[In] Integrate[F^(c*(a + b*x))/(d + e*x)^4, x]`

```
[Out] (F^(a*c)*((b^3*c^3*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F]^3)/F^((b*c*d)/e) - (e*F^(b*c*x)*(2*e^2 + b*c*e*(d + e*x)*Log[F] + b^2*c^2*(d + e*x)^2*Log[F]^2))/(d + e*x)^3))/(6*e^4)
```

**Maple [A]**

time = 0.07, size = 199, normalized size = 1.55

method	result
risch	$ -\frac{c^3 b^3 \ln(F)^3 F^{bcx} F^{ca}}{3e^4 \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^3} - \frac{c^3 b^3 \ln(F)^3 F^{bcx} F^{ca}}{6e^4 \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^2} - \frac{c^3 b^3 \ln(F)^3 F^{bcx} F^{ca}}{6e^4 \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)} - \frac{c^3 b^3 \ln(F)^3 F^{\frac{c(ae-bd)}{e}} \text{expIntegral}(1, \dots)}{6e^4} $

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(F^(c*(b*x+a))/(e*x+d)^4, x, method=_RETURNVERBOSE)`

```
[Out] -1/3*c^3*b^3*ln(F)^3/e^4*F^(b*c*x)*F^(c*a)/(b*c*x*ln(F)+1/e*ln(F)*b*c*d)^3-1/6*c^3*b^3*ln(F)^3/e^4*F^(b*c*x)*F^(c*a)/(b*c*x*ln(F)+1/e*ln(F)*b*c*d)^2-1/6*c^3*b^3*ln(F)^3/e^4*F^(b*c*x)*F^(c*a)/(b*c*x*ln(F)+1/e*ln(F)*b*c*d)-1/6*c^3*b^3*ln(F)^3/e^4*F^(c*(a*e-b*d)/e)*Ei(1, -b*c*x*ln(F)-c*a*ln(F)-(-ln(F)*a*c*e+ln(F)*b*c*d)/e)
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(F^(c\*(b\*x+a))/(e\*x+d)^4,x, algorithm="maxima")**[Out]** integrate(F^((b\*x + a)\*c)/(x\*e + d)^4, x)**Fricas [A]**

time = 0.35, size = 200, normalized size = 1.56

$$\frac{(b^3c^3x^3e^3+3b^3c^2dx^2e^2+3b^3c^2d^2xe+b^3c^2d^3)Ei((bcxe+bcd)e^{(-1)}\log(F))\log(F)^3 - ((b^2c^2x^2e^3+2b^2c^2dxe^2+b^2c^2d^2e)\log(F)^2+(bcxe^3+bcd^2)\log(F)+2e^3)F^{bcx+ac}}{F^{(bcd-ace)e^{(-1)}}} \quad \frac{6(x^3e^7+3dx^2e^6+3d^2xe^5+d^3e^4)}{6(x^3e^7+3dx^2e^6+3d^2xe^5+d^3e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(F^(c\*(b\*x+a))/(e\*x+d)^4,x, algorithm="fricas")

**[Out]** 1/6\*((b^3\*c^3\*x^3\*e^3 + 3\*b^3\*c^3\*d\*x^2\*e^2 + 3\*b^3\*c^3\*d^2\*x\*e + b^3\*c^3\*d^3)\*Ei((b\*c\*x\*e + b\*c\*d)\*e^(-1)\*log(F))\*log(F)^3/F^((b\*c\*d - a\*c\*e)\*e^(-1)) - ((b^2\*c^2\*x^2\*e^3 + 2\*b^2\*c^2\*d\*x\*e^2 + b^2\*c^2\*d^2\*e)\*log(F)^2 + (b\*c\*x\*e^3 + b\*c\*d\*e^2)\*log(F) + 2\*e^3)\*F^(b\*c\*x + a\*c))/(x^3\*e^7 + 3\*d\*x^2\*e^6 + 3\*d^2\*x\*e^5 + d^3\*e^4)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{c(a+bx)}}{(d+ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(F\*\*(c\*(b\*x+a))/(e\*x+d)\*\*4,x)**[Out]** Integral(F\*\*(c\*(a + b\*x))/(d + e\*x)\*\*4, x)**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(F^(c\*(b\*x+a))/(e\*x+d)^4,x, algorithm="giac")**[Out]** integrate(F^((b\*x + a)\*c)/(x\*e + d)^4, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c(a+bx)}}{(d+ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))/(d + e\*x)^4,x)

[Out] int(F^(c\*(a + b\*x))/(d + e\*x)^4, x)



### 3.11 $\int \frac{F^{c(a+bx)}}{(d+ex)^5} dx$

**Optimal.** Leaf size=161

$$\frac{F^{c(a+bx)}}{4e(d+ex)^4} - \frac{bcF^{c(a+bx)} \log(F)}{12e^2(d+ex)^3} - \frac{b^2c^2F^{c(a+bx)} \log^2(F)}{24e^3(d+ex)^2} - \frac{b^3c^3F^{c(a+bx)} \log^3(F)}{24e^4(d+ex)} + \frac{b^4c^4F^{c(a-\frac{bd}{e})} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{24e^5}$$

[Out]  $-1/4 * F^{(c*(b*x+a))} / e / (e*x+d)^4 - 1/12 * b*c * F^{(c*(b*x+a))} * \ln(F) / e^2 / (e*x+d)^3 - 1/24 * b^2 * c^2 * F^{(c*(b*x+a))} * \ln(F)^2 / e^3 / (e*x+d)^2 - 1/24 * b^3 * c^3 * F^{(c*(b*x+a))} * \ln(F)^3 / e^4 / (e*x+d) + 1/24 * b^4 * c^4 * F^{(c*(a-b*d/e))} * \text{Ei}(b*c*(e*x+d)*\ln(F)/e) * \ln(F)^4 / e^5$

**Rubi [A]**

time = 0.09, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2208, 2209}

$$\frac{b^4c^4 \log^4(F) F^{c(a-\frac{bd}{e})} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{24e^5} - \frac{b^3c^3 \log^3(F) F^{c(a+bx)}}{24e^4(d+ex)} - \frac{b^2c^2 \log^2(F) F^{c(a+bx)}}{24e^3(d+ex)^2} - \frac{bc \log(F) F^{c(a+bx)}}{12e^2(d+ex)^3} - \frac{F^{c(a+bx)}}{4e(d+ex)^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c*(a + b*x)} / (d + e*x)^5, x]$

[Out]  $-1/4 * F^{(c*(a + b*x))} / (e*(d + e*x)^4) - (b*c * F^{(c*(a + b*x))} * \text{Log}[F]) / (12 * e^2 * (d + e*x)^3) - (b^2 * c^2 * F^{(c*(a + b*x))} * \text{Log}[F]^2) / (24 * e^3 * (d + e*x)^2) - (b^3 * c^3 * F^{(c*(a + b*x))} * \text{Log}[F]^3) / (24 * e^4 * (d + e*x)) + (b^4 * c^4 * F^{(c*(a - (b*d)/e)}) * \text{ExpIntegralEi}[(b*c*(d + e*x)*\text{Log}[F])/e] * \text{Log}[F]^4) / (24 * e^5)$

Rule 2208

$\text{Int}[(b_*) * (F_*)^{((g_*) * ((e_*) + (f_*) * (x_*)))^{(n_*) * ((c_*) + (d_*) * (x_*))^{(m_*)}, x\_Symbol] :> \text{Simp}[(c + d*x)^{(m + 1)} * ((b * F^{(g*(e + f*x))})^n / (d * (m + 1))), x] - \text{Dist}[f * g * n * (\text{Log}[F] / (d * (m + 1))), \text{Int}[(c + d*x)^{(m + 1)} * (b * F^{(g*(e + f*x))})^n, x], x] /; \text{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \text{LtQ}[m, -1] \&\& \text{IntegerQ}[2*m] \&\& !\text{TrueQ}[\$UseGamma]$

Rule 2209

$\text{Int}[(F_*)^{((g_*) * ((e_*) + (f_*) * (x_*))) / ((c_*) + (d_*) * (x_*))}, x\_Symbol] :> \text{Simp}[(F^{(g*(e - c*(f/d))}) / d * \text{ExpIntegralEi}[f * g * (c + d*x) * (\text{Log}[F] / d)], x] /; \text{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\text{TrueQ}[\$UseGamma]$

Rubi steps

$$\begin{aligned}
\int \frac{F^{c(a+bx)}}{(d+ex)^5} dx &= -\frac{F^{c(a+bx)}}{4e(d+ex)^4} + \frac{(bc \log(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^4} dx}{4e} \\
&= -\frac{F^{c(a+bx)}}{4e(d+ex)^4} - \frac{bcF^{c(a+bx)} \log(F)}{12e^2(d+ex)^3} + \frac{(b^2c^2 \log^2(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^3} dx}{12e^2} \\
&= -\frac{F^{c(a+bx)}}{4e(d+ex)^4} - \frac{bcF^{c(a+bx)} \log(F)}{12e^2(d+ex)^3} - \frac{b^2c^2F^{c(a+bx)} \log^2(F)}{24e^3(d+ex)^2} + \frac{(b^3c^3 \log^3(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^2} dx}{24e^3} \\
&= -\frac{F^{c(a+bx)}}{4e(d+ex)^4} - \frac{bcF^{c(a+bx)} \log(F)}{12e^2(d+ex)^3} - \frac{b^2c^2F^{c(a+bx)} \log^2(F)}{24e^3(d+ex)^2} - \frac{b^3c^3F^{c(a+bx)} \log^3(F)}{24e^4(d+ex)} + \frac{(b^4c^4 F^{c(a+bx)}) \int \frac{1}{d+ex} dx}{24e^4} \\
&= -\frac{F^{c(a+bx)}}{4e(d+ex)^4} - \frac{bcF^{c(a+bx)} \log(F)}{12e^2(d+ex)^3} - \frac{b^2c^2F^{c(a+bx)} \log^2(F)}{24e^3(d+ex)^2} - \frac{b^3c^3F^{c(a+bx)} \log^3(F)}{24e^4(d+ex)} + \frac{b^4c^4 F^{c(a+bx)}}{24e^4}
\end{aligned}$$

**Mathematica [A]**

time = 0.15, size = 121, normalized size = 0.75

$$\frac{F^{ac} \left( b^4 c^4 F^{-\frac{bcd}{e}} \text{Ei} \left( \frac{bc(d+ex) \log(F)}{e} \right) \log^4(F) - \frac{e F^{bcx} (6e^3 + 2bce^2(d+ex) \log(F) + b^2c^2e(d+ex)^2 \log^2(F) + b^3c^3(d+ex)^3 \log^3(F))}{(d+ex)^4} \right)}{24e^5}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))/(d + e\*x)^5, x]

[Out] (F^(a\*c)\*((b^4\*c^4\*ExpIntegralEi[(b\*c\*(d + e\*x)\*Log[F])/e]\*Log[F]^4)/F^((b\*c\*d)/e) - (e\*F^(b\*c\*x)\*(6\*e^3 + 2\*b\*c\*e^2\*(d + e\*x)\*Log[F] + b^2\*c^2\*e\*(d + e\*x)^2\*Log[F]^2 + b^3\*c^3\*(d + e\*x)^3\*Log[F]^3))/(d + e\*x)^4))/(24\*e^5)

**Maple [A]**

time = 0.07, size = 243, normalized size = 1.51

method	result
risch	$ -\frac{c^4 b^4 \ln(F)^4 F^{bcx} F^{ca}}{4e^5 \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^4} - \frac{c^4 b^4 \ln(F)^4 F^{bcx} F^{ca}}{12e^5 \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^3} - \frac{c^4 b^4 \ln(F)^4 F^{bcx} F^{ca}}{24e^5 \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^2} - \frac{c^4 b^4 \ln(F)^4 F^{bcx} F^{ca}}{24e^5 \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)} - \frac{c^4 b^4 \ln(F)^4 F^{bcx} F^{ca}}{24e^5} $

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))/(e\*x+d)^5, x, method=\_RETURNVERBOSE)

[Out] -1/4\*c^4\*b^4\*ln(F)^4/e^5\*F^(b\*c\*x)\*F^(c\*a)/(b\*c\*x\*ln(F)+1/e\*ln(F)\*b\*c\*d)^4-1/12\*c^4\*b^4\*ln(F)^4/e^5\*F^(b\*c\*x)\*F^(c\*a)/(b\*c\*x\*ln(F)+1/e\*ln(F)\*b\*c\*d)^3-1/24\*c^4\*b^4\*ln(F)^4/e^5\*F^(b\*c\*x)\*F^(c\*a)/(b\*c\*x\*ln(F)+1/e\*ln(F)\*b\*c\*d)^2-1/24\*c^4\*b^4\*ln(F)^4/e^5\*F^(b\*c\*x)\*F^(c\*a)/(b\*c\*x\*ln(F)+1/e\*ln(F)\*b\*c\*d)-1/24\*c^4\*b^4\*ln(F)^4/e^5\*F^(b\*c\*x)\*F^(c\*a)/(b\*c\*x\*ln(F)+1/e\*ln(F)\*b\*c\*d)-1/24\*c^4\*b^4\*ln(F)^4/e^5\*F^(b\*c\*x)\*F^(c\*a)/(b\*c\*x\*ln(F)+1/e\*ln(F)\*b\*c\*d)

$24*c^4*b^4*\ln(F)^4/e^5*F^(c*(a*e-b*d)/e)*Ei(1,-b*c*x*\ln(F)-c*a*\ln(F)-(-\ln(F))*a*c*e+\ln(F)*b*c*d)/e$

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^5,x, algorithm="maxima")

[Out] integrate(F^((b\*x + a)\*c)/(x\*e + d)^5, x)

**Fricas** [A]

time = 0.40, size = 285, normalized size = 1.77

$$\frac{(b^4e^4x^4+4b^4e^4dx^3+6b^4e^4d^2x^2+4b^4e^4d^3x+4b^4e^4d^4)Ei((bcxe+bd)e^{-1}\log(F))\log(F)^4 - ((b^3c^3x^3e^4 + 3b^3c^3dx^2e^3 + 3b^3c^3d^2xe^2 + b^3c^3d^3e)\log(F)^3 + (b^2c^2x^2e^4 + 2b^2c^2dxe^3 + b^2c^2d^2e^2)\log(F)^2 + 2(bcxe^4 + bcd^3e^3)\log(F) + 6e^4)F^{bcx+ac}}{24(x^4e^9 + 4dx^3e^8 + 6d^2x^2e^7 + 4d^3xe^6 + d^4e^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^5,x, algorithm="fricas")

[Out]  $1/24*((b^4*c^4*x^4*e^4 + 4*b^4*c^4*d*x^3*e^3 + 6*b^4*c^4*d^2*x^2*e^2 + 4*b^4*c^4*d^3*x*e + b^4*c^4*d^4)*Ei((b*c*x*e + b*c*d)*e^{-1}*\log(F))*\log(F)^4/F^{((b*c*d - a*c*e)*e^{-1})} - ((b^3*c^3*x^3*e^4 + 3*b^3*c^3*d*x^2*e^3 + 3*b^3*c^3*d^2*x*e^2 + b^3*c^3*d^3*e)*\log(F)^3 + (b^2*c^2*x^2*e^4 + 2*b^2*c^2*d*x*e^3 + b^2*c^2*d^2*e^2)*\log(F)^2 + 2*(b*c*x*e^4 + b*c*d*e^3)*\log(F) + 6*e^4)*F^{(b*c*x + a*c)})/(x^4*e^9 + 4*d*x^3*e^8 + 6*d^2*x^2*e^7 + 4*d^3*x*e^6 + d^4*e^5)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{c(a+bx)}}{(d+ex)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))/(e\*x+d)\*\*5,x)

[Out] Integral(F\*\*(c\*(a + b\*x))/(d + e\*x)\*\*5, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(x*e + d)^5, x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c(a+bx)}}{(d+ex)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/(d + e*x)^5,x)
```

```
[Out] int(F^(c*(a + b*x))/(d + e*x)^5, x)
```

### 3.12 $\int F^{c(a+bx)}(d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4) dx$

**Optimal.** Leaf size=141

$$\frac{24e^4F^{c(a+bx)}}{b^5c^5\log^5(F)} - \frac{24e^3F^{c(a+bx)}(d+ex)}{b^4c^4\log^4(F)} + \frac{12e^2F^{c(a+bx)}(d+ex)^2}{b^3c^3\log^3(F)} - \frac{4eF^{c(a+bx)}(d+ex)^3}{b^2c^2\log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^4}{bc\log(F)}$$

[Out]  $24e^4F^{c(bx+a)}/b^5/c^5/\ln(F)^5 - 24e^3F^{c(bx+a)}(ex+d)/b^4/c^4/\ln(F)^4 + 12e^2F^{c(bx+a)}(ex+d)^2/b^3/c^3/\ln(F)^3 - 4eF^{c(bx+a)}(ex+d)^3/b^2/c^2/\ln(F)^2 + F^{c(bx+a)}(ex+d)^4/b/c/\ln(F)$

**Rubi [A]**

time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$ ,

Rules used = {2218, 2207, 2225}

$$\frac{24e^4F^{c(a+bx)}}{b^5c^5\log^5(F)} - \frac{24e^3(d+ex)F^{c(a+bx)}}{b^4c^4\log^4(F)} + \frac{12e^2(d+ex)^2F^{c(a+bx)}}{b^3c^3\log^3(F)} - \frac{4e(d+ex)^3F^{c(a+bx)}}{b^2c^2\log^2(F)} + \frac{(d+ex)^4F^{c(a+bx)}}{bc\log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))\*(d^4 + 4\*d^3\*e\*x + 6\*d^2\*e^2\*x^2 + 4\*d\*e^3\*x^3 + e^4\*x^4), x]

[Out]  $(24e^4F^{c(a+bx)})/(b^5c^5\text{Log}[F]^5) - (24e^3F^{c(a+bx)}(d+ex))/(b^4c^4\text{Log}[F]^4) + (12e^2F^{c(a+bx)}(d+ex)^2)/(b^3c^3\text{Log}[F]^3) - (4eF^{c(a+bx)}(d+ex)^3)/(b^2c^2\text{Log}[F]^2) + (F^{c(a+bx)}(d+ex)^4)/(bc\text{Log}[F])$

Rule 2207

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(c + d\*x)^m\*((b\*F^(g\*(e + f\*x)))^n/(f\*g\*n\*Log[F])), x] - Dist[d\*(m/(f\*g\*n\*Log[F])), Int[(c + d\*x)^(m-1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

Rule 2218

Int[((a\_.) + (b\_.)\*(F\_)^((g\_.)\*(v\_)))^(n\_.)]^(p\_.)\*(u\_)^(m\_.), x\_Symbol] := Int[NormalizePowerOfLinear[u, x]^m\*(a + b\*(F^(g\*ExpandToSum[v, x]))^n)^p, x] /; FreeQ[{F, a, b, g, n, p}, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && !(LinearMatchQ[v, x] && PowerOfLinearMatchQ[u, x]) && IntegerQ[m]

Rule 2225

Int[(F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

### Rubi steps

$$\begin{aligned}
\int F^{c(a+bx)}(d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4) dx &= \int F^{c(a+bx)}(d+ex)^4 dx \\
&= \frac{F^{c(a+bx)}(d+ex)^4}{bc \log(F)} - \frac{(4e) \int F^{c(a+bx)}(d+ex)^3 dx}{bc \log(F)} \\
&= -\frac{4eF^{c(a+bx)}(d+ex)^3}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^4}{bc \log(F)} + \frac{(12e^2)}{b^3c^3 \log^3(F)} \\
&= \frac{12e^2F^{c(a+bx)}(d+ex)^2}{b^3c^3 \log^3(F)} - \frac{4eF^{c(a+bx)}(d+ex)^3}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^4}{bc \log(F)} \\
&= -\frac{24e^3F^{c(a+bx)}(d+ex)}{b^4c^4 \log^4(F)} + \frac{12e^2F^{c(a+bx)}(d+ex)^2}{b^3c^3 \log^3(F)} \\
&= \frac{24e^4F^{c(a+bx)}}{b^5c^5 \log^5(F)} - \frac{24e^3F^{c(a+bx)}(d+ex)}{b^4c^4 \log^4(F)} + \frac{12e^2F^{c(a+bx)}}{b^3c^3 \log^3(F)}
\end{aligned}$$

### Mathematica [A]

time = 0.02, size = 100, normalized size = 0.71

$$\frac{F^{c(a+bx)}(24e^4 - 24bce^3(d+ex)\log(F) + 12b^2c^2e^2(d+ex)^2\log^2(F) - 4b^3c^3e(d+ex)^3\log^3(F) + b^4c^4(d+ex)^4\log^4(F))}{b^5c^5\log^5(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d^4 + 4\*d^3\*e\*x + 6\*d^2\*e^2\*x^2 + 4\*d\*e^3\*x^3 + e^4\*x^4), x]

[Out] (F^(c\*(a + b\*x))\*(24\*e^4 - 24\*b\*c\*e^3\*(d + e\*x)\*Log[F] + 12\*b^2\*c^2\*e^2\*(d + e\*x)^2\*Log[F]^2 - 4\*b^3\*c^3\*e\*(d + e\*x)^3\*Log[F]^3 + b^4\*c^4\*(d + e\*x)^4\*Log[F]^4))/(b^5\*c^5\*Log[F]^5)

### Maple [A]

time = 0.02, size = 260, normalized size = 1.84

method	result
gospers	$\frac{(e^4x^4c^4b^4 \ln(F)^4 + 4 \ln(F)^4 b^4 c^4 d e^3 x^3 + 6 \ln(F)^4 b^4 c^4 d^2 e^2 x^2 + 4 \ln(F)^4 b^4 c^4 d^3 e x + \ln(F)^4 b^4 c^4 d^4 - 4 \ln(F)^3 b^3 c^3 e^4 x^3 - 12 \ln(F)^3 b^3 c^3 d e^3 x^2 + 4 \ln(F)^3 b^3 c^3 d^2 e^2 x + 4 \ln(F)^3 b^3 c^3 d^3 e - 4 \ln(F)^3 b^3 c^3 d^4 + 4 \ln(F)^2 b^2 c^2 e^4 x^3 + 12 \ln(F)^2 b^2 c^2 d e^3 x^2 + 12 \ln(F)^2 b^2 c^2 d^2 e^2 x + 12 \ln(F)^2 b^2 c^2 d^3 e - 4 \ln(F)^2 b^2 c^2 d^4 + 4 \ln(F) b c e^4 x^3 + 12 \ln(F) b c d e^3 x^2 + 12 \ln(F) b c d^2 e^2 x + 12 \ln(F) b c d^3 e - 4 \ln(F) b c d^4 + 4 e^4 x^4 + 4 d e^3 x^3 + 6 d^2 e^2 x^2 + 4 d^3 e x + d^4) e^{c(bx+a) \ln(F)}}{c^5 b^5 \ln(F)^5} + \frac{e^4 x^4 e^{c(bx+a) \ln(F)}}{c b \ln(F)} + \frac{4e(c^3 b^3 \ln(F)^3)}{c^3 b^3 \ln(F)^3}$
risch	$\frac{(e^4x^4c^4b^4 \ln(F)^4 + 4 \ln(F)^4 b^4 c^4 d e^3 x^3 + 6 \ln(F)^4 b^4 c^4 d^2 e^2 x^2 + 4 \ln(F)^4 b^4 c^4 d^3 e x + \ln(F)^4 b^4 c^4 d^4 - 4 \ln(F)^3 b^3 c^3 e^4 x^3 - 12 \ln(F)^3 b^3 c^3 d e^3 x^2 + 4 \ln(F)^3 b^3 c^3 d^2 e^2 x + 4 \ln(F)^3 b^3 c^3 d^3 e - 4 \ln(F)^3 b^3 c^3 d^4 + 4 \ln(F)^2 b^2 c^2 e^4 x^3 + 12 \ln(F)^2 b^2 c^2 d e^3 x^2 + 12 \ln(F)^2 b^2 c^2 d^2 e^2 x + 12 \ln(F)^2 b^2 c^2 d^3 e - 4 \ln(F)^2 b^2 c^2 d^4 + 4 \ln(F) b c e^4 x^3 + 12 \ln(F) b c d e^3 x^2 + 12 \ln(F) b c d^2 e^2 x + 12 \ln(F) b c d^3 e - 4 \ln(F) b c d^4 + 4 e^4 x^4 + 4 d e^3 x^3 + 6 d^2 e^2 x^2 + 4 d^3 e x + d^4) e^{c(bx+a) \ln(F)}}{c^5 b^5 \ln(F)^5} + \frac{e^4 x^4 e^{c(bx+a) \ln(F)}}{c b \ln(F)} + \frac{4e(c^3 b^3 \ln(F)^3)}{c^3 b^3 \ln(F)^3}$
norman	$\frac{(\ln(F)^4 b^4 c^4 d^4 - 4 \ln(F)^3 b^3 c^3 d^3 e + 12 c^2 b^2 \ln(F)^2 d^2 e^2 - 24 d e^3 c b \ln(F) + 24 e^4) e^{c(bx+a) \ln(F)}}{c^5 b^5 \ln(F)^5} + \frac{e^4 x^4 e^{c(bx+a) \ln(F)}}{c b \ln(F)} + \frac{4e(c^3 b^3 \ln(F)^3)}{c^3 b^3 \ln(F)^3}$

meijerg	$-\frac{F^{ca}e^4 \left( 24 - \frac{(5b^4c^4x^4 \ln(F)^4 - 20b^3c^3x^3 \ln(F)^3 + 60b^2c^2x^2 \ln(F)^2 - 120bcx \ln(F) + 120)e^{bcx \ln(F)}}{5} \right)}{c^5b^5 \ln(F)^5} + \frac{4F^{ca}e^3d \left( 6 - \frac{(-4b^3c^3x^3 \ln(F)^3 + 12b^2c^2x^2 \ln(F)^2 - 12bcx \ln(F) + 120)e^{bcx \ln(F)}}{5} \right)}{c^4}$
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a))*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4),x,method=_RETURNVERBOSE)`

[Out]  $(e^{4*x^4*c^4*b^4*\ln(F)^4+4*\ln(F)^4*b^4*c^4*d*e^3*x^3+6*\ln(F)^4*b^4*c^4*d^2*e^2*x^2+4*\ln(F)^4*b^4*c^4*d^3*e*x+d^4} * x^{12} * \ln(F)^{12} * b^3 * c^3 * d^2 * e^2 * x^4 * \ln(F)^3 * b^3 * c^3 * e^4 * x^3 - 12 * \ln(F)^3 * b^3 * c^3 * d * e^3 * x^2 - 12 * \ln(F)^3 * b^3 * c^3 * d^2 * e^2 * x - 4 * \ln(F)^3 * b^3 * c^3 * d^3 * e + 12 * \ln(F)^2 * b^2 * c^2 * d * e^4 * x^2 + 24 * \ln(F)^2 * b^2 * c^2 * d * e^3 * x + 12 * c^2 * b^2 * \ln(F)^2 * d^2 * e^2 - 24 * \ln(F) * b * c * e^4 * x - 24 * d * e^3 * c * b * \ln(F) + 24 * e^4) * F^{(c*(b*x+a))} / c^5 / b^5 / \ln(F)^5$

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(143) = 286.

time = 0.51, size = 311, normalized size = 2.21

$$\frac{F^{bcx \ln(F)} - 4(F^{bcx \ln(F)} - F^{bcx \ln(F)})d^{bcx \ln(F)+1}}{bc \log(F)^2} + \frac{6(F^{bcx \ln(F)} - 2F^{bcx \ln(F)} + 2F^{bcx \ln(F)})d^{bcx \ln(F)+2}}{b^2c^2 \log(F)^2} + \frac{4(F^{bcx \ln(F)} - 3F^{bcx \ln(F)} + 3F^{bcx \ln(F)})d^{bcx \ln(F)+3}}{b^3c^3 \log(F)^3} + \frac{(F^{bcx \ln(F)} - 4F^{bcx \ln(F)} + 12F^{bcx \ln(F)} - 24F^{bcx \ln(F)} + 24F^{bcx \ln(F)})d^{bcx \ln(F)+4}}{b^4c^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4),x,algorithm="maxima")`

[Out]  $F^{(b*c*x + a*c)*d^4/(b*c*\log(F))} + 4*(F^{(a*c)*b*c*x*\log(F)} - F^{(a*c)})*d^3*e^{(b*c*x*\log(F) + 1)/(b^2*c^2*\log(F)^2)} + 6*(F^{(a*c)*b^2*c^2*x^2*\log(F)^2} - 2*F^{(a*c)*b*c*x*\log(F)} + 2*F^{(a*c)})*d^2*e^{(b*c*x*\log(F) + 2)/(b^3*c^3*\log(F)^3)} + 4*(F^{(a*c)*b^3*c^3*x^3*\log(F)^3} - 3*F^{(a*c)*b^2*c^2*x^2*\log(F)^2} + 6*F^{(a*c)*b*c*x*\log(F)} - 6*F^{(a*c)})*d*e^{(b*c*x*\log(F) + 3)/(b^4*c^4*\log(F)^4)} + (F^{(a*c)*b^4*c^4*x^4*\log(F)^4} - 4*F^{(a*c)*b^3*c^3*x^3*\log(F)^3} + 12*F^{(a*c)*b^2*c^2*x^2*\log(F)^2} - 24*F^{(a*c)*b*c*x*\log(F)} + 24*F^{(a*c)})*e^{(b*c*x*\log(F) + 4)/(b^5*c^5*\log(F)^5)}$

**Fricas** [A]

time = 0.42, size = 217, normalized size = 1.54

$$\frac{((b^4c^4x^4e^4 + 4b^4c^4d^2x^2e^2 + 6b^4c^4d^2x^2e^2 + 4b^4c^4d^3xe + b^4c^4d^4) \log(F)^4 - 4(b^3c^3x^3e^4 + 3b^3c^3d^2x^2e^2 + 3b^3c^3d^2xe + b^3c^3d^3e) \log(F)^3 + 12(b^2c^2x^2e^4 + 2b^2c^2dxe^2 + b^2c^2d^2e) \log(F)^2 - 24(b^2c^2d^3e + b^2c^2d^4e) \log(F) + 24e^4) F^{bcx+ac}}{b^5c^5 \log(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4),x,algorithm="fricas")`

[Out]  $((b^4*c^4*x^4*e^4 + 4*b^4*c^4*d*x^3*e^3 + 6*b^4*c^4*d^2*x^2*e^2 + 4*b^4*c^4*d^3*x*e + b^4*c^4*d^4)*\log(F)^4 - 4*(b^3*c^3*x^3*e^4 + 3*b^3*c^3*d*x^2*e^3$

$$+ 3*b^3*c^3*d^2*x*e^2 + b^3*c^3*d^3*e)*\log(F)^3 + 12*(b^2*c^2*x^2*e^4 + 2*b^2*c^2*d*x*e^3 + b^2*c^2*d^2*e^2)*\log(F)^2 - 24*(b*c*x*e^4 + b*c*d*e^3)*\log(F) + 24*e^4)*F^{(b*c*x + a*c)}/(b^5*c^5*\log(F)^5)$$

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 350 vs.  $2(139) = 278$ .

time = 0.10, size = 350, normalized size = 2.48

$$\left\{ \begin{array}{l} \frac{F^{(a+b)}(b^5c^5d^5 \log(F)^5 + 4b^4c^4d^4 \log(F)^4 + 6b^3c^3d^3 \log(F)^3 + 4b^2c^2d^2 \log(F)^2 + 4b^5c^5d^5 \log(F)^5 - 4b^4c^4d^4 \log(F)^4 - 12b^3c^3d^3 \log(F)^3 - 12b^2c^2d^2 \log(F)^2 - 4b^5c^5d^5 \log(F)^5 + 12b^4c^4d^4 \log(F)^4 + 12b^3c^3d^3 \log(F)^3 - 24b^2c^2d^2 \log(F)^2 - 24b^5c^5d^5 \log(F)^5 + 24e^4)}{d^4x + 2d^3cx^2 + 2d^2c^2x^3 + d^2x^4 + \frac{c^2x^5}{5}} \end{array} \right. \begin{array}{l} \text{for } b^5c^5 \log(F)^5 \neq 0 \\ \text{otherwise} \end{array}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(e\*\*4\*x\*\*4+4\*d\*e\*\*3\*x\*\*3+6\*d\*\*2\*e\*\*2\*x\*\*2+4\*d\*\*3\*e\*x+d\*\*4),x)

[Out] Piecewise((F\*\*(c\*(a + b\*x))\*(b\*\*4\*c\*\*4\*d\*\*4\*log(F)\*\*4 + 4\*b\*\*4\*c\*\*4\*d\*\*3\*e\*x\*log(F)\*\*4 + 6\*b\*\*4\*c\*\*4\*d\*\*2\*e\*\*2\*x\*\*2\*log(F)\*\*4 + 4\*b\*\*4\*c\*\*4\*d\*e\*\*3\*x\*\*3\*log(F)\*\*4 + b\*\*4\*c\*\*4\*e\*\*4\*x\*\*4\*log(F)\*\*4 - 4\*b\*\*3\*c\*\*3\*d\*\*3\*e\*log(F)\*\*3 - 12\*b\*\*3\*c\*\*3\*d\*\*2\*e\*\*2\*x\*log(F)\*\*3 - 12\*b\*\*3\*c\*\*3\*d\*e\*\*3\*x\*\*2\*log(F)\*\*3 - 4\*b\*\*3\*c\*\*3\*e\*\*4\*x\*\*3\*log(F)\*\*3 + 12\*b\*\*2\*c\*\*2\*d\*\*2\*e\*\*2\*log(F)\*\*2 + 24\*b\*\*2\*c\*\*2\*d\*e\*\*3\*x\*log(F)\*\*2 + 12\*b\*\*2\*c\*\*2\*e\*\*4\*x\*\*2\*log(F)\*\*2 - 24\*b\*c\*d\*e\*\*3\*log(F) - 24\*b\*c\*e\*\*4\*x\*log(F) + 24\*e\*\*4)/(b\*\*5\*c\*\*5\*log(F)\*\*5), Ne(b\*\*5\*c\*\*5\*log(F)\*\*5, 0)), (d\*\*4\*x + 2\*d\*\*3\*e\*x\*\*2 + 2\*d\*\*2\*e\*\*2\*x\*\*3 + d\*e\*\*3\*x\*\*4 + e\*\*4\*x\*\*5/5, True))

**Giac [C]** Result contains complex when optimal does not.

time = 3.66, size = 7859, normalized size = 55.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e^4\*x^4+4\*d\*e^3\*x^3+6\*d^2\*e^2\*x^2+4\*d^3\*e\*x+d^4),x, algorithm="giac")

[Out] -((4\*(pi^3\*b^4\*c^4\*x^4\*log(abs(F))\*sgn(F) - pi\*b^4\*c^4\*x^4\*log(abs(F)))^3\*sgn(F) - pi^3\*b^4\*c^4\*x^4\*log(abs(F)) + pi\*b^4\*c^4\*x^4\*log(abs(F)))^3 - pi^3\*b^3\*c^3\*x^3\*sgn(F) + 3\*pi\*b^3\*c^3\*x^3\*log(abs(F))^2\*sgn(F) + pi^3\*b^3\*c^3\*x^3 - 3\*pi\*b^3\*c^3\*x^3\*log(abs(F))^2 - 6\*pi\*b^2\*c^2\*x^2\*log(abs(F))\*sgn(F) + 6\*pi\*b^2\*c^2\*x^2\*log(abs(F)) + 6\*pi\*b\*c\*x\*sgn(F) - 6\*pi\*b\*c\*x\*(pi^5\*b^5\*c^5\*sgn(F) - 10\*pi^3\*b^5\*c^5\*log(abs(F))^2\*sgn(F) + 5\*pi\*b^5\*c^5\*log(abs(F)))^4\*sgn(F) - pi^5\*b^5\*c^5 + 10\*pi^3\*b^5\*c^5\*log(abs(F))^2 - 5\*pi\*b^5\*c^5\*log(abs(F))^4)/((pi^5\*b^5\*c^5\*sgn(F) - 10\*pi^3\*b^5\*c^5\*log(abs(F))^2\*sgn(F) + 5\*pi\*b^5\*c^5\*log(abs(F))^4\*sgn(F) - pi^5\*b^5\*c^5 + 10\*pi^3\*b^5\*c^5\*log(abs(F)))^2 - 5\*pi\*b^5\*c^5\*log(abs(F))^4)^2 + (5\*pi^4\*b^5\*c^5\*log(abs(F))\*sgn(F) - 10\*pi^2\*b^5\*c^5\*log(abs(F))^3\*sgn(F) - 5\*pi^4\*b^5\*c^5\*log(abs(F)) + 10\*pi^2\*b^5\*c^5\*log(abs(F))^3 - 2\*b^5\*c^5\*log(abs(F))^5)^2) - (pi^4\*b^4\*c^4\*x^4\*s



$$\begin{aligned}
& \text{gn}(F) - 6\pi^2 b^4 c^4 x^4 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^4 b^4 c^4 x^4 + 6\pi^2 \\
& b^4 c^4 x^4 \log(\text{abs}(F))^2 - 2b^4 c^4 x^4 \log(\text{abs}(F))^4 + 12\pi^2 b^3 c^3 x^3 \\
& x^3 \log(\text{abs}(F)) \text{sgn}(F) - 12\pi^2 b^3 c^3 x^3 \log(\text{abs}(F)) + 8b^3 c^3 x^3 \log(\text{abs}(F))^3 \\
& - 12\pi^2 b^2 c^2 x^2 \text{sgn}(F) + 12\pi^2 b^2 c^2 x^2 - 24b^2 c^2 x^2 \log(\text{abs}(F))^2 + 48b \\
& c x \log(\text{abs}(F)) - 48) * (5\pi^4 b^5 c^5 \log(\text{abs}(F)) \text{sgn}(F) - 10\pi^2 b^5 c^5 \log(\text{abs}(F))^3 \text{sgn}(F) \\
& - 5\pi^4 b^5 c^5 \log(\text{abs}(F)) + 10\pi^2 b^5 c^5 \log(\text{abs}(F))^3 - 2b^5 c^5 \log(\text{abs}(F))^5) / ((\pi^5 b^5 c^5 \text{sgn}(F) \\
& - 10\pi^3 b^5 c^5 \log(\text{abs}(F))^2 \text{sgn}(F) + 5\pi b^5 c^5 \log(\text{abs}(F))^4 \text{sgn}(F) - \pi^5 b^5 c^5 + 10\pi^3 b^5 c^5 \log(\text{abs}(F))^2 \\
& - 5\pi b^5 c^5 \log(\text{abs}(F))^4)^2 + (5\pi^4 b^5 c^5 \log(\text{abs}(F)) \text{sgn}(F) - 10\pi^2 b^5 c^5 \log(\text{abs}(F))^3 \text{sgn}(F) \\
& - 5\pi^4 b^5 c^5 \log(\text{abs}(F)) + 10\pi^2 b^5 c^5 \log(\text{abs}(F))^3 - 2b^5 c^5 \log(\text{abs}(F))^5)^2) * \cos(-1/2\pi b c x \text{sgn}(F) + 1/2\pi b c x - 1/2\pi \\
& a c \text{sgn}(F) + 1/2\pi a c) - ((\pi^4 b^4 c^4 x^4 \text{sgn}(F) - 6\pi^2 b^4 c^4 x^4 \log(\text{abs}(F))^2 - 2b^4 c^4 x^4 \log(\text{abs}(F))^4 + 12\pi^2 b^3 c^3 x^3 \log(\text{abs}(F)) \text{sgn}(F) \\
& - 12\pi^2 b^3 c^3 x^3 \log(\text{abs}(F)) + 8b^3 c^3 x^3 \log(\text{abs}(F))^3 - 12\pi^2 b^2 c^2 x^2 \log(\text{abs}(F))^2 + 48b \\
& c x \log(\text{abs}(F)) - 48) * (\pi^5 b^5 c^5 \text{sgn}(F) - 10\pi^3 b^5 c^5 \log(\text{abs}(F))^2 \text{sgn}(F) + 5\pi b^5 c^5 \log(\text{abs}(F))^4 \text{sgn}(F) - \pi^5 b^5 c^5 + 10\pi^3 b^5 c^5 \log(\text{abs}(F))^2 \\
& - 5\pi b^5 c^5 \log(\text{abs}(F))^4) / ((\pi^5 b^5 c^5 \text{sgn}(F) - 10\pi^3 b^5 c^5 \log(\text{abs}(F))^2 \text{sgn}(F) + 5\pi b^5 c^5 \log(\text{abs}(F))^4 \text{sgn}(F) - \pi^5 b^5 c^5 + 10\pi^3 b^5 c^5 \log(\text{abs}(F))^2 \\
& - 5\pi b^5 c^5 \log(\text{abs}(F))^4)^2 + (5\pi^4 b^5 c^5 \log(\text{abs}(F)) \text{sgn}(F) - 10\pi^2 b^5 c^5 \log(\text{abs}(F))^3 \text{sgn}(F) - 5\pi^4 b^5 c^5 \log(\text{abs}(F)) + 10\pi^2 b^5 c^5 \log(\text{abs}(F))^3 \\
& - 2b^5 c^5 \log(\text{abs}(F))^5)^2) + 4 * (\pi^3 b^4 c^4 x^4 \log(\text{abs}(F)) \text{sgn}(F) - \pi b^4 c^4 x^4 \log(\text{abs}(F))^3 \text{sgn}(F) - \pi^3 b^4 c^4 x^4 \log(\text{abs}(F)) + \pi b^4 c^4 x^4 \log(\text{abs}(F))^3 \\
& - \pi^3 b^3 c^3 x^3 \text{sgn}(F) + 3\pi b^3 c^3 x^3 \log(\text{abs}(F))^2 \text{sgn}(F) + \pi^3 b^3 c^3 x^3 - 3\pi b^3 c^3 x^3 \log(\text{abs}(F))^2 - 6\pi b^2 c^2 x^2 \log(\text{abs}(F)) \text{sgn}(F) + 6\pi b^2 c^2 x^2 \log(\text{abs}(F)) \\
& + 6\pi b c x \text{sgn}(F) - 6\pi b c x) * (5\pi^4 b^5 c^5 \log(\text{abs}(F)) \text{sgn}(F) - 10\pi^2 b^5 c^5 \log(\text{abs}(F))^3 \text{sgn}(F) - 5\pi^4 b^5 c^5 \log(\text{abs}(F)) + 10\pi^2 b^5 c^5 \log(\text{abs}(F))^3 - 2b^5 c^5 \log(\text{abs}(F))^5) / ((\pi^5 b^5 c^5 \text{sgn}(F) - 10\pi^3 b^5 c^5 \log(\text{abs}(F))^2 \text{sgn}(F) + 5\pi b^5 c^5 \log(\text{abs}(F))^4 \text{sgn}(F) - \pi^5 b^5 c^5 + 10\pi^3 b^5 c^5 \log(\text{abs}(F))^2 - 5\pi b^5 c^5 \log(\text{abs}(F))^4)^2 + (5\pi^4 b^5 c^5 \log(\text{abs}(F)) \text{sgn}(F) - 10\pi^2 b^5 c^5 \log(\text{abs}(F))^3 \text{sgn}(F) - 5\pi^4 b^5 c^5 \log(\text{abs}(F)) + 10\pi^2 b^5 c^5 \log(\text{abs}(F))^3 - 2b^5 c^5 \log(\text{abs}(F))^5)^2) * \sin(-1/2\pi b c x \text{sgn}(F) + 1/2\pi b c x - 1/2\pi a c \text{sgn}(F) + 1/2\pi a c) * e^{(b c x \log(\text{abs}(F)) + a c \log(\text{abs}(F)) + 4) - 8I * ((I\pi^4 b^4 c^4 x^4 \text{sgn}(F) - 4\pi^3 b^4 c^4 x^4 \log(\text{abs}(F)) \text{sgn}(F) - 6I\pi^2 b^4 c^4 x^4 \log(\text{abs}(F))^2 \text{sgn}(F) + 4\pi b^4 c^4 x^4 \log(\text{abs}(F))^3 \text{sgn}(F) - I\pi^4 b^4 c^4 x^4 + 4\pi^3 b^4 c^4 x^4 \log(\text{abs}(F)) + 6I\pi^2 b^4 c^4 x^4 \log(\text{abs}(F))^2 - 4\pi b^4 c^4 x^4 \log(\text{abs}(F))^3 - 2I b^4 c^4 x^4 \log(\text{abs}(F))^4 + 4\pi^3 b^3 c^3 x^3 \text{sgn}(F) + 12I\pi^2 b^3 c^3 x^3 \log(\text{abs}(F)) \text{sgn}(F) - 12\pi b^3 c^3 x^3 \log(\text{abs}(F))^2 \text{sgn}(F) - 4\pi^3 b^3 c^3 x^3 - 12I\pi^2 b^3 c^3 x^3 \log(\text{abs}(F)) + 12\pi b^3 c^3 x^3 \log(\text{abs}(F))^2 + 8I b^3 c^3 x^3 \log(\text{abs}(F))^3 - 12I\pi^2 b^2 c^2 x^2 \text{sgn}(F)
\end{aligned}$$

$$\begin{aligned} & \operatorname{gn}(F) + 24\pi b^2 c^2 x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) + 12I\pi^2 b^2 c^2 x^2 - 24\pi \\ & i b^2 c^2 x^2 \log(\operatorname{abs}(F)) - 24I b^2 c^2 x^2 \log(\operatorname{abs}(F))^2 - 24\pi i b c x \operatorname{sgn}(F) + 24\pi i b c x \\ & + 48I b c x \log(\operatorname{abs}(F)) - 48I e^{(1/2I\pi i b c x \operatorname{sgn}(F))} - 1/2I\pi i b c x + 1/2I\pi i a c \operatorname{sgn}(F) \\ & - 1/2I\pi i a c / (16I\pi^5 b^5 c^5 \operatorname{sgn}(F) - 80\pi^4 b^5 c^5 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 160I\pi^3 b^5 c^5 \log(\operatorname{abs}(F))^2 \\ & \operatorname{sgn}(F) + 160\pi^2 b^5 c^5 \log(\operatorname{abs}(F))^3 \operatorname{sgn}(F) + 80I\pi b^5 c^5 \log(\operatorname{abs}(F))^4 \operatorname{sgn}(F) - 16I\pi^5 b^5 c^5 \\ & + 80\pi^4 b^5 c^5 \log(\operatorname{abs}(F)) + 160I\pi^3 b^5 c^5 \log(\operatorname{abs}(F))^2 - 160\pi^2 b^5 c^5 \log(\operatorname{abs}(F))^3 - 80I\pi b^5 c^5 \\ & \log(\operatorname{abs}(F))^4 + 32b^5 c^5 \log(\operatorname{abs}(F))^5) - (I\pi^4 b^4 c^4 x^4 \operatorname{sgn}(F) + 4\pi^3 b^4 c^4 x^4 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) \\ & - 6I\pi^4 \dots \end{aligned}$$

**Mupad [B]**

time = 3.49, size = 260, normalized size = 1.84

$$\frac{F^{5+5ix} (b^5 e^{4d} \ln(F)^5 + 4b^5 e^d d^2 x \ln(F)^4 + 6b^5 e^d d^2 e^2 x^2 \ln(F)^3 + 4b^5 e^d d^2 e^3 x^3 \ln(F)^2 + b^5 e^d e^4 x^4 \ln(F) - 4b^5 e^d d^2 e \ln(F)^5 - 12b^5 e^d d^2 e^2 x \ln(F)^4 - 12b^5 e^d d^2 e^3 x^2 \ln(F)^3 - 4b^5 e^d e^2 x^3 \ln(F)^2 + 12b^5 e^d d^2 e^2 \ln(F)^2 + 24b^5 e^d d^2 e x \ln(F)^2 + 12b^5 e^d e^2 x^2 \ln(F)^2 - 24b^5 e^d x \ln(F) + 24e^5)}{b^5 e^5 \ln(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{int}(F^{(c(a + bx))} (d^4 + e^4 x^4 + 4d^2 e^3 x^3 + 6d^2 e^2 x^2 + 4d^3 e x), x)$

[Out]  $(F^{(ac + bcx)} (24e^4 + b^4 c^4 d^4 \log(F)^4 - 24b^3 c^3 e^4 x \log(F) - 4b^3 c^3 d^3 e \log(F)^3 + 12b^2 c^2 d^2 e^2 \log(F)^2 + 12b^2 c^2 e^4 x^2 \log(F)^2 - 4b^3 c^3 e^4 x^3 \log(F)^3 + b^4 c^4 e^4 x^4 \log(F)^4 - 24b^3 c^3 d^3 e \log(F) + 6b^4 c^4 d^2 e^2 x^2 \log(F)^4 + 24b^2 c^2 d^2 e^3 x \log(F)^2 + 4b^4 c^4 d^3 e x \log(F)^4 - 12b^3 c^3 d^2 e^2 x \log(F)^3 - 12b^3 c^3 d^3 e \log(F)^3 + 4b^4 c^4 d^2 e^3 x^3 \log(F)^4)) / (b^5 c^5 \log(F)^5)$

### 3.13 $\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3) dx$

**Optimal.** Leaf size=110

$$-\frac{6e^3 F^{c(a+bx)}}{b^4 c^4 \log^4(F)} + \frac{6e^2 F^{c(a+bx)}(d+ex)}{b^3 c^3 \log^3(F)} - \frac{3e F^{c(a+bx)}(d+ex)^2}{b^2 c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^3}{bc \log(F)}$$

[Out]  $-6e^3 F^{c(a+bx)}/b^4/c^4/\ln(F)^4 + 6e^2 F^{c(a+bx)}(d+ex)/b^3/c^3/\ln(F)^3 - 3e F^{c(a+bx)}(d+ex)^2/b^2/c^2/\ln(F)^2 + F^{c(a+bx)}(d+ex)^3/bc/\ln(F)$

**Rubi [A]**

time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 37,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$ , Rules used = {2218, 2207, 2225}

$$-\frac{6e^3 F^{c(a+bx)}}{b^4 c^4 \log^4(F)} + \frac{6e^2(d+ex)F^{c(a+bx)}}{b^3 c^3 \log^3(F)} - \frac{3e(d+ex)^2 F^{c(a+bx)}}{b^2 c^2 \log^2(F)} + \frac{(d+ex)^3 F^{c(a+bx)}}{bc \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3), x]$

[Out]  $(-6e^3 F^{c(a+bx)})/(b^4 c^4 \text{Log}[F]^4) + (6e^2 F^{c(a+bx)}(d+ex))/(b^3 c^3 \text{Log}[F]^3) - (3e F^{c(a+bx)}(d+ex)^2)/(b^2 c^2 \text{Log}[F]^2) + (F^{c(a+bx)}(d+ex)^3)/(bc \text{Log}[F])$

Rule 2207

$\text{Int}[(b \cdot F^{c(a+bx)})^{(g \cdot (e \cdot x) + f \cdot x)}]^{(n \cdot (c \cdot x) + d \cdot x)^m}, x\_Symbol] :> \text{Simp}[(c + dx)^m \cdot (b \cdot F^{c(a+bx)})^n / (f \cdot g \cdot n \cdot \text{Log}[F])], x] - \text{Dist}[d \cdot m / (f \cdot g \cdot n \cdot \text{Log}[F]), \text{Int}[(c + dx)^{m-1} \cdot (b \cdot F^{c(a+bx)})^n], x], x] /;$  FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !TrueQ[UseGamma]

Rule 2218

$\text{Int}[(a \cdot F^{c(a+bx)}) + (b \cdot F^{c(a+bx)})^{(g \cdot (v \cdot x))}]^{(n \cdot (p \cdot u)^m)}, x\_Symbol] :> \text{Int}[\text{NormalizePowerOfLinear}[u, x]^m \cdot (a + b \cdot (F^{c(a+bx)})^{(g \cdot \text{ExpandToSum}[v, x])})^n]^p, x] /;$  FreeQ[{F, a, b, g, n, p}, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && !(LinearMatchQ[v, x] && PowerOfLinearMatchQ[u, x]) && IntegerQ[m]

Rule 2225

$\text{Int}[(F^{c(a+bx)})^{(c \cdot (a \cdot x) + b \cdot x)}]^{(n \cdot x)}, x\_Symbol] :> \text{Simp}[F^{c(a+bx)}]^{(n \cdot x)} / (b \cdot c \cdot n \cdot \text{Log}[F]), x] /;$  FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned}
\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3) dx &= \int F^{c(a+bx)}(d + ex)^3 dx \\
&= \frac{F^{c(a+bx)}(d + ex)^3}{bc \log(F)} - \frac{(3e) \int F^{c(a+bx)}(d + ex)^2 dx}{bc \log(F)} \\
&= -\frac{3eF^{c(a+bx)}(d + ex)^2}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d + ex)^3}{bc \log(F)} + \frac{(6e^2) \int F^{c(a+bx)}(d + ex) dx}{b^2c^2 \log^2(F)} \\
&= \frac{6e^2F^{c(a+bx)}(d + ex)}{b^3c^3 \log^3(F)} - \frac{3eF^{c(a+bx)}(d + ex)^2}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d + ex)^3}{bc \log(F)} \\
&= -\frac{6e^3F^{c(a+bx)}}{b^4c^4 \log^4(F)} + \frac{6e^2F^{c(a+bx)}(d + ex)}{b^3c^3 \log^3(F)} - \frac{3eF^{c(a+bx)}(d + ex)^2}{b^2c^2 \log^2(F)}
\end{aligned}$$

**Mathematica [A]**

time = 0.03, size = 78, normalized size = 0.71

$$\frac{F^{c(a+bx)}(-6e^3 + 6bce^2(d + ex) \log(F) - 3b^2c^2e(d + ex)^2 \log^2(F) + b^3c^3(d + ex)^3 \log^3(F))}{b^4c^4 \log^4(F)}$$

Antiderivative was successfully verified.

`[In] Integrate[F^(c*(a + b*x))*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2 + e^3*x^3), x]``[Out] (F^(c*(a + b*x))*(-6*e^3 + 6*b*c*e^2*(d + e*x)*Log[F] - 3*b^2*c^2*e*(d + e*x)^2*Log[F]^2 + b^3*c^3*(d + e*x)^3*Log[F]^3))/(b^4*c^4*Log[F]^4)`**Maple [A]**

time = 0.02, size = 165, normalized size = 1.50

method	result
gospers	$\frac{(e^3x^3c^3b^3 \ln(F)^3 + 3 \ln(F)^3b^3c^3de^2x^2 + 3 \ln(F)^3b^3c^3d^2ex + c^3b^3 \ln(F)^3d^3 - 3 \ln(F)^2b^2c^2e^3x^2 - 6 \ln(F)^2b^2c^2de^2x - 3 \ln(F)^2b^2c^2d^2e + 6e^3)x^3 + (6e^2c^3b^3 \ln(F)^3 + 6e^2c^3b^3d^2e + 6e^2c^3b^3d^2ex + 6e^2c^3b^3d^2e^2x^2 - 3 \ln(F)^2b^2c^2e^3x^2 - 6 \ln(F)^2b^2c^2de^2x - 3 \ln(F)^2b^2c^2d^2e + 6e^3)x^2 + (6e^2c^3b^3 \ln(F)^3 + 6e^2c^3b^3d^2e + 6e^2c^3b^3d^2ex + 6e^2c^3b^3d^2e^2x^2 - 3 \ln(F)^2b^2c^2e^3x^2 - 6 \ln(F)^2b^2c^2de^2x - 3 \ln(F)^2b^2c^2d^2e + 6e^3)x + 6e^3}{c^4b^4 \ln(F)^4}$
risch	$\frac{(e^3x^3c^3b^3 \ln(F)^3 + 3 \ln(F)^3b^3c^3de^2x^2 + 3 \ln(F)^3b^3c^3d^2ex + c^3b^3 \ln(F)^3d^3 - 3 \ln(F)^2b^2c^2e^3x^2 - 6 \ln(F)^2b^2c^2de^2x - 3 \ln(F)^2b^2c^2d^2e + 6e^3)x^3 + (6e^2c^3b^3 \ln(F)^3 + 6e^2c^3b^3d^2e + 6e^2c^3b^3d^2ex + 6e^2c^3b^3d^2e^2x^2 - 3 \ln(F)^2b^2c^2e^3x^2 - 6 \ln(F)^2b^2c^2de^2x - 3 \ln(F)^2b^2c^2d^2e + 6e^3)x^2 + (6e^2c^3b^3 \ln(F)^3 + 6e^2c^3b^3d^2e + 6e^2c^3b^3d^2ex + 6e^2c^3b^3d^2e^2x^2 - 3 \ln(F)^2b^2c^2e^3x^2 - 6 \ln(F)^2b^2c^2de^2x - 3 \ln(F)^2b^2c^2d^2e + 6e^3)x + 6e^3}{c^4b^4 \ln(F)^4}$
norman	$\frac{(c^3b^3 \ln(F)^3d^3 - 3 \ln(F)^2b^2c^2d^2e + 6 \ln(F)bcd^2e^2 - 6e^3)e^{c(bx+a) \ln(F)}}{c^4b^4 \ln(F)^4} + \frac{e^3x^3e^{c(bx+a) \ln(F)}}{cb \ln(F)} + \frac{3e(\ln(F)^2b^2c^2d^2 - 2 \ln(F)bced + 2e^2)}{c^3b^3 \ln(F)^3}$
meijerg	$\frac{F^{ca}e^3 \left( 6 - \frac{(-4b^3c^3x^3 \ln(F)^3 + 12b^2c^2x^2 \ln(F)^2 - 24bcx \ln(F) + 24)e^{bcx \ln(F)}}{4} \right)}{c^4b^4 \ln(F)^4} - \frac{3F^{ca}e^2d \left( 2 - \frac{(3b^2c^2x^2 \ln(F)^2 - 6bcx \ln(F) + 6)e^{bcx \ln(F)}}{3} \right)}{c^3b^3 \ln(F)^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*(e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3),x,method=\_RETURNVERBOSE)

[Out] (e^3\*x^3\*c^3\*b^3\*ln(F)^3+3\*ln(F)^3\*b^3\*c^3\*d\*e^2\*x^2+3\*ln(F)^3\*b^3\*c^3\*d^2\*e\*x+c^3\*b^3\*ln(F)^3\*d^3-3\*ln(F)^2\*b^2\*c^2\*e^3\*x^2-6\*ln(F)^2\*b^2\*c^2\*d\*e^2\*x-3\*ln(F)^2\*b^2\*c^2\*d^2\*e+6\*ln(F)\*b\*c\*e^3\*x+6\*ln(F)\*b\*c\*d\*e^2-6\*e^3)\*F^(c\*(b\*x+a))/c^4/b^4/ln(F)^4

**Maxima** [A]

time = 0.30, size = 208, normalized size = 1.89

$$\frac{F^{bcx+ac}d^3}{bc \log(F)} + \frac{3(F^{ac}bcx \log(F) - F^{ac})d^2 e^{bcx \log(F)+1}}{b^2 c^2 \log(F)^2} + \frac{3(F^{ac}b^2 c^2 x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})d e^{bcx \log(F)+2}}{b^3 c^3 \log(F)^3} + \frac{(F^{ac}b^3 c^3 x^3 \log(F)^3 - 3F^{ac}b^2 c^2 x^2 \log(F)^2 + 6F^{ac}bcx \log(F) - 6F^{ac})e^{bcx \log(F)+3}}{b^4 c^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3),x, algorithm="maxima")

[Out] F^(b\*c\*x + a\*c)\*d^3/(b\*c\*log(F)) + 3\*(F^(a\*c)\*b\*c\*x\*log(F) - F^(a\*c))\*d^2\*e^(b\*c\*x\*log(F) + 1)/(b^2\*c^2\*log(F)^2) + 3\*(F^(a\*c)\*b^2\*c^2\*x^2\*log(F)^2 - 2\*F^(a\*c)\*b\*c\*x\*log(F) + 2\*F^(a\*c))\*d\*e^(b\*c\*x\*log(F) + 2)/(b^3\*c^3\*log(F)^3) + (F^(a\*c)\*b^3\*c^3\*x^3\*log(F)^3 - 3\*F^(a\*c)\*b^2\*c^2\*x^2\*log(F)^2 + 6\*F^(a\*c)\*b\*c\*x\*log(F) - 6\*F^(a\*c))\*e^(b\*c\*x\*log(F) + 3)/(b^4\*c^4\*log(F)^4)

**Fricas** [A]

time = 0.39, size = 142, normalized size = 1.29

$$\frac{((b^3 c^3 x^3 e^3 + 3 b^3 c^3 d x^2 e^2 + 3 b^3 c^3 d^2 x e + b^3 c^3 d^3) \log(F)^3 - 3 (b^2 c^2 x^2 e^3 + 2 b^2 c^2 d x e^2 + b^2 c^2 d^2 e) \log(F)^2 + 6 (bcx e^3 + bcde^2) \log(F) - 6 e^3) F^{bcx+ac}}{b^4 c^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3),x, algorithm="fricas")

[Out] ((b^3\*c^3\*x^3\*e^3 + 3\*b^3\*c^3\*d\*x^2\*e^2 + 3\*b^3\*c^3\*d^2\*x\*e + b^3\*c^3\*d^3)\*log(F)^3 - 3\*(b^2\*c^2\*x^2\*e^3 + 2\*b^2\*c^2\*d\*x\*e^2 + b^2\*c^2\*d^2\*e)\*log(F)^2 + 6\*(b\*c\*x\*e^3 + b\*c\*d\*e^2)\*log(F) - 6\*e^3)\*F^(b\*c\*x + a\*c)/(b^4\*c^4\*log(F)^4)

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(107) = 214.

time = 0.08, size = 231, normalized size = 2.10

$$\begin{cases} \frac{F^{c(\alpha+bx)}(b^3 c^3 d^3 \log(F)^3 + 3 b^3 c^3 d^2 e x \log(F)^3 + 3 b^3 c^3 d e^2 x^2 \log(F)^3 + b^3 c^3 e^3 x^3 \log(F)^3 - 3 b^2 c^2 d^2 e \log(F)^2 - 6 b^2 c^2 d e^2 x \log(F)^2 - 3 b^2 c^2 e^3 x^2 \log(F)^2 + 6 b c d e^2 \log(F) + 6 b c e^3 x \log(F) - 6 e^3)}{b^4 c^4 \log(F)^4} & \text{for } b^4 c^4 \log(F)^4 \neq 0 \\ d^3 x + \frac{3 d^2 e x^2}{2} + d e^2 x^3 + \frac{e^3 x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(e\*\*3\*x\*\*3+3\*d\*e\*\*2\*x\*\*2+3\*d\*\*2\*e\*x+d\*\*3),x)

```
[Out] Piecewise((F**(c*(a + b*x))*(b**3*c**3*d**3*log(F)**3 + 3*b**3*c**3*d**2*e*
x*log(F)**3 + 3*b**3*c**3*d*e**2*x**2*log(F)**3 + b**3*c**3*e**3*x**3*log(F)
)**3 - 3*b**2*c**2*d**2*e*log(F)**2 - 6*b**2*c**2*d*e**2*x*log(F)**2 - 3*b*
*2*c**2*e**3*x**2*log(F)**2 + 6*b*c*d*e**2*log(F) + 6*b*c*e**3*x*log(F) - 6
*e**3)/(b**4*c**4*log(F)**4), Ne(b**4*c**4*log(F)**4, 0)), (d**3*x + 3*d**2
*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4, True))
```

**Giac** [C] Result contains complex when optimal does not.  
time = 3.53, size = 4685, normalized size = 42.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3),x, algorithm="g
iac")
```

```
[Out] ((4*(pi^3*b^3*c^3*x^3*sgn(F) - 3*pi*b^3*c^3*x^3*log(abs(F))^2*sgn(F) - pi^3
*b^3*c^3*x^3 + 3*pi*b^3*c^3*x^3*log(abs(F))^2 + 6*pi*b^2*c^2*x^2*log(abs(F)
)*sgn(F) - 6*pi*b^2*c^2*x^2*log(abs(F)) - 6*pi*b*c*x*sgn(F) + 6*pi*b*c*x)*(
pi^3*b^4*c^4*log(abs(F))*sgn(F) - pi*b^4*c^4*log(abs(F))^3*sgn(F) - pi^3*b^
4*c^4*log(abs(F)) + pi*b^4*c^4*log(abs(F))^3)/((pi^4*b^4*c^4*sgn(F) - 6*pi^
2*b^4*c^4*log(abs(F))^2*sgn(F) - pi^4*b^4*c^4 + 6*pi^2*b^4*c^4*log(abs(F))^
2 - 2*b^4*c^4*log(abs(F))^4)^2 + 16*(pi^3*b^4*c^4*log(abs(F))*sgn(F) - pi*b
^4*c^4*log(abs(F))^3*sgn(F) - pi^3*b^4*c^4*log(abs(F)) + pi*b^4*c^4*log(abs
(F))^3)^2) - (pi^4*b^4*c^4*sgn(F) - 6*pi^2*b^4*c^4*log(abs(F))^2*sgn(F) - p
i^4*b^4*c^4 + 6*pi^2*b^4*c^4*log(abs(F))^2 - 2*b^4*c^4*log(abs(F))^4)*(3*pi
^2*b^3*c^3*x^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*x^3*log(abs(F)) + 2*b^3*
c^3*x^3*log(abs(F))^3 - 3*pi^2*b^2*c^2*x^2*sgn(F) + 3*pi^2*b^2*c^2*x^2 - 6*
b^2*c^2*x^2*log(abs(F))^2 + 12*b*c*x*log(abs(F)) - 12)/((pi^4*b^4*c^4*sgn(F)
) - 6*pi^2*b^4*c^4*log(abs(F))^2*sgn(F) - pi^4*b^4*c^4 + 6*pi^2*b^4*c^4*log
(abs(F))^2 - 2*b^4*c^4*log(abs(F))^4)^2 + 16*(pi^3*b^4*c^4*log(abs(F))*sgn(
F) - pi*b^4*c^4*log(abs(F))^3*sgn(F) - pi^3*b^4*c^4*log(abs(F)) + pi*b^4*c^
4*log(abs(F))^3)^2)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*s
gn(F) + 1/2*pi*a*c) + ((pi^4*b^4*c^4*sgn(F) - 6*pi^2*b^4*c^4*log(abs(F))^2*
sgn(F) - pi^4*b^4*c^4 + 6*pi^2*b^4*c^4*log(abs(F))^2 - 2*b^4*c^4*log(abs(F)
))^4*(pi^3*b^3*c^3*x^3*sgn(F) - 3*pi*b^3*c^3*x^3*log(abs(F))^2*sgn(F) - pi^
3*b^3*c^3*x^3 + 3*pi*b^3*c^3*x^3*log(abs(F))^2 + 6*pi*b^2*c^2*x^2*log(abs(F)
))*sgn(F) - 6*pi*b^2*c^2*x^2*log(abs(F)) - 6*pi*b*c*x*sgn(F) + 6*pi*b*c*x)/
((pi^4*b^4*c^4*sgn(F) - 6*pi^2*b^4*c^4*log(abs(F))^2*sgn(F) - pi^4*b^4*c^4
+ 6*pi^2*b^4*c^4*log(abs(F))^2 - 2*b^4*c^4*log(abs(F))^4)^2 + 16*(pi^3*b^4*
c^4*log(abs(F))*sgn(F) - pi*b^4*c^4*log(abs(F))^3*sgn(F) - pi^3*b^4*c^4*log
(abs(F)) + pi*b^4*c^4*log(abs(F))^3)^2) + 4*(pi^3*b^4*c^4*log(abs(F))*sgn(F)
) - pi*b^4*c^4*log(abs(F))^3*sgn(F) - pi^3*b^4*c^4*log(abs(F)) + pi*b^4*c^4
*log(abs(F))^3)*(3*pi^2*b^3*c^3*x^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*x^3
*log(abs(F)) + 2*b^3*c^3*x^3*log(abs(F))^3 - 3*pi^2*b^2*c^2*x^2*sgn(F) + 3*
```

$$\begin{aligned} & \pi^2 b^2 c^2 x^2 - 6 b^2 c^2 x^2 \log(\text{abs}(F))^2 + 12 b c x \log(\text{abs}(F)) - 12 \\ & / ((\pi^4 b^4 c^4 \text{sgn}(F) - 6 \pi^2 b^4 c^4 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^4 b^4 c^4 \\ & + 6 \pi^2 b^4 c^4 \log(\text{abs}(F))^2 - 2 b^4 c^4 \log(\text{abs}(F))^4)^2 + 16 (\pi^3 b^4 \\ & c^4 \log(\text{abs}(F)) \text{sgn}(F) - \pi b^4 c^4 \log(\text{abs}(F))^3 \text{sgn}(F) - \pi^3 b^4 c^4 \log \\ & (\text{abs}(F)) + \pi b^4 c^4 \log(\text{abs}(F))^3)^2) * \sin(-1/2 \pi b c x \text{sgn}(F) + 1/2 \pi \\ & b c x - 1/2 \pi a c \text{sgn}(F) + 1/2 \pi a c) * e^{(b c x \log(\text{abs}(F)) + a c \log(\text{abs} \\ & (F)) + 3) - 1/2 I * ((\pi^3 b^3 c^3 x^3 \text{sgn}(F) + 3 I \pi^2 b^3 c^3 x^3 \log(\text{abs} \\ & (F)) \text{sgn}(F) - 3 \pi b^3 c^3 x^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 c^3 x^3 - 3 I \\ & \pi^2 b^3 c^3 x^3 \log(\text{abs}(F)) + 3 \pi b^3 c^3 x^3 \log(\text{abs}(F))^2 + 2 I b^3 c^3 \\ & x^3 \log(\text{abs}(F))^3 - 3 I \pi^2 b^2 c^2 x^2 \text{sgn}(F) + 6 \pi b^2 c^2 x^2 \log(\text{abs} \\ & (F)) \text{sgn}(F) + 3 I \pi^2 b^2 c^2 x^2 - 6 \pi b^2 c^2 x^2 \log(\text{abs}(F)) - 6 I b^2 \\ & c^2 x^2 \log(\text{abs}(F))^2 - 6 \pi b c x \text{sgn}(F) + 6 \pi b c x + 12 I b c x \log \\ & (\text{abs}(F)) - 12 I) * e^{(1/2 I \pi b c x \text{sgn}(F) - 1/2 I \pi b c x + 1/2 I \pi a c \text{sgn} \\ & (F) - 1/2 I \pi a c) / (\pi^4 b^4 c^4 \text{sgn}(F) + 4 I \pi^3 b^4 c^4 \log(\text{abs}(F)) \text{sgn} \\ & (F) - 6 \pi^2 b^4 c^4 \log(\text{abs}(F))^2 \text{sgn}(F) - 4 I \pi b^4 c^4 \log(\text{abs}(F))^3 \text{sgn} \\ & (F) - \pi^4 b^4 c^4 - 4 I \pi^3 b^4 c^4 \log(\text{abs}(F)) + 6 \pi^2 b^4 c^4 \log(\text{abs} \\ & (F))^2 + 4 I \pi b^4 c^4 \log(\text{abs}(F))^3 - 2 b^4 c^4 \log(\text{abs}(F))^4) + (\pi^3 b^3 \\ & c^3 x^3 \text{sgn}(F) - 3 I \pi^2 b^3 c^3 x^3 \log(\text{abs}(F)) \text{sgn}(F) - 3 \pi b^3 c^3 x^3 \\ & \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 c^3 x^3 + 3 I \pi^2 b^3 c^3 x^3 \log(\text{abs}(F)) \\ & + 3 \pi b^3 c^3 x^3 \log(\text{abs}(F))^2 - 2 I b^3 c^3 x^3 \log(\text{abs}(F))^3 + 3 I \pi \\ & \pi^2 b^2 c^2 x^2 \text{sgn}(F) + 6 \pi b^2 c^2 x^2 \log(\text{abs}(F)) \text{sgn}(F) - 3 I \pi^2 b^2 \\ & c^2 x^2 - 6 \pi b^2 c^2 x^2 \log(\text{abs}(F)) + 6 I b^2 c^2 x^2 \log(\text{abs}(F))^2 - \\ & 6 \pi b c x \text{sgn}(F) + 6 \pi b c x - 12 I b c x \log(\text{abs}(F)) + 12 I) * e^{(-1/2 I \pi \\ & b c x \text{sgn}(F) + 1/2 I \pi b c x - 1/2 I \pi a c \text{sgn}(F) + 1/2 I \pi a c) / (\pi^4 \\ & b^4 c^4 \text{sgn}(F) - 4 I \pi^3 b^4 c^4 \log(\text{abs}(F)) \text{sgn}(F) - 6 \pi^2 b^4 c^4 \log \\ & (\text{abs}(F))^2 \text{sgn}(F) + 4 I \pi b^4 c^4 \log(\text{abs}(F))^3 \text{sgn}(F) - \pi^4 b^4 c^4 + 4 I \\ & \pi^3 b^4 c^4 \log(\text{abs}(F)) + 6 \pi^2 b^4 c^4 \log(\text{abs}(F))^2 - 4 I \pi b^4 c^4 \log \\ & (\text{abs}(F))^3 - 2 b^4 c^4 \log(\text{abs}(F))^4) * e^{(b c x \log(\text{abs}(F)) + a c \log(\text{abs} \\ & (F)) + 3) + 3 * (((\pi^2 b^2 c^2 d x^2 \text{sgn}(F) - \pi^2 b^2 c^2 d x^2 + 2 b^2 c^2 \\ & d x^2 \log(\text{abs}(F))^2 - 4 b c d x \log(\text{abs}(F)) + 4 d) * (3 \pi^2 b^3 c^3 \log(\text{abs} \\ & (F)) \text{sgn}(F) - 3 \pi^2 b^3 c^3 \log(\text{abs}(F)) + 2 b^3 c^3 \log(\text{abs}(F))^3) / ((\pi^3 b^3 \\ & c^3 \text{sgn}(F) - 3 \pi b^3 c^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 c^3 + 3 \pi b^3 \\ & c^3 \log(\text{abs}(F))^2)^2 + (3 \pi^2 b^3 c^3 \log(\text{abs}(F)) \text{sgn}(F) - 3 \pi^2 b^3 c^3 \\ & \log(\text{abs}(F)) + 2 b^3 c^3 \log(\text{abs}(F))^3)^2) - 2 * (\pi^3 b^3 c^3 \text{sgn}(F) - 3 \pi \\ & b^3 c^3 \log(\text{abs}(F))^2 \text{sgn}(F) - \pi^3 b^3 c^3 + 3 \pi b^3 c^3 \log(\text{abs}(F))^2) * \\ & (\pi b^2 c^2 d x^2 \log(\text{abs}(F)) \text{sgn}(F) - \pi b^2 c^2 d x^2 \log(\text{abs}(F)) - \pi b c \\ & d x \text{sgn}(F) + \pi b c d x) / ((\pi^3 b^3 c^3 \text{sgn}(F) - 3 \pi b^3 c^3 \log(\text{abs}(F)) \\ & ^2 \text{sgn}(F) - \pi^3 b^3 c^3 + 3 \pi b^3 c^3 \log(\text{abs} \dots \end{aligned}$$

Mupad [B]

time = 3.43, size = 165, normalized size = 1.50

$$\frac{F^{a+bc} (b^3 c^3 d^3 \ln(F)^3 + 3 b^3 c^3 d^2 e x \ln(F)^3 + 3 b^3 c^3 d e^2 x^2 \ln(F)^3 + b^3 c^3 e^3 x^3 \ln(F)^3 - 3 b^2 c^2 d^2 e \ln(F)^2 - 6 b^2 c^2 d e^2 x \ln(F)^2 - 3 b^2 c^2 e^3 x^2 \ln(F)^2 + 6 b c d e^2 \ln(F) + 6 b c e^3 x \ln(F) - 6 e^3)}{b^4 c^4 \ln(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))\*(d^3 + e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x),x)

[Out] (F^(a\*c + b\*c\*x)\*(b^3\*c^3\*d^3\*log(F)^3 - 6\*e^3 + 6\*b\*c\*e^3\*x\*log(F) - 3\*b^2\*c^2\*d^2\*e\*log(F)^2 - 3\*b^2\*c^2\*e^3\*x^2\*log(F)^2 + b^3\*c^3\*e^3\*x^3\*log(F)^3 + 6\*b\*c\*d\*e^2\*log(F) - 6\*b^2\*c^2\*d\*e^2\*x\*log(F)^2 + 3\*b^3\*c^3\*d^2\*e\*x\*log(F)^3 + 3\*b^3\*c^3\*d\*e^2\*x^2\*log(F)^3))/(b^4\*c^4\*log(F)^4)



### 3.14 $\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2) dx$

Optimal. Leaf size=79

$$\frac{2e^2 F^{c(a+bx)}}{b^3 c^3 \log^3(F)} - \frac{2e F^{c(a+bx)}(d+ex)}{b^2 c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^2}{bc \log(F)}$$

[Out]  $2e^2 F^{c(bx+a)}/b^3/c^3/\ln(F)^3 - 2e F^{c(bx+a)}(e*x+d)/b^2/c^2/\ln(F)^2 + F^{c(bx+a)}(e*x+d)^2/b/c/\ln(F)$

Rubi [A]

time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {27, 2207, 2225}

$$\frac{2e^2 F^{c(a+bx)}}{b^3 c^3 \log^3(F)} - \frac{2e(d+ex)F^{c(a+bx)}}{b^2 c^2 \log^2(F)} + \frac{(d+ex)^2 F^{c(a+bx)}}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))\*(d^2 + 2\*d\*e\*x + e^2\*x^2), x]

[Out]  $(2e^2 F^{c(a + b*x)})/(b^3 c^3 \text{Log}[F]^3) - (2e F^{c(a + b*x)}(d + e*x))/(b^2 c^2 \text{Log}[F]^2) + (F^{c(a + b*x)}(d + e*x)^2)/(b*c \text{Log}[F])$

Rule 27

Int[(u\_.)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[u\*Cancel[(b/2 + c\*x)^(2\*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4\*a\*c, 0] && IntegerQ[p]

Rule 2207

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(c + d\*x)^m\*((b\*F^(g\*(e + f\*x)))^n/(f\*g\*n\*Log[F])), x] - Dist[d\*(m/(f\*g\*n\*Log[F])), Int[(c + d\*x)^(m-1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !TrueQ[UseGamma]

Rule 2225

Int[(F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned}
\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2) dx &= \int F^{c(a+bx)}(d + ex)^2 dx \\
&= \frac{F^{c(a+bx)}(d + ex)^2}{bc \log(F)} - \frac{(2e) \int F^{c(a+bx)}(d + ex) dx}{bc \log(F)} \\
&= -\frac{2eF^{c(a+bx)}(d + ex)}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d + ex)^2}{bc \log(F)} + \frac{(2e^2) \int F^{c(a+bx)} dx}{b^2c^2 \log^2(F)} \\
&= \frac{2e^2 F^{c(a+bx)}}{b^3c^3 \log^3(F)} - \frac{2eF^{c(a+bx)}(d + ex)}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d + ex)^2}{bc \log(F)}
\end{aligned}$$

**Mathematica [A]**

time = 0.02, size = 56, normalized size = 0.71

$$\frac{F^{c(a+bx)}(2e^2 - 2bce(d + ex) \log(F) + b^2c^2(d + ex)^2 \log^2(F))}{b^3c^3 \log^3(F)}$$

Antiderivative was successfully verified.

`[In] Integrate[F^(c*(a + b*x))*(d^2 + 2*d*e*x + e^2*x^2), x]``[Out] (F^(c*(a + b*x))*(2*e^2 - 2*b*c*e*(d + e*x)*Log[F] + b^2*c^2*(d + e*x)^2*Log[F]^2))/(b^3*c^3*Log[F]^3)`**Maple [A]**

time = 0.02, size = 91, normalized size = 1.15

method	result	size
gospers	$\frac{(e^2x^2c^2b^2 \ln(F)^2 + 2 \ln(F)^2 b^2 c^2 dex + \ln(F)^2 b^2 c^2 d^2 - 2 \ln(F) bc e^2 x - 2 \ln(F) bced + 2e^2) F^{c(bx+a)}}{c^3 b^3 \ln(F)^3}$	91
risch	$\frac{(e^2x^2c^2b^2 \ln(F)^2 + 2 \ln(F)^2 b^2 c^2 dex + \ln(F)^2 b^2 c^2 d^2 - 2 \ln(F) bc e^2 x - 2 \ln(F) bced + 2e^2) F^{c(bx+a)}}{c^3 b^3 \ln(F)^3}$	91
norman	$\frac{(\ln(F)^2 b^2 c^2 d^2 - 2 \ln(F) bced + 2e^2) e^{c(bx+a) \ln(F)}}{c^3 b^3 \ln(F)^3} + \frac{e^2 x^2 e^{c(bx+a) \ln(F)}}{cb \ln(F)} + \frac{2e(\ln(F) bcd - e) x e^{c(bx+a) \ln(F)}}{c^2 b^2 \ln(F)^2}$	112
meijerg	$-\frac{F^{ca} e^2 \left( 2 - \frac{(3b^2 c^2 x^2 \ln(F)^2 - 6bcx \ln(F) + 6) e^{bcx \ln(F)}}{3} \right)}{c^3 b^3 \ln(F)^3} + \frac{2F^{ca} ed \left( 1 - \frac{(-2bcx \ln(F) + 2) e^{bcx \ln(F)}}{2} \right)}{c^2 b^2 \ln(F)^2} - \frac{F^{ca} d^2 (1 - e^{bcx \ln(F)})}{cb \ln(F)}$	127

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(F^(c*(b*x+a))*(e^2*x^2+2*d*e*x+d^2), x, method=_RETURNVERBOSE)``[Out] (e^2*x^2*c^2*b^2*ln(F)^2+2*ln(F)^2*b^2*c^2*d*e*x+ln(F)^2*b^2*c^2*d^2-2*ln(F)*b*c*e^2*x-2*ln(F)*b*c*e*d+2*e^2)*F^(c*(b*x+a))/c^3/b^3/ln(F)^3`

**Maxima [A]**

time = 0.29, size = 125, normalized size = 1.58

$$\frac{F^{bcx+ac}d^2}{bc \log(F)} + \frac{2(F^{ac}bcx \log(F) - F^{ac})de^{(bcx \log(F)+1)}}{b^2c^2 \log(F)^2} + \frac{(F^{ac}b^2c^2x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})e^{(bcx \log(F)+2)}}{b^3c^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(F^(c\*(b\*x+a))\*(e^2\*x^2+2\*d\*e\*x+d^2),x, algorithm="maxima")

**[Out]** F^(b\*c\*x + a\*c)\*d^2/(b\*c\*log(F)) + 2\*(F^(a\*c)\*b\*c\*x\*log(F) - F^(a\*c))\*d\*e^(b\*c\*x\*log(F) + 1)/(b^2\*c^2\*log(F)^2) + (F^(a\*c)\*b^2\*c^2\*x^2\*log(F)^2 - 2\*F^(a\*c)\*b\*c\*x\*log(F) + 2\*F^(a\*c))\*e^(b\*c\*x\*log(F) + 2)/(b^3\*c^3\*log(F)^3)

**Fricas [A]**

time = 0.38, size = 83, normalized size = 1.05

$$\frac{((b^2c^2x^2e^2 + 2b^2c^2dxe + b^2c^2d^2) \log(F)^2 - 2(bcxe^2 + bcde) \log(F) + 2e^2)F^{bcx+ac}}{b^3c^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(F^(c\*(b\*x+a))\*(e^2\*x^2+2\*d\*e\*x+d^2),x, algorithm="fricas")

**[Out]** ((b^2\*c^2\*x^2\*e^2 + 2\*b^2\*c^2\*d\*x\*e + b^2\*c^2\*d^2)\*log(F)^2 - 2\*(b\*c\*x\*e^2 + b\*c\*d\*e)\*log(F) + 2\*e^2)\*F^(b\*c\*x + a\*c)/(b^3\*c^3\*log(F)^3)

**Sympy [A]**

time = 0.07, size = 133, normalized size = 1.68

$$\begin{cases} \frac{F^{c(a+bx)}(b^2c^2d^2 \log(F)^2 + 2b^2c^2dex \log(F)^2 + b^2c^2e^2x^2 \log(F)^2 - 2bcde \log(F) - 2bce^2x \log(F) + 2e^2)}{b^3c^3 \log(F)^3} & \text{for } b^3c^3 \log(F)^3 \neq 0 \\ d^2x + dex^2 + \frac{e^2x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(F\*\*(c\*(b\*x+a))\*(e\*\*2\*x\*\*2+2\*d\*e\*x+d\*\*2),x)

**[Out]** Piecewise((F\*\*(c\*(a + b\*x))\*(b\*\*2\*c\*\*2\*d\*\*2\*log(F)\*\*2 + 2\*b\*\*2\*c\*\*2\*d\*e\*x\*log(F)\*\*2 + b\*\*2\*c\*\*2\*e\*\*2\*x\*\*2\*log(F)\*\*2 - 2\*b\*c\*d\*e\*log(F) - 2\*b\*c\*e\*\*2\*x\*log(F) + 2\*e\*\*2)/(b\*\*3\*c\*\*3\*log(F)\*\*3), Ne(b\*\*3\*c\*\*3\*log(F)\*\*3, 0)), (d\*\*2\*x + d\*e\*x\*\*2 + e\*\*2\*x\*\*3/3, True))

**Giac [C]** Result contains complex when optimal does not.

time = 2.94, size = 2490, normalized size = 31.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e^2*x^2+2*d*e*x+d^2),x, algorithm="giac")
[Out] (((3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)*(pi^2*b^2*c^2*x^2*sgn(F) - pi^2*b^2*c^2*x^2 + 2*b^2*c^2*x^2*log(abs(F))^2 - 4*b*c*x*log(abs(F)) + 4)/((pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)^2 + (3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)^2) - 2*(pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)*(pi*b^2*c^2*x^2*log(abs(F))*sgn(F) - pi*b^2*c^2*x^2*log(abs(F)) - pi*b*c*x*sgn(F) + pi*b*c*x)/((pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)^2 + (3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)^2))*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c) + ((pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)*(pi^2*b^2*c^2*x^2*sgn(F) - pi^2*b^2*c^2*x^2 + 2*b^2*c^2*x^2*log(abs(F))^2 - 4*b*c*x*log(abs(F)) + 4)/((pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)^2 + (3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)^2) + 2*(3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)*(pi*b^2*c^2*x^2*log(abs(F))*sgn(F) - pi*b^2*c^2*x^2*log(abs(F)) - pi*b*c*x*sgn(F) + pi*b*c*x)/((pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)^2 + (3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)^2))*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)) + 2) - 2*I*((-I*pi^2*b^2*c^2*x^2*sgn(F) + 2*pi*b^2*c^2*x^2*log(abs(F))*sgn(F) + I*pi^2*b^2*c^2*x^2 - 2*pi*b^2*c^2*x^2*log(abs(F)) - 2*I*b^2*c^2*x^2*log(abs(F))^2 - 2*pi*b*c*x*sgn(F) + 2*pi*b*c*x + 4*I*b*c*x*log(abs(F)) - 4*I)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(-4*I*pi^3*b^3*c^3*sgn(F) + 12*pi^2*b^3*c^3*log(abs(F))*sgn(F) + 12*I*pi*b^3*c^3*log(abs(F))^2*sgn(F) + 4*I*pi^3*b^3*c^3 - 12*pi^2*b^3*c^3*log(abs(F)) - 12*I*pi*b^3*c^3*log(abs(F))^2 + 8*b^3*c^3*log(abs(F))^3) - (-I*pi^2*b^2*c^2*x^2*sgn(F) - 2*pi*b^2*c^2*x^2*log(abs(F))*sgn(F) + I*pi^2*b^2*c^2*x^2 + 2*pi*b^2*c^2*x^2*log(abs(F)) - 2*I*b^2*c^2*x^2*log(abs(F))^2 + 2*pi*b*c*x*sgn(F) - 2*pi*b*c*x + 4*I*b*c*x*log(abs(F)) - 4*I)*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(4*I*pi^3*b^3*c^3*sgn(F) + 12*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 12*I*pi*b^3*c^3*log(abs(F))^2*sgn(F) - 4*I*pi^3*b^3*c^3 - 12*pi^2*b^3*c^3*log(abs(F)) + 12*I*pi*b^3*c^3*log(abs(F))^2 + 8*b^3*c^3*log(abs(F))^3))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)) + 2) + 2*(2*((pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))*(pi*b*c*d*x*sgn(F) - pi*b*c*d*x)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2) + (pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)*(b*c*d*x*log(abs(F)) - d)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs
```

$(F)^2)^2 + 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))^2)*$   
 $\cos(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c) +$   
 $((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)*(\pi*b*c*d*$   
 $x*\text{sgn}(F) - \pi*b*c*d*x)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log$   
 $(\text{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))^2$   
 $- 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))*(b*c*d*x*\log$   
 $(\text{abs}(F)) - d)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))$   
 $)^2)^2 + 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))^2)*\sin$   
 $(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c)*e^{(b$   
 $*c*x*\log(\text{abs}(F)) + a*c*\log(\text{abs}(F)) + 1) - I*((\pi*b*c*d*x*\text{sgn}(F) - \pi*b*c*d*$   
 $x - 2*I*b*c*d*x*\log(\text{abs}(F)) + 2*I*d)*e^{(1/2*I*\pi*b*c*x*\text{sgn}(F) - 1/2*I*\pi*b*$   
 $c*x + 1/2*I*\pi*a*c*\text{sgn}(F) - 1/2*I*\pi*a*c)/(\pi^2*b^2*c^2*\text{sgn}(F) + 2*I*\pi*b^2$   
 $*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi^2*b^2*c^2 - 2*I*\pi*b^2*c^2*\log(\text{abs}(F)) + 2*b^2$   
 $*c^2*\log(\text{abs}(F))^2) + (\pi*b*c*d*x*\text{sgn}(F) - \pi*b*c*d*x + 2*I*b*c*d*x*\log(\text{abs}$   
 $(F)) - 2*I*d)*e^{(-1/2*I*\pi*b*c*x*\text{sgn}(F) + 1/2*I*\pi*b*c*x - 1/2*I*\pi*a*c*\text{sgn}$   
 $(F) + 1/2*I*\pi*a*c)/(\pi^2*b^2*c^2*\text{sgn}(F) - 2*I*\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F)$   
 $) - \pi^2*b^2*c^2 + 2*I*\pi*b^2*c^2*\log(\text{abs}(F)) + 2*b^2*c^2*\log(\text{abs}(F))^2))*e$   
 $^{(b*c*x*\log(\text{abs}(F)) + a*c*\log(\text{abs}(F)) + 1) + 2*(2*b*c*d^2*\cos(-1/2*\pi*b*c*x$   
 $*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c)*\log(\text{abs}(F))/(4*b^2$   
 $*c^2*\log(\text{abs}(F))^2 + (\pi*b*c*\text{sgn}(F) - \pi*b*c)^2) - (\pi*b*c*\text{sgn}(F) - \pi*b*c)$   
 $*d^2*\sin(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a$   
 $*c)/(4*b^2*c^2*\log(\text{abs}(F))^2 + (\pi*b*c*\text{sgn}(F) - \pi*b*c)^2))*e^{(b*c*x*\log(\text{ab}$   
 $s(F)) + a*c*\log(\text{abs}(F))) + I*(I*d^2*e^{(1/2*I*\pi*b*c*x*\text{sgn}(F) - 1/2*I*\pi*b*c$   
 $*x + 1/2*I*\pi*a*c*\text{sgn}(F) - 1/2*I*\pi*a*c)/(I*\pi*...$

### Mupad [B]

time = 3.40, size = 91, normalized size = 1.15

$$\frac{F^{ac+bcx} (b^2 c^2 d^2 \ln(F)^2 + 2 b^2 c^2 d e x \ln(F)^2 + b^2 c^2 e^2 x^2 \ln(F)^2 - 2 b c d e \ln(F) - 2 b c e^2 x \ln(F) + 2 e^2)}{b^3 c^3 \ln(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))\*(d^2 + e^2\*x^2 + 2\*d\*e\*x), x)

[Out] (F^(a\*c + b\*c\*x)\*(2\*e^2 + b^2\*c^2\*d^2\*log(F)^2 - 2\*b\*c\*e^2\*x\*log(F) + b^2\*c^2\*e^2\*x^2\*log(F)^2 - 2\*b\*c\*d\*e\*log(F) + 2\*b^2\*c^2\*d\*e\*x\*log(F)^2))/(b^3\*c^3\*log(F)^3)

### 3.15 $\int \frac{F^{c(a+bx)}}{d^2+2dex+e^2x^2} dx$

Optimal. Leaf size=57

$$-\frac{F^{c(a+bx)}}{e(d+ex)} + \frac{bcF^{c\left(a-\frac{bd}{e}\right)} \operatorname{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log(F)}{e^2}$$

[Out]  $-F^{c(bx+a)}/e/(ex+d)+bcF^{c(a-bd/e)}*Ei(bc(d+ex)*\ln(F)/e)*\ln(F)/e^2$

Rubi [A]

time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$ ,

Rules used = {27, 2208, 2209}

$$\frac{bc \log(F) F^{c\left(a-\frac{bd}{e}\right)} \operatorname{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{e^2} - \frac{F^{c(a+bx)}}{e(d+ex)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{c(a+bx)}/(d^2+2d*ex+e^2*x^2), x]$

[Out]  $-(F^{c(a+bx)}/(e*(d+ex))) + (bc*F^{c(a-(bd)/e)}*ExpIntegralEi[bc*(d+ex)*Log[F])/e)*Log[F])/e^2$

Rule 27

$\operatorname{Int}[(u_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)}, x\_Symbol] \rightarrow \operatorname{Int}[u\_Cancel[(b/2 + c*x)^(2*p)/c^p], x] /; \operatorname{FreeQ}\{a, b, c\}, x] \&\& \operatorname{EqQ}[b^2 - 4*a*c, 0] \&\& \operatorname{IntegerQ}[p]$

Rule 2208

$\operatorname{Int}[(b_.)*(F_.)^{((g_.)*((e_.) + (f_.)*(x_)))})^{(n_.)*((c_.) + (d_.)*(x_))}^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m+1)}*((b*F^{(g*(e+f*x))})^n/(d*(m+1))), x] - \operatorname{Dist}[f*g*n*(\operatorname{Log}[F]/(d*(m+1))), \operatorname{Int}[(c + d*x)^{(m+1)}*(b*F^{(g*(e+f*x))})^n, x], x] /; \operatorname{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{IntegerQ}[2*m] \&\& !\operatorname{TrueQ}[\$UseGamma]$

Rule 2209

$\operatorname{Int}[(F_.)^{((g_.)*((e_.) + (f_.)*(x_)))}/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[(F^{(g*(e-c*(f/d))})/d)*ExpIntegralEi[f*g*(c+d*x)*(\operatorname{Log}[F]/d)], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\operatorname{TrueQ}[\$UseGamma]$

Rubi steps

$$\begin{aligned}
\int \frac{F^{c(a+bx)}}{d^2 + 2dex + e^2x^2} dx &= \int \frac{F^{c(a+bx)}}{(d+ex)^2} dx \\
&= -\frac{F^{c(a+bx)}}{e(d+ex)} + \frac{(bc \log(F)) \int \frac{F^{c(a+bx)}}{d+ex} dx}{e} \\
&= -\frac{F^{c(a+bx)}}{e(d+ex)} + \frac{bcF^{c\left(a-\frac{bd}{e}\right)} \operatorname{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log(F)}{e^2}
\end{aligned}$$

**Mathematica [A]**

time = 0.02, size = 55, normalized size = 0.96

$$\frac{F^{ac} \left( -\frac{eF^{bcx}}{d+ex} + bcF^{-\frac{bcd}{e}} \operatorname{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log(F) \right)}{e^2}$$

Antiderivative was successfully verified.

`[In] Integrate[F^(c*(a + b*x))/(d^2 + 2*d*e*x + e^2*x^2), x]``[Out] (F^(a*c)*(-(e*F^(b*c*x))/(d + e*x)) + (b*c*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F])/F^((b*c*d)/e))/e^2`**Maple [A]**

time = 0.09, size = 99, normalized size = 1.74

method	result	size
risch	$-\frac{cb \ln(F) F^{bcx} F^{ca}}{e^2 \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)} - \frac{cb \ln(F) F^{\frac{c(ae-bd)}{e}} \operatorname{expIntegral}\left(1, -bcx \ln(F) - ca \ln(F) - \frac{-\ln(F) ace + \ln(F) bcd}{e}\right)}{e^2}$	99

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(F^(c*(b*x+a))/(e^2*x^2+2*d*e*x+d^2), x, method=_RETURNVERBOSE)``[Out] -c*b*ln(F)/e^2*F^(b*c*x)*F^(c*a)/(b*c*x*ln(F)+1/e*ln(F)*b*c*d)-c*b*ln(F)/e^2*F^(c*(a*e-b*d)/e)*Ei(1, -b*c*x*ln(F)-c*a*ln(F)-(-ln(F)*a*c*e+ln(F)*b*c*d)/e)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e^2\*x^2+2\*d\*e\*x+d^2),x, algorithm="maxima")

[Out] integrate(F^((b\*x + a)\*c)/(x^2\*e^2 + 2\*d\*x\*e + d^2), x)

**Fricas** [A]

time = 0.36, size = 77, normalized size = 1.35

$$-\frac{F^{bcx+ac}e - \frac{(bcxe+bcd)\text{Ei}((bcxe+bcd)e^{(-1)\log(F)})\log(F)}{F^{(bcd-ace)e^{(-1)}}}}{xe^3 + de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e^2\*x^2+2\*d\*e\*x+d^2),x, algorithm="fricas")

[Out] -(F^(b\*c\*x + a\*c)\*e - (b\*c\*x\*e + b\*c\*d)\*Ei((b\*c\*x\*e + b\*c\*d)\*e^(-1)\*log(F)) \*log(F)/F^((b\*c\*d - a\*c\*e)\*e^(-1)))/(x\*e^3 + d\*e^2)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{c(a+bx)}}{(d+ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))/(e\*\*2\*x\*\*2+2\*d\*e\*x+d\*\*2),x)

[Out] Integral(F\*\*(c\*(a + b\*x))/(d + e\*x)\*\*2, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e^2\*x^2+2\*d\*e\*x+d^2),x, algorithm="giac")

[Out] integrate(F^((b\*x + a)\*c)/(x^2\*e^2 + 2\*d\*x\*e + d^2), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{F^{c(a+bx)}}{d^2 + 2de x + e^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))/(d^2 + e^2\*x^2 + 2\*d\*e\*x),x)

[Out] int(F^(c\*(a + b\*x))/(d^2 + e^2\*x^2 + 2\*d\*e\*x), x)



$$3.16 \quad \int \frac{F^{c(a+bx)}}{d^3 + 3d^2ex + 3de^2x^2 + e^3x^3} dx$$

Optimal. Leaf size=95

$$-\frac{F^{c(a+bx)}}{2e(d+ex)^2} - \frac{bcF^{c(a+bx)} \log(F)}{2e^2(d+ex)} + \frac{b^2c^2F^{c(a-\frac{bd}{e})} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log^2(F)}{2e^3}$$

[Out]  $-1/2 * F^{(c*(b*x+a))} / e / (e*x+d)^2 - 1/2 * b*c * F^{(c*(b*x+a))} * \ln(F) / e^2 / (e*x+d) + 1/2 * b^2 * c^2 * F^{(c*(a-b*d/e))} * \text{Ei}(b*c*(e*x+d) * \ln(F) / e) * \ln(F)^2 / e^3$

Rubi [A]

time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2218, 2208, 2209}

$$\frac{b^2c^2 \log^2(F) F^{c(a-\frac{bd}{e})} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{2e^3} - \frac{bc \log(F) F^{c(a+bx)}}{2e^2(d+ex)} - \frac{F^{c(a+bx)}}{2e(d+ex)^2}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))/(d^3 + 3\*d^2\*e\*x + 3\*d\*e^2\*x^2 + e^3\*x^3), x]

[Out]  $-1/2 * F^{(c*(a + b*x))} / (e*(d + e*x)^2) - (b*c * F^{(c*(a + b*x))} * \text{Log}[F]) / (2 * e^2 * (d + e*x)) + (b^2 * c^2 * F^{(c*(a - (b*d)/e)}) * \text{ExpIntegralEi}[(b*c*(d + e*x) * \text{Log}[F]) / e] * \text{Log}[F]^2) / (2 * e^3)$

Rule 2208

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)\*((b \* F^(g\*(e + f\*x)))^n / (d\*(m + 1))), x] - Dist[f \* g \* n \* (Log[F] / (d\*(m + 1))), Int[(c + d\*x)^(m + 1) \* (b \* F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

Rule 2209

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))) / ((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - c\*(f/d))) / d \* ExpIntegralEi[f \* g \* (c + d\*x) \* (Log[F] / d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

Rule 2218

Int[((a\_.) + (b\_.)\*((F\_)^((g\_.)\*(v\_))))^(n\_.)]^(p\_.)\*(u\_)^(m\_.), x\_Symbol] :> Int[NormalizePowerOfLinear[u, x]^m \* (a + b \* (F^(g \* ExpandToSum[v, x]))^n)^p, x] /; FreeQ[{F, a, b, g, n, p}, x] && LinearQ[v, x] && PowerOfLinearQ[u, x]

] && !(LinearMatchQ[v, x] && PowerOfLinearMatchQ[u, x]) && IntegerQ[m]

Rubi steps

$$\begin{aligned}
 \int \frac{F^{c(a+bx)}}{d^3 + 3d^2ex + 3de^2x^2 + e^3x^3} dx &= \int \frac{F^{c(a+bx)}}{(d+ex)^3} dx \\
 &= -\frac{F^{c(a+bx)}}{2e(d+ex)^2} + \frac{(bc \log(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^2} dx}{2e} \\
 &= -\frac{F^{c(a+bx)}}{2e(d+ex)^2} - \frac{bcF^{c(a+bx)} \log(F)}{2e^2(d+ex)} + \frac{(b^2c^2 \log^2(F)) \int \frac{F^{c(a+bx)}}{d+ex} dx}{2e^2} \\
 &= -\frac{F^{c(a+bx)}}{2e(d+ex)^2} - \frac{bcF^{c(a+bx)} \log(F)}{2e^2(d+ex)} + \frac{b^2c^2F^{c\left(a-\frac{bd}{e}\right)} \text{Ei}\left(\frac{bc(d+ex) \log(F)}{e}\right) \log^2(F)}{2e^3}
 \end{aligned}$$

**Mathematica [A]**

time = 0.04, size = 88, normalized size = 0.93

$$\frac{F^{c\left(a-\frac{bd}{e}\right)} \left( b^2c^2(d+ex)^2 \text{Ei}\left(\frac{bc(d+ex) \log(F)}{e}\right) \log^2(F) - eF^{\frac{bc(d+ex)}{e}} (e + bc(d+ex) \log(F)) \right)}{2e^3(d+ex)^2}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))/(d^3 + 3\*d^2\*e\*x + 3\*d\*e^2\*x^2 + e^3\*x^3), x]

[Out] (F^(c\*(a - (b\*d)/e))\*(b^2\*c^2\*(d + e\*x)^2\*ExpIntegralEi[(b\*c\*(d + e\*x)\*Log[F])/e]\*Log[F]^2 - e\*F^((b\*c\*(d + e\*x))/e)\*(e + b\*c\*(d + e\*x)\*Log[F]))/(2\*e^3\*(d + e\*x)^2)

**Maple [A]**

time = 0.05, size = 155, normalized size = 1.63

method	result
risch	$  -\frac{c^2b^2 \ln(F)^2 F^{bcx} F^{ca}}{2e^3 \left( bcx \ln(F) + \frac{\ln(F)bcd}{e} \right)^2} - \frac{c^2b^2 \ln(F)^2 F^{bcx} F^{ca}}{2e^3 \left( bcx \ln(F) + \frac{\ln(F)bcd}{e} \right)} - \frac{c^2b^2 \ln(F)^2 F^{\frac{c(ae-bd)}{e}} \expIntegral\left(1, -bcx \ln(F) - ca \ln(F) - \frac{-\ln(F)a}{e}\right)}{2e^3}  $

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))/(e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3), x, method=\_RETURNVERBOSE)

[Out] -1/2\*c^2\*b^2\*ln(F)^2/e^3\*F^(b\*c\*x)\*F^(c\*a)/(b\*c\*x\*ln(F)+1/e\*ln(F)\*b\*c\*d)^2-1/2\*c^2\*b^2\*ln(F)^2/e^3\*F^(b\*c\*x)\*F^(c\*a)/(b\*c\*x\*ln(F)+1/e\*ln(F)\*b\*c\*d)-1/2

$*c^2*b^2*\ln(F)^2/e^3*F^{(c*(a*e-b*d)/e)*Ei(1,-b*c*x*\ln(F)-c*a*\ln(F)-(-\ln(F))*a*c*e+\ln(F)*b*c*d)/e}$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3),x, algorithm="maxima")

[Out] integrate(F^((b\*x + a)\*c)/(x^3\*e^3 + 3\*d\*x^2\*e^2 + 3\*d^2\*x\*e + d^3), x)

**Fricas [A]**

time = 0.37, size = 130, normalized size = 1.37

$$\frac{\frac{(b^2c^2x^2e^2+2b^2c^2dxe+b^2c^2d^2)Ei((bcxe+bcd)e^{(-1)}\log(F))\log(F)^2}{F^{(bcd-ace)e^{(-1)}}} - ((bcxe^2 + bcde)\log(F) + e^2)F^{bcx+ac}}{2(x^2e^5 + 2dxe^4 + d^2e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3),x, algorithm="fricas")

[Out] 1/2\*((b^2\*c^2\*x^2\*e^2 + 2\*b^2\*c^2\*d\*x\*e + b^2\*c^2\*d^2)\*Ei((b\*c\*x\*e + b\*c\*d)\*e^(-1)\*log(F))\*log(F)^2/F^((b\*c\*d - a\*c\*e)\*e^(-1)) - ((b\*c\*x\*e^2 + b\*c\*d\*e)\*log(F) + e^2)\*F^(b\*c\*x + a\*c))/(x^2\*e^5 + 2\*d\*x\*e^4 + d^2\*e^3)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{c(a+bx)}}{(d+ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))/(e\*\*3\*x\*\*3+3\*d\*e\*\*2\*x\*\*2+3\*d\*\*2\*e\*x+d\*\*3),x)

[Out] Integral(F\*\*(c\*(a + b\*x))/(d + e\*x)\*\*3, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3),x, algorithm="giac")

[Out] integrate(F^((b\*x + a)\*c)/(x^3\*e^3 + 3\*d\*x^2\*e^2 + 3\*d^2\*x\*e + d^3), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c(a+bx)}}{d^3 + 3d^2ex + 3de^2x^2 + e^3x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))/(d^3 + e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x),x)

[Out] int(F^(c\*(a + b\*x))/(d^3 + e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x), x)

$$3.17 \quad \int \frac{F^{c(a+bx)}}{d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4} dx$$

Optimal. Leaf size=128

$$\frac{F^{c(a+bx)}}{3e(d+ex)^3} - \frac{bcF^{c(a+bx)} \log(F)}{6e^2(d+ex)^2} - \frac{b^2c^2F^{c(a+bx)} \log^2(F)}{6e^3(d+ex)} + \frac{b^3c^3F^{c(a-\frac{bd}{e})} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log^3(F)}{6e^4}$$

[Out]  $-1/3 * F^{c*(b*x+a)} / e / (e*x+d)^3 - 1/6 * b*c * F^{c*(b*x+a)} * \ln(F) / e^2 / (e*x+d)^2 - 1/6 * b^2 * c^2 * F^{c*(b*x+a)} * \ln(F)^2 / e^3 / (e*x+d) + 1/6 * b^3 * c^3 * F^{c*(a-b*d/e)} * \text{Ei}(b*c*(e*x+d) * \ln(F) / e) * \ln(F)^3 / e^4$

Rubi [A]

time = 0.10, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 50,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.060$ , Rules used = {2218, 2208, 2209}

$$\frac{b^3c^3 \log^3(F) F^{c(a-\frac{bd}{e})} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{6e^4} - \frac{b^2c^2 \log^2(F) F^{c(a+bx)}}{6e^3(d+ex)} - \frac{bc \log(F) F^{c(a+bx)}}{6e^2(d+ex)^2} - \frac{F^{c(a+bx)}}{3e(d+ex)^3}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))/(d^4 + 4\*d^3\*e\*x + 6\*d^2\*e^2\*x^2 + 4\*d\*e^3\*x^3 + e^4\*x^4), x]

[Out]  $-1/3 * F^{c*(a + b*x)} / (e*(d + e*x)^3) - (b*c * F^{c*(a + b*x)} * \text{Log}[F]) / (6 * e^2 * (d + e*x)^2) - (b^2 * c^2 * F^{c*(a + b*x)} * \text{Log}[F]^2) / (6 * e^3 * (d + e*x)) + (b^3 * c^3 * F^{c*(a - (b*d)/e)} * \text{ExpIntegralEi}[(b*c*(d + e*x) * \text{Log}[F]) / e] * \text{Log}[F]^3) / (6 * e^4)$

Rule 2208

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)\*((b \* F^(g\*(e + f\*x)))^n / (d\*(m + 1))), x] - Dist[f\*g\*n\*(Log[F]/(d\*(m + 1))), Int[(c + d\*x)^(m + 1)\*(b \* F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntEgerQ[2\*m] && !TrueQ[\$UseGamma]

Rule 2209

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))) / ((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - c\*(f/d)))/d \* ExpIntegralEi[f\*g\*(c + d\*x)\*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

Rule 2218

Int[((a\_.) + (b\_.)\*((F\_)^((g\_.)\*(v\_))))^(n\_.)]^(p\_.)\*(u\_)^(m\_.), x\_Symbol] :> Int[NormalizePowerOfLinear[u, x]^m\*(a + b\*(F^(g\*ExpandToSum[v, x]))^n)^p,

x] /; FreeQ[{F, a, b, g, n, p}, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && !(LinearMatchQ[v, x] && PowerOfLinearMatchQ[u, x]) && IntegerQ[m]

Rubi steps

$$\begin{aligned}
 \int \frac{F^{c(a+bx)}}{d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4} dx &= \int \frac{F^{c(a+bx)}}{(d+ex)^4} dx \\
 &= -\frac{F^{c(a+bx)}}{3e(d+ex)^3} + \frac{(bc \log(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^3} dx}{3e} \\
 &= -\frac{F^{c(a+bx)}}{3e(d+ex)^3} - \frac{bcF^{c(a+bx)} \log(F)}{6e^2(d+ex)^2} + \frac{(b^2c^2 \log^2(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^2} dx}{6e^2} \\
 &= -\frac{F^{c(a+bx)}}{3e(d+ex)^3} - \frac{bcF^{c(a+bx)} \log(F)}{6e^2(d+ex)^2} - \frac{b^2c^2F^{c(a+bx)} \log^2(F)}{6e^3(d+ex)} + \\
 &= -\frac{F^{c(a+bx)}}{3e(d+ex)^3} - \frac{bcF^{c(a+bx)} \log(F)}{6e^2(d+ex)^2} - \frac{b^2c^2F^{c(a+bx)} \log^2(F)}{6e^3(d+ex)} +
 \end{aligned}$$

**Mathematica [A]**

time = 0.02, size = 99, normalized size = 0.77

$$\frac{F^{ac} \left( b^3 c^3 F^{-\frac{bcd}{e}} \operatorname{Ei} \left( \frac{bc(d+ex) \log(F)}{e} \right) \log^3(F) - \frac{eF^{bcx} (2e^2 + bce(d+ex) \log(F) + b^2c^2(d+ex)^2 \log^2(F))}{(d+ex)^3} \right)}{6e^4}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))/(d^4 + 4\*d^3\*e\*x + 6\*d^2\*e^2\*x^2 + 4\*d\*e^3\*x^3 + e^4\*x^4), x]

[Out] (F^(a\*c)\*((b^3\*c^3\*ExpIntegralEi[(b\*c\*(d + e\*x)\*Log[F])/e]\*Log[F]^3)/F^((b\*c\*d)/e) - (eF^(b\*c\*x)\*(2\*e^2 + b\*c\*e\*(d + e\*x)\*Log[F] + b^2\*c^2\*(d + e\*x)^2\*Log[F]^2))/(d + e\*x)^3))/(6\*e^4)

**Maple [A]**

time = 0.08, size = 199, normalized size = 1.55

method	result
risch	$  -\frac{c^3 b^3 \ln(F)^3 F^{bcx} F^{ca}}{3e^4 \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^3} - \frac{c^3 b^3 \ln(F)^3 F^{bcx} F^{ca}}{6e^4 \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^2} - \frac{c^3 b^3 \ln(F)^3 F^{bcx} F^{ca}}{6e^4 \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)} - \frac{c^3 b^3 \ln(F)^3 F^{\frac{c(ae-bd)}{e}} \operatorname{expIntegral}(1)}{6e^4}  $

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a)))/(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4),x,method=_RETURNVERBOSE)`

[Out] 
$$-1/3*c^3*b^3*\ln(F)^3/e^4*F^(b*c*x)*F^(c*a)/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d)^3-1/6*c^3*b^3*\ln(F)^3/e^4*F^(b*c*x)*F^(c*a)/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d)^2-1/6*c^3*b^3*\ln(F)^3/e^4*F^(b*c*x)*F^(c*a)/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d)-1/6*c^3*b^3*\ln(F)^3/e^4*F^(c*(a*e-b*d)/e)*Ei(1,-b*c*x*\ln(F)-c*a*\ln(F)-(-\ln(F)*a*c*e+\ln(F)*b*c*d)/e)$$

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a)))/(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4),x,algorithm="maxima")`

[Out] `integrate(F^((b*x + a)*c)/(x^4*e^4 + 4*d*x^3*e^3 + 6*d^2*x^2*e^2 + 4*d^3*x*e + d^4), x)`

**Fricas** [A]

time = 0.38, size = 200, normalized size = 1.56

$$\frac{(b^3c^3x^3e^3+3b^3c^3dx^2e^2+3b^3c^3d^2xe+b^3c^3d^3)Ei((bcxe+bcd)e^{-1}\log(F))\log(F)^3 - ((b^2c^2x^2e^3+2b^2c^2dxe^2+b^2c^2d^2e)\log(F)^2+(bcxe^3+bcde^2)\log(F)+2e^3)F^{bcx+ac}}{F^{(bcd-ace)e^{-1}} 6(x^3e^7+3dx^2e^6+3d^2xe^5+d^3e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a)))/(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4),x,algorithm="fricas")`

[Out] 
$$1/6*((b^3*c^3*x^3*e^3 + 3*b^3*c^3*d*x^2*e^2 + 3*b^3*c^3*d^2*x*e + b^3*c^3*d^3)*Ei((b*c*x*e + b*c*d)*e^{-1}*\log(F))*\log(F)^3/F^((b*c*d - a*c*e)*e^{-1}) - ((b^2*c^2*x^2*e^3 + 2*b^2*c^2*d*x*e^2 + b^2*c^2*d^2*e)*\log(F)^2 + (b*c*x*e^3 + b*c*d*e^2)*\log(F) + 2*e^3)*F^(b*c*x + a*c))/(x^3*e^7 + 3*d*x^2*e^6 + 3*d^2*x*e^5 + d^3*e^4)$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{c(a+bx)}}{(d+ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(c*(b*x+a)))/(e**4*x**4+4*d*e**3*x**3+6*d**2*e**2*x**2+4*d**3*e*x+d**4),x)`

[Out] Integral(F\*\*(c\*(a + b\*x))/(d + e\*x)\*\*4, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e^4\*x^4+4\*d\*e^3\*x^3+6\*d^2\*e^2\*x^2+4\*d^3\*e\*x+d^4), x, algorithm="giac")

[Out] integrate(F^((b\*x + a)\*c)/(x^4\*e^4 + 4\*d\*x^3\*e^3 + 6\*d^2\*x^2\*e^2 + 4\*d^3\*x\*e + d^4), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c(a+bx)}}{d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))/(d^4 + e^4\*x^4 + 4\*d\*e^3\*x^3 + 6\*d^2\*e^2\*x^2 + 4\*d^3\*e\*x), x)

[Out] int(F^(c\*(a + b\*x))/(d^4 + e^4\*x^4 + 4\*d\*e^3\*x^3 + 6\*d^2\*e^2\*x^2 + 4\*d^3\*e\*x), x)



$$3.18 \quad \int \frac{F^{c(a+bx)}}{d^5 + 5d^4ex + 10d^3e^2x^2 + 10d^2e^3x^3 + 5de^4x^4 + e^5x^5} dx$$

Optimal. Leaf size=161

$$\frac{F^{c(a+bx)}}{4e(d+ex)^4} - \frac{bcF^{c(a+bx)} \log(F)}{12e^2(d+ex)^3} - \frac{b^2c^2F^{c(a+bx)} \log^2(F)}{24e^3(d+ex)^2} - \frac{b^3c^3F^{c(a+bx)} \log^3(F)}{24e^4(d+ex)} + \frac{b^4c^4F^{c(a-\frac{bd}{e})} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{24e^5}$$

[Out]  $-1/4 * F^{(c*(b*x+a))} / e / (e*x+d)^4 - 1/12 * b*c * F^{(c*(b*x+a))} * \ln(F) / e^2 / (e*x+d)^3 - 1/24 * b^2 * c^2 * F^{(c*(b*x+a))} * \ln(F)^2 / e^3 / (e*x+d)^2 - 1/24 * b^3 * c^3 * F^{(c*(b*x+a))} * \ln(F)^3 / e^4 / (e*x+d) + 1/24 * b^4 * c^4 * F^{(c*(a-b*d/e))} * \text{Ei}(b*c*(e*x+d)*\ln(F)/e) * \ln(F)^4 / e^5$

Rubi [A]

time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 61,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {2218, 2208, 2209}

$$\frac{b^4c^4 \log^4(F) F^{c(a-\frac{bd}{e})} \text{Ei}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{24e^5} - \frac{b^3c^3 \log^3(F) F^{c(a+bx)}}{24e^4(d+ex)} - \frac{b^2c^2 \log^2(F) F^{c(a+bx)}}{24e^3(d+ex)^2} - \frac{bc \log(F) F^{c(a+bx)}}{12e^2(d+ex)^3} - \frac{F^{c(a+bx)}}{4e(d+ex)^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{(c*(a + b*x))} / (d^5 + 5*d^4*e*x + 10*d^3*e^2*x^2 + 10*d^2*e^3*x^3 + 5*d*e^4*x^4 + e^5*x^5), x]$

[Out]  $-1/4 * F^{(c*(a + b*x))} / (e*(d + e*x)^4) - (b*c * F^{(c*(a + b*x))} * \text{Log}[F]) / (12 * e^2 * (d + e*x)^3) - (b^2 * c^2 * F^{(c*(a + b*x))} * \text{Log}[F]^2) / (24 * e^3 * (d + e*x)^2) - (b^3 * c^3 * F^{(c*(a + b*x))} * \text{Log}[F]^3) / (24 * e^4 * (d + e*x)) + (b^4 * c^4 * F^{(c*(a - (b*d)/e)}) * \text{ExpIntegralEi}[(b*c*(d + e*x)*\text{Log}[F])/e] * \text{Log}[F]^4) / (24 * e^5)$

Rule 2208

$\text{Int}[(b_*) * (F_*)^{((g_*) * ((e_*) + (f_*) * (x_))))^{(n_*)} * ((c_*) + (d_*) * (x_))^{(m_*)}, x\_Symbol] :> \text{Simp}[(c + d*x)^{(m+1)} * ((b * F^{(g*(e + f*x))})^n / (d * (m+1))), x] - \text{Dist}[f * g * n * (\text{Log}[F] / (d * (m+1))), \text{Int}[(c + d*x)^{(m+1)} * (b * F^{(g*(e + f*x))})^n, x], x] /; \text{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \text{LtQ}[m, -1] \&\& \text{IntegerQ}[2*m] \&\& !\text{TrueQ}[\$UseGamma]$

Rule 2209

$\text{Int}[(F_*)^{((g_*) * ((e_*) + (f_*) * (x_))) / ((c_*) + (d_*) * (x_))}, x\_Symbol] :> \text{Simp}[(F^{(g*(e - c*(f/d))}) / d * \text{ExpIntegralEi}[f * g * (c + d*x) * (\text{Log}[F] / d)], x] /; \text{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\text{TrueQ}[\$UseGamma]$

Rule 2218

```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*(u_)^(m_.), x_Symbol] :
> Int[NormalizePowerOfLinear[u, x]^m*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p,
x] /; FreeQ[{F, a, b, g, n, p}, x] && LinearQ[v, x] && PowerOfLinearQ[u, x
] && !(LinearMatchQ[v, x] && PowerOfLinearMatchQ[u, x]) && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{F^{c(a+bx)}}{d^5 + 5d^4ex + 10d^3e^2x^2 + 10d^2e^3x^3 + 5de^4x^4 + e^5x^5} dx &= \int \frac{F^{c(a+bx)}}{(d+ex)^5} dx \\
&= -\frac{F^{c(a+bx)}}{4e(d+ex)^4} + \frac{(bc \log(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^4} dx}{4e} \\
&= -\frac{F^{c(a+bx)}}{4e(d+ex)^4} - \frac{bcF^{c(a+bx)} \log(F)}{12e^2(d+ex)^3} + \frac{(b^2c^2 \log^2(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^3} dx}{24e^3(d+ex)^2} \\
&= -\frac{F^{c(a+bx)}}{4e(d+ex)^4} - \frac{bcF^{c(a+bx)} \log(F)}{12e^2(d+ex)^3} - \frac{b^2c^2 F^{c(a+bx)} \log^2(F)}{24e^3(d+ex)^2} \\
&= -\frac{F^{c(a+bx)}}{4e(d+ex)^4} - \frac{bcF^{c(a+bx)} \log(F)}{12e^2(d+ex)^3} - \frac{b^2c^2 F^{c(a+bx)} \log^2(F)}{24e^3(d+ex)^2} \\
&= -\frac{F^{c(a+bx)}}{4e(d+ex)^4} - \frac{bcF^{c(a+bx)} \log(F)}{12e^2(d+ex)^3} - \frac{b^2c^2 F^{c(a+bx)} \log^2(F)}{24e^3(d+ex)^2}
\end{aligned}$$

**Mathematica [A]**

time = 0.02, size = 121, normalized size = 0.75

$$\frac{F^{ac} \left( b^4 c^4 F^{-\frac{bcd}{e}} \operatorname{Ei} \left( \frac{bc(d+ex) \log(F)}{e} \right) \log^4(F) - \frac{eF^{bcx} (6e^3 + 2bce^2(d+ex) \log(F) + b^2c^2e(d+ex)^2 \log^2(F) + b^3c^3(d+ex)^3 \log^3(F))}{(d+ex)^4} \right)}{24e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(d^5 + 5*d^4*e*x + 10*d^3*e^2*x^2 + 10*d^2*e^3*x^3 + 5*d*e^4*x^4 + e^5*x^5), x]
```

```
[Out] (F^(a*c)*((b^4*c^4*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F]^4)/F^(b*c*d/e) - (e*F^(b*c*x)*(6*e^3 + 2*b*c*e^2*(d + e*x)*Log[F] + b^2*c^2*e*(d + e*x)^2*Log[F]^2 + b^3*c^3*(d + e*x)^3*Log[F]^3))/(d + e*x)^4))/(24*e^5)
```

**Maple [A]**

time = 0.08, size = 243, normalized size = 1.51

method	result
--------	--------

risch	$-\frac{c^4 b^4 \ln(F)^4 F^{bcx} F^{ca}}{4e^5 \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^4} - \frac{c^4 b^4 \ln(F)^4 F^{bcx} F^{ca}}{12e^5 \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^3} - \frac{c^4 b^4 \ln(F)^4 F^{bcx} F^{ca}}{24e^5 \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)^2} - \frac{c^4 b^4 \ln(F)^4 F^{bcx} F^{ca}}{24e^5 \left( bcx \ln(F) + \frac{\ln(F) bcd}{e} \right)}$
-------	-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a)))/(e^5*x^5+5*d*e^4*x^4+10*d^2*e^3*x^3+10*d^3*e^2*x^2+5*d^4*e*x+d^5),x,method=_RETURNVERBOSE)`

[Out] 
$$-1/4*c^4*b^4*\ln(F)^4/e^5*F^{(b*c*x)*F^{(c*a)}}/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d)^4 - 1/12*c^4*b^4*\ln(F)^4/e^5*F^{(b*c*x)*F^{(c*a)}}/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d)^3 - 1/24*c^4*b^4*\ln(F)^4/e^5*F^{(b*c*x)*F^{(c*a)}}/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d)^2 - 1/24*c^4*b^4*\ln(F)^4/e^5*F^{(b*c*x)*F^{(c*a)}}/(b*c*x*\ln(F)+1/e*\ln(F)*b*c*d) - 1/24*c^4*b^4*\ln(F)^4/e^5*F^{(c*(a*e-b*d)/e)*Ei(1,-b*c*x*\ln(F)-c*a*\ln(F)-(-\ln(F))*a*c*e+\ln(F)*b*c*d)/e}$$

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a)))/(e^5*x^5+5*d*e^4*x^4+10*d^2*e^3*x^3+10*d^3*e^2*x^2+5*d^4*e*x+d^5),x,algorithm="maxima")`

[Out] `integrate(F^((b*x + a)*c)/(x^5*e^5 + 5*d*x^4*e^4 + 10*d^2*x^3*e^3 + 10*d^3*x^2*e^2 + 5*d^4*x*e + d^5), x)`

**Fricas** [A]

time = 0.37, size = 285, normalized size = 1.77

$$\frac{(b^4 c^4 x^4 e^4 + 4 b^4 c^4 d x^3 e^3 + 6 b^4 c^4 d^2 x^2 e^2 + 4 b^4 c^4 d^3 x e + b^4 c^4 d^4) \operatorname{Ei}\left(\frac{bcx+bd}{e} \ln(F)\right) \log(F)^4}{F^{(bcx+bd)e^{-1}}} - \frac{((b^3 c^3 x^3 e^4 + 3 b^3 c^3 d x^2 e^3 + 3 b^3 c^3 d^2 x e^2 + b^3 c^3 d^3 e) \log(F)^3 + (b^2 c^2 x^2 e^4 + 2 b^2 c^2 d x e^3 + b^2 c^2 d^2 e^2) \log(F)^2 + 2(bcxe^4 + bcde^3) \log(F) + 6e^4) F^{bcx+ac}}{24(x^4 e^9 + 4 dx^3 e^8 + 6 d^2 x^2 e^7 + 4 d^3 x e^6 + d^4 e^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a)))/(e^5*x^5+5*d*e^4*x^4+10*d^2*e^3*x^3+10*d^3*e^2*x^2+5*d^4*e*x+d^5),x,algorithm="fricas")`

[Out] 
$$1/24*((b^4*c^4*x^4*e^4 + 4*b^4*c^4*d*x^3*e^3 + 6*b^4*c^4*d^2*x^2*e^2 + 4*b^4*c^4*d^3*x*e + b^4*c^4*d^4)*Ei((b*c*x*e + b*c*d)*e^{-1})*\log(F)^4/F^{(b*c*d - a*c*e)*e^{-1}} - ((b^3*c^3*x^3*e^4 + 3*b^3*c^3*d*x^2*e^3 + 3*b^3*c^3*d^2*x*e^2 + b^3*c^3*d^3*e)*\log(F)^3 + (b^2*c^2*x^2*e^4 + 2*b^2*c^2*d*x*e^3 + b^2*c^2*d^2*e^2)*\log(F)^2 + 2*(b*c*x*e^4 + b*c*d*e^3)*\log(F) + 6*e^4)*F^{(b*c*x + a*c)}}/(x^4*e^9 + 4*d*x^3*e^8 + 6*d^2*x^2*e^7 + 4*d^3*x*e^6 + d^4*e^5)$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{c(a+bx)}}{(d+ex)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))/(e**5*x**5+5*d*e**4*x**4+10*d**2*e**3*x**3+10*d**3*
e**2*x**2+5*d**4*e*x+d**5), x)
```

```
[Out] Integral(F**(c*(a + b*x))/(d + e*x)**5, x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(e^5*x^5+5*d*e^4*x^4+10*d^2*e^3*x^3+10*d^3*e^2*x^2+
5*d^4*e*x+d^5), x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(x^5*e^5 + 5*d*x^4*e^4 + 10*d^2*x^3*e^3 + 10*d^3*
x^2*e^2 + 5*d^4*x*e + d^5), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c(a+bx)}}{d^5 + 5d^4ex + 10d^3e^2x^2 + 10d^2e^3x^3 + 5de^4x^4 + e^5x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/(d^5 + e^5*x^5 + 5*d*e^4*x^4 + 10*d^3*e^2*x^2 + 10*d^2*
e^3*x^3 + 5*d^4*e*x), x)
```

```
[Out] int(F^(c*(a + b*x))/(d^5 + e^5*x^5 + 5*d*e^4*x^4 + 10*d^3*e^2*x^2 + 10*d^2*
e^3*x^3 + 5*d^4*e*x), x)
```

### 3.19 $\int F^{c(a+bx)}((d+ex)^n)^m dx$

**Optimal.** Leaf size=72

$$\frac{F^{c\left(a-\frac{bd}{e}\right)}((d+ex)^n)^m \Gamma\left(1+mn, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-mn}}{bc \log(F)}$$

[Out]  $F^{c(a-b*d/e)}*((e*x+d)^n)^m*\text{GAMMA}(m*n+1, -b*c*(e*x+d)*\ln(F)/e)/b/c/\ln(F)/(-b*c*(e*x+d)*\ln(F)/e)^{(m*n)}$

**Rubi [A]**

time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {1973, 2212}

$$\frac{((d+ex)^n)^m F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-mn} \text{Gamma}\left(mn+1, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c(a+bx)}*((d+ex)^n)^m, x]$

[Out]  $(F^{c(a-(b*d)/e)}*((d+ex)^n)^m*\text{Gamma}[1+m*n, -((b*c*(d+ex)*\text{Log}[F])/e)])/(b*c*\text{Log}[F]*(-((b*c*(d+ex)*\text{Log}[F])/e))^{(m*n)})$

Rule 1973

$\text{Int}[(u_*)*((c_*)*((a_*) + (b_*)*(x_)^{(n_)})^{(q_)})^{(p_)}, x\_Symbol] \rightarrow \text{Dist}[\text{Simp}[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^{(p*q)}], \text{Int}[u*(1 + b*(x^n/a))^{(p*q)}, x], x] /;$  FreeQ[{a, b, c, n, p, q}, x] && !GeQ[a, 0]

Rule 2212

$\text{Int}[(F_)^{((g_)*((e_*) + (f_)*(x_)))}*((c_*) + (d_)*(x_))^{(m_)}, x\_Symbol] \rightarrow \text{Simp}[(-F^{(g*(e - c*(f/d)))})*((c + d*x)^{\text{FracPart}[m]}/(d*((-f)*g*(\text{Log}[F]/d)))^{(\text{IntPart}[m] + 1)*((-f)*g*\text{Log}[F]*((c + d*x)/d))^{\text{FracPart}[m]})]*\text{Gamma}[m + 1, ((-f)*g*(\text{Log}[F]/d))*(c + d*x)], x] /;$  FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

Rubi steps

$$\begin{aligned} \int F^{c(a+bx)}((d+ex)^n)^m dx &= (d+ex)^{-mn} ((d+ex)^n)^m \int F^{ac+bcx} (d+ex)^{mn} dx \\ &= \frac{F^{c\left(a-\frac{bd}{e}\right)}((d+ex)^n)^m \Gamma\left(1+mn, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-mn}}{bc \log(F)} \end{aligned}$$

**Mathematica [A]**

time = 0.03, size = 72, normalized size = 1.00

$$\frac{F^{c\left(a-\frac{bd}{e}\right)}((d+ex)^n)^m \Gamma\left(1+mn, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-mn}}{bc \log(F)}$$

Antiderivative was successfully verified.

`[In] Integrate[F^(c*(a + b*x))*((d + e*x)^n)^m,x]`

```
[Out] (F^(c*(a - (b*d)/e))*((d + e*x)^n)^m*Gamma[1 + m*n, -((b*c*(d + e*x)*Log[F]
)/e)]/(b*c*Log[F]*(-(b*c*(d + e*x)*Log[F])/e))^(m*n))
```

**Maple [F]**

time = 0.01, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}((ex+d)^n)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(F^(c*(b*x+a))*((e*x+d)^n)^m,x)``[Out] int(F^(c*(b*x+a))*((e*x+d)^n)^m,x)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(c*(b*x+a))*((e*x+d)^n)^m,x, algorithm="maxima")``[Out] integrate(((x*e + d)^n)^m*F^((b*x + a)*c), x)`**Fricas [A]**

time = 0.10, size = 68, normalized size = 0.94

$$\frac{e^{(-mne \log(-bce^{(-1)} \log(F)) + (bcd - ace) \log(F))e^{(-1)}} \Gamma(mn + 1, -(bcxe + bcd)e^{(-1)} \log(F))}{bc \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(c*(b*x+a))*((e*x+d)^n)^m,x, algorithm="fricas")`

```
[Out] e^(-(m*n*e*log(-b*c*e^(-1)*log(F)) + (b*c*d - a*c*e)*log(F))*e^(-1))*gamma(
m*n + 1, -(b*c*x*e + b*c*d)*e^(-1)*log(F))/(b*c*log(F))
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}((d+ex)^n)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*((e\*x+d)\*\*n)\*\*m,x)

[Out] Integral(F\*\*(c\*(a + b\*x))\*((d + e\*x)\*\*n)\*\*m, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*((e\*x+d)^n)^m,x, algorithm="giac")

[Out] integrate(((x\*e + d)^n)^m\*F^((b\*x + a)\*c), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int F^{c(a+bx)}((d+ex)^n)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))\*((d + e\*x)^n)^m,x)

[Out] int(F^(c\*(a + b\*x))\*((d + e\*x)^n)^m, x)

### 3.20 $\int F^{c(a+bx)}(d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4)^m dx$

Optimal. Leaf size=71

$$\frac{F^{c\left(a-\frac{bd}{e}\right)}((d+ex)^4)^m \Gamma\left(1+4m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-4m}}{bc \log(F)}$$

[Out]  $F^{c*(a-b*d/e)}*((e*x+d)^4)^m*\text{GAMMA}(1+4*m, -b*c*(e*x+d)*\ln(F)/e)/b/c/\ln(F)/(-b*c*(e*x+d)*\ln(F)/e)^{(4*m)}$

Rubi [A]

time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 50,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {2219, 2212}

$$\frac{((d+ex)^4)^m F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-4m} \text{Gamma}\left(4m+1, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))\*(d^4 + 4\*d^3\*e\*x + 6\*d^2\*e^2\*x^2 + 4\*d\*e^3\*x^3 + e^4\*x^4)^m, x]

[Out]  $(F^{c*(a - (b*d)/e)}*((d + e*x)^4)^m*\text{Gamma}[1 + 4*m, -((b*c*(d + e*x)*\text{Log}[F])/e)])/ (b*c*\text{Log}[F]*(-((b*c*(d + e*x)*\text{Log}[F])/e))^{(4*m)})$

Rule 2212

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d))^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d)^FracPart[m]))*Gamma[m + 1, ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

Rule 2219

```
Int[((a_) + (b_)*((F_)^((g_)*(v_)))^(n_))^(p_)*(u_)^(m_), x_Symbol]
> Module[{uu = NormalizePowerOfLinear[u, x], z}, Simp[z = If[PowerQ[uu] && FreeQ[uu[[2]], x], uu[[1]]^(m*uu[[2]]), uu^m]; (uu^m/z)*Int[z*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p, x], x] /; FreeQ[{F, a, b, g, m, n, p}, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && !(LinearMatchQ[v, x] && PowerOfLinearMatchQ[u, x]) && !IntegerQ[m]
```

Rubi steps



$$\int F^{c(a+bx)}(d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4)^m dx = (d + ex)^{-4m} ((d + ex)^4)^m \int F^{c(a+bx)}(d + ex)^{4m} \\ = \frac{F^{c(a-\frac{bd}{e})}((d + ex)^4)^m \Gamma\left(1 + 4m, -\frac{bc(d+ex)\log(F)}{e}\right)}{bc \log(F)}$$

**Mathematica [A]**

time = 0.03, size = 71, normalized size = 1.00

$$\frac{F^{c(a-\frac{bd}{e})}((d + ex)^4)^m \Gamma\left(1 + 4m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-4m}}{bc \log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(d^4 + 4*d^3*e*x + 6*d^2*e^2*x^2 + 4*d*e^3*x^3 + e^4*x^4)^m, x]
```

```
[Out] (F^(c*(a - (b*d)/e))*((d + e*x)^4)^m*Gamma[1 + 4*m, -((b*c*(d + e*x)*Log[F])/e)]/(b*c*Log[F]*(-(b*c*(d + e*x)*Log[F])/e))^(4*m))
```

**Maple [F]**

time = 0.02, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}(e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4)^m, x)
```

```
[Out] int(F^(c*(b*x+a))*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4)^m, x)
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4)^m, x, algorithm="maxima")
```

```
[Out] integrate((x^4*e^4 + 4*d*x^3*e^3 + 6*d^2*x^2*e^2 + 4*d^3*x*e + d^4)^m*F^((b*x + a)*c), x)
```

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4)^m, x, algorithm="fricas")
```

```
[Out] integral((x^4*e^4 + 4*d*x^3*e^3 + 6*d^2*x^2*e^2 + 4*d^3*x*e + d^4)^m*F^(b*c*x + a*c), x)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)} (d+ex)^4{}^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(e**4*x**4+4*d*e**3*x**3+6*d**2*e**2*x**2+4*d**3*e*x+d**4)**m, x)
```

```
[Out] Integral(F**(c*(a + b*x))*((d + e*x)**4)**m, x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4)^m, x, algorithm="giac")
```

```
[Out] integrate((x^4*e^4 + 4*d*x^3*e^3 + 6*d^2*x^2*e^2 + 4*d^3*x*e + d^4)^m*F^((b*x + a)*c), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int F^{c(a+bx)} (d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*(d^4 + e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x)^m, x)
```

```
[Out] int(F^(c*(a + b*x))*(d^4 + e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x)^m, x)
```

### 3.21 $\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^m dx$

Optimal. Leaf size=71

$$\frac{F^{c\left(a-\frac{bd}{e}\right)}((d+ex)^3)^m \Gamma\left(1+3m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-3m}}{bc \log(F)}$$

[Out]  $F^{c*(a-b*d/e)}*((e*x+d)^3)^m*\text{GAMMA}(1+3*m, -b*c*(e*x+d)*\ln(F)/e)/b/c/\ln(F)/(-b*c*(e*x+d)*\ln(F)/e)^{(3*m)}$

Rubi [A]

time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$ , Rules used = {2219, 2212}

$$\frac{((d+ex)^3)^m F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-3m} \text{Gamma}\left(3m+1, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c*(a+b*x)}*(d^3+3*d^2*e*x+3*d*e^2*x^2+e^3*x^3)^m, x]$

[Out]  $(F^{c*(a-(b*d)/e)}*((d+e*x)^3)^m*\text{Gamma}[1+3*m, -((b*c*(d+e*x)*\text{Log}[F])/e)])/(b*c*\text{Log}[F]*(-((b*c*(d+e*x)*\text{Log}[F])/e))^{(3*m)})$

Rule 2212

```
Int[(F_)^((g_)*(e_) + (f_)*(x_))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*(-f)*g*(Log[F]/d))
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]]*Gamma[m + 1,
((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2219

```
Int[((a_) + (b_)*((F_)^((g_)*(v_)))^(n_))^(p_)*(u_)^(m_), x_Symbol] :
> Module[{uu = NormalizePowerOfLinear[u, x], z}, Simp[z = If[PowerQ[uu] &&
FreeQ[uu[[2]], x], uu[[1]]^(m*uu[[2]]), uu^m]; (uu^m/z)*Int[z*(a + b*(F^(g*
ExpandToSum[v, x]))^n)^p, x], x] /; FreeQ[{F, a, b, g, m, n, p}, x] && Lin
earQ[v, x] && PowerOfLinearQ[u, x] && !(LinearMatchQ[v, x] && PowerOfLinea
rMatchQ[u, x]) && !IntegerQ[m]
```

Rubi steps

$$\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^m dx = (d + ex)^{-3m} ((d + ex)^3)^m \int F^{c(a+bx)}(d + ex)^{3m} dx$$

$$= \frac{F^{c\left(a-\frac{bd}{e}\right)}((d + ex)^3)^m \Gamma\left(1 + 3m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)}{e}\right)}{bc \log(F)}$$

**Mathematica [A]**

time = 0.03, size = 71, normalized size = 1.00

$$\frac{F^{c\left(a-\frac{bd}{e}\right)}((d + ex)^3)^m \Gamma\left(1 + 3m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-3m}}{bc \log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2 + e^3*x^3)^m,x]
```

```
[Out] (F^(c*(a - (b*d)/e))*((d + e*x)^3)^m*Gamma[1 + 3*m, -((b*c*(d + e*x)*Log[F])/e)]/(b*c*Log[F]*(-(b*c*(d + e*x)*Log[F])/e))^(3*m))
```

**Maple [F]**

time = 0.02, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m,x)
```

```
[Out] int(F^(c*(b*x+a))*(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m,x)
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m,x, algorithm="maxima")
```

```
[Out] integrate((x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3)^m*F^((b*x + a)*c), x)
```

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3)^m,x, algorithm="fricas")

[Out] integral((x^3\*e^3 + 3\*d\*x^2\*e^2 + 3\*d^2\*x\*e + d^3)^m\*F^(b\*c\*x + a\*c), x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}((d+ex)^3)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(e\*\*3\*x\*\*3+3\*d\*e\*\*2\*x\*\*2+3\*d\*\*2\*e\*x+d\*\*3)\*\*m,x)

[Out] Integral(F\*\*(c\*(a + b\*x))\*((d + e\*x)\*\*3)\*\*m, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3)^m,x, algorithm="giac")

[Out] integrate((x^3\*e^3 + 3\*d\*x^2\*e^2 + 3\*d^2\*x\*e + d^3)^m\*F^((b\*x + a)\*c), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))\*(d^3 + e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x)^m,x)

[Out] int(F^(c\*(a + b\*x))\*(d^3 + e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x)^m, x)

### 3.22 $\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2)^m dx$

**Optimal.** Leaf size=71

$$\frac{F^{c\left(a-\frac{bd}{e}\right)}((d+ex)^2)^m \Gamma\left(1+2m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-2m}}{bc \log(F)}$$

[Out]  $F^{c*(a-b*d/e)}*((e*x+d)^2)^m * \text{GAMMA}(1+2*m, -b*c*(e*x+d)*\ln(F)/e) / b/c/\ln(F) / (-b*c*(e*x+d)*\ln(F)/e)^{(2*m)}$

**Rubi [A]**

time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {2219, 2212}

$$\frac{((d+ex)^2)^m F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-2m} \text{Gamma}\left(2m+1, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))\*(d^2 + 2\*d\*e\*x + e^2\*x^2)^m, x]

[Out]  $(F^{c*(a - (b*d)/e)}*((d + e*x)^2)^m * \text{Gamma}[1 + 2*m, -((b*c*(d + e*x)*\text{Log}[F])/e)]) / (b*c*\text{Log}[F]*(-((b*c*(d + e*x)*\text{Log}[F])/e))^{(2*m)})$

Rule 2212

```
Int[(F_)^((g_.)*(e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)))^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d)^FracPart[m]))*Gamma[m + 1, ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

Rule 2219

```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*(u_)^(m_.), x_Symbol]
:> Module[{uu = NormalizePowerOfLinear[u, x], z}, Simp[z = If[PowerQ[uu] && FreeQ[uu[[2]], x], uu[[1]]^(m*uu[[2]]), uu^m]; (uu^m/z)*Int[z*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p, x]] /; FreeQ[{F, a, b, g, m, n, p}, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && !(LinearMatchQ[v, x] && PowerOfLinearMatchQ[u, x]) && !IntegerQ[m]
```

Rubi steps

$$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2)^m dx = (d + ex)^{-2m} ((d + ex)^2)^m \int F^{c(a+bx)}(d + ex)^{2m} dx$$

$$= \frac{F^{c\left(a-\frac{bd}{e}\right)}((d + ex)^2)^m \Gamma\left(1 + 2m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-2m}}{bc \log(F)}$$

**Mathematica [A]**

time = 0.03, size = 71, normalized size = 1.00

$$\frac{F^{c\left(a-\frac{bd}{e}\right)}((d + ex)^2)^m \Gamma\left(1 + 2m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-2m}}{bc \log(F)}$$

Antiderivative was successfully verified.

`[In] Integrate[F^(c*(a + b*x))*(d^2 + 2*d*e*x + e^2*x^2)^m, x]``[Out] (F^(c*(a - (b*d)/e))*((d + e*x)^2)^m*Gamma[1 + 2*m, -((b*c*(d + e*x)*Log[F])/e)])/ (b*c*Log[F]*(-(b*c*(d + e*x)*Log[F])/e))^(2*m))`**Maple [F]**

time = 0.03, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}(e^2x^2 + 2dex + d^2)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(F^(c*(b*x+a))*(e^2*x^2+2*d*e*x+d^2)^m, x)``[Out] int(F^(c*(b*x+a))*(e^2*x^2+2*d*e*x+d^2)^m, x)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(c*(b*x+a))*(e^2*x^2+2*d*e*x+d^2)^m, x, algorithm="maxima")``[Out] integrate((x^2*e^2 + 2*d*x*e + d^2)^m*F^((b*x + a)*c), x)`**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e^2\*x^2+2\*d\*e\*x+d^2)^m,x, algorithm="fricas")

[Out] integral((x^2\*e^2 + 2\*d\*x\*e + d^2)^m\*F^(b\*c\*x + a\*c), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}((d+ex)^2)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(e\*\*2\*x\*\*2+2\*d\*e\*x+d\*\*2)\*\*m,x)

[Out] Integral(F\*\*(c\*(a + b\*x))\*((d + e\*x)\*\*2)\*\*m, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e^2\*x^2+2\*d\*e\*x+d^2)^m,x, algorithm="giac")

[Out] integrate((x^2\*e^2 + 2\*d\*x\*e + d^2)^m\*F^((b\*x + a)\*c), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))\*(d^2 + e^2\*x^2 + 2\*d\*e\*x)^m,x)

[Out] int(F^(c\*(a + b\*x))\*(d^2 + e^2\*x^2 + 2\*d\*e\*x)^m, x)



### 3.23 $\int F^{c(a+bx)}(d+ex)^m dx$

Optimal. Leaf size=67

$$\frac{F^{c(a-\frac{bd}{e})}(d+ex)^m \Gamma\left(1+m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-m}}{bc \log(F)}$$

[Out]  $F^{c*(a-b*d/e)}*(e*x+d)^m*\text{GAMMA}(1+m, -b*c*(e*x+d)*\ln(F)/e)/b/c/\ln(F)/((-b*c*(e*x+d)*\ln(F)/e)^m)$

Rubi [A]

time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ ,

Rules used = {2212}

$$\frac{(d+ex)^m F^{c(a-\frac{bd}{e})} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-m} \text{Gamma}\left(m+1, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c*(a+b*x)}*(d+e*x)^m, x]$

[Out]  $(F^{c*(a-(b*d)/e)}*(d+e*x)^m*\text{Gamma}[1+m, -((b*c*(d+e*x)*\text{Log}[F])/e)])/(b*c*\text{Log}[F]*(-((b*c*(d+e*x)*\text{Log}[F])/e))^m)$

Rule 2212

```
Int[(F_)^((g_.)*((e_.)+(f_.)*(x_)))*((c_.)+(d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e-c*(f/d))))*((c+d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d))
)^(IntPart[m]+1)*((-f)*g*Log[F]*((c+d*x)/d))^FracPart[m])*Gamma[m+1,
((-f)*g*(Log[F]/d))*(c+d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rubi steps

$$\int F^{c(a+bx)}(d+ex)^m dx = \frac{F^{c(a-\frac{bd}{e})}(d+ex)^m \Gamma\left(1+m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-m}}{bc \log(F)}$$

Mathematica [A]

time = 0.01, size = 67, normalized size = 1.00

$$\frac{F^{c(a-\frac{bd}{e})}(d+ex)^m \Gamma\left(1+m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-m}}{bc \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d + e\*x)^m,x]

[Out] (F^(c\*(a - (b\*d)/e))\*(d + e\*x)^m\*Gamma[1 + m, -((b\*c\*(d + e\*x)\*Log[F])/e)]) / (b\*c\*Log[F]\*(-(b\*c\*(d + e\*x)\*Log[F])/e))^m

**Maple [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}(ex+d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*(e\*x+d)^m,x)

[Out] int(F^(c\*(b\*x+a))\*(e\*x+d)^m,x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^m,x, algorithm="maxima")

[Out] integrate((x\*e + d)^m\*F^((b\*x + a)\*c), x)

**Fricas [A]**

time = 0.09, size = 65, normalized size = 0.97

$$\frac{e^{(-me \log(-bce^{(-1)} \log(F)) + (bcd - ace) \log(F))e^{(-1)}} \Gamma(m + 1, -(bcxe + bcd)e^{(-1)} \log(F))}{bc \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^m,x, algorithm="fricas")

[Out] e^(- (m\*e\*log(-b\*c\*e^(-1)\*log(F)) + (b\*c\*d - a\*c\*e)\*log(F))\*e^(-1))\*gamma(m + 1, -(b\*c\*x\*e + b\*c\*d)\*e^(-1)\*log(F))/(b\*c\*log(F))

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}(d+ex)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(e\*x+d)\*\*m,x)

[Out] Integral(F\*\*(c\*(a + b\*x))\*(d + e\*x)\*\*m, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^m,x, algorithm="giac")

[Out] integrate((x\*e + d)^m\*F^((b\*x + a)\*c), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int F^{c(a+bx)} (d+ex)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))\*(d + e\*x)^m,x)

[Out] int(F^(c\*(a + b\*x))\*(d + e\*x)^m, x)

### 3.24 $\int F^{c(a+bx)}(d+ex)^{-m} dx$

Optimal. Leaf size=69

$$\frac{F^{c\left(a-\frac{bd}{e}\right)}(d+ex)^{-m}\Gamma\left(1-m, -\frac{bc(d+ex)\log(F)}{e}\right)\left(-\frac{bc(d+ex)\log(F)}{e}\right)^m}{bc\log(F)}$$

[Out]  $F^{c*(a-b*d/e)}*GAMMA(1-m, -b*c*(e*x+d)*ln(F)/e)*(-b*c*(e*x+d)*ln(F)/e)^m/b/c/((e*x+d)^m)/ln(F)$

Rubi [A]

time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ ,

Rules used = {2212}

$$\frac{(d+ex)^{-m}F^{c\left(a-\frac{bd}{e}\right)}\left(-\frac{bc\log(F)(d+ex)}{e}\right)^m\Gamma\left(1-m, -\frac{bc\log(F)(d+ex)}{e}\right)}{bc\log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))/(d + e\*x)^m, x]

[Out]  $(F^{c*(a - (b*d)/e)}*Gamma[1 - m, -((b*c*(d + e*x)*Log[F])/e)]*(-((b*c*(d + e*x)*Log[F])/e))^m)/(b*c*(d + e*x)^m*Log[F])$

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*(-f)*g*(Log[F]/d))
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d)^FracPart[m])*Gamma[m + 1,
((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rubi steps

$$\int F^{c(a+bx)}(d+ex)^{-m} dx = \frac{F^{c\left(a-\frac{bd}{e}\right)}(d+ex)^{-m}\Gamma\left(1-m, -\frac{bc(d+ex)\log(F)}{e}\right)\left(-\frac{bc(d+ex)\log(F)}{e}\right)^m}{bc\log(F)}$$

Mathematica [A]

time = 0.03, size = 69, normalized size = 1.00

$$\frac{F^{c\left(a-\frac{bd}{e}\right)}(d+ex)^{-m}\Gamma\left(1-m, -\frac{bc(d+ex)\log(F)}{e}\right)\left(-\frac{bc(d+ex)\log(F)}{e}\right)^m}{bc\log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))/(d + e\*x)^m, x]

[Out] (F^(c\*(a - (b\*d)/e))\*Gamma[1 - m, -((b\*c\*(d + e\*x)\*Log[F])/e)]\*(-((b\*c\*(d + e\*x)\*Log[F])/e))^m)/(b\*c\*(d + e\*x)^m\*Log[F])

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}(ex+d)^{-m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))/((e\*x+d)^m), x)

[Out] int(F^(c\*(b\*x+a))/((e\*x+d)^m), x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/((e\*x+d)^m), x, algorithm="maxima")

[Out] integrate(F^((b\*x + a)\*c)/(x\*e + d)^m, x)

Fricas [A]

time = 0.11, size = 67, normalized size = 0.97

$$\frac{e^{((me \log(-bce^{(-1)} \log(F)) - (bcd - ace) \log(F))e^{(-1)})} \Gamma(-m + 1, -(bcxe + bcd)e^{(-1)} \log(F))}{bc \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/((e\*x+d)^m), x, algorithm="fricas")

[Out] e^((m\*e\*log(-b\*c\*e^(-1)\*log(F)) - (b\*c\*d - a\*c\*e)\*log(F))\*e^(-1))\*gamma(-m + 1, -(b\*c\*x\*e + b\*c\*d)\*e^(-1)\*log(F))/(b\*c\*log(F))

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))/((e\*x+d)\*\*m), x)

[Out] Timed out

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/((e\*x+d)^m),x, algorithm="giac")

[Out] integrate(F^((b\*x + a)\*c)/(x\*e + d)^m, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c(a+bx)}}{(d+ex)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))/(d + e\*x)^m,x)

[Out] int(F^(c\*(a + b\*x))/(d + e\*x)^m, x)

### 3.25 $\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2)^{-m} dx$

Optimal. Leaf size=73

$$\frac{F^{c\left(a-\frac{bd}{e}\right)}((d+ex)^2)^{-m} \Gamma\left(1-2m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{2m}}{bc \log(F)}$$

[Out]  $F^{c(a-b*d/e)} * \text{GAMMA}(1-2*m, -b*c*(e*x+d)*\ln(F)/e) * (-b*c*(e*x+d)*\ln(F)/e)^{(2*m)}/b/c/(((e*x+d)^2)^m)/\ln(F)$

Rubi [A]

time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {2219, 2212}

$$\frac{((d+ex)^2)^{-m} F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{2m} \text{Gamma}\left(1-2m, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c(a + b*x)} / (d^2 + 2*d*e*x + e^2*x^2)^m, x]$

[Out]  $(F^{c(a - (b*d)/e)} * \text{Gamma}[1 - 2*m, -((b*c*(d + e*x)*\text{Log}[F])/e)]) * (-((b*c*(d + e*x)*\text{Log}[F])/e))^{(2*m)}) / (b*c*((d + e*x)^2)^m * \text{Log}[F])$

Rule 2212

```
Int[(F_)^((g_)*(e_) + (f_)*(x_))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d))^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1, ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

Rule 2219

```
Int[((a_) + (b_)*((F_)^((g_)*(v_)))^(n_))^(p_)*(u_)^(m_), x_Symbol]
> Module[{uu = NormalizePowerOfLinear[u, x], z}, Simp[z = If[PowerQ[uu] && FreeQ[uu[[2]], x], uu[[1]]^(m*uu[[2]]), uu^m]; (uu^m/z)*Int[z*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p, x], x] /; FreeQ[{F, a, b, g, m, n, p}, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && !(LinearMatchQ[v, x] && PowerOfLinearMatchQ[u, x]) && !IntegerQ[m]
```

Rubi steps

$$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2)^{-m} dx = (d + ex)^{2m} ((d + ex)^2)^{-m} \int F^{c(a+bx)}(d + ex)^{-2m} dx$$

$$= \frac{F^{c\left(a-\frac{bd}{e}\right)}((d + ex)^2)^{-m} \Gamma\left(1 - 2m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{2m}}{bc \log(F)}$$

**Mathematica [A]**

time = 0.03, size = 73, normalized size = 1.00

$$\frac{F^{c\left(a-\frac{bd}{e}\right)}((d + ex)^2)^{-m} \Gamma\left(1 - 2m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{2m}}{bc \log(F)}$$

Antiderivative was successfully verified.

`[In] Integrate[F^(c*(a + b*x))/(d^2 + 2*d*e*x + e^2*x^2)^m, x]``[Out] (F^(c*(a - (b*d)/e))*Gamma[1 - 2*m, -((b*c*(d + e*x)*Log[F])/e)]*(-((b*c*(d + e*x)*Log[F])/e)^(2*m)))/(b*c*((d + e*x)^2)^m*Log[F])`**Maple [F]**

time = 0.03, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}(e^2x^2 + 2dex + d^2)^{-m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(F^(c*(b*x+a))/((e^2*x^2+2*d*e*x+d^2)^m), x)``[Out] int(F^(c*(b*x+a))/((e^2*x^2+2*d*e*x+d^2)^m), x)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(c*(b*x+a))/((e^2*x^2+2*d*e*x+d^2)^m), x, algorithm="maxima")``[Out] integrate(F^((b*x + a)*c)/(x^2*e^2 + 2*d*x*e + d^2)^m, x)`**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/((e^2\*x^2+2\*d\*e\*x+d^2)^m),x, algorithm="fricas")

[Out] integral(F^(b\*c\*x + a\*c)/(x^2\*e^2 + 2\*d\*x\*e + d^2)^m, x)

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))/((e\*\*2\*x\*\*2+2\*d\*e\*x+d\*\*2)\*\*m),x)

[Out] Timed out

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/((e^2\*x^2+2\*d\*e\*x+d^2)^m),x, algorithm="giac")

[Out] integrate(F^((b\*x + a)\*c)/(x^2\*e^2 + 2\*d\*x\*e + d^2)^m, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c(a+bx)}}{(d^2 + 2dex + e^2x^2)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))/(d^2 + e^2\*x^2 + 2\*d\*e\*x)^m,x)

[Out] int(F^(c\*(a + b\*x))/(d^2 + e^2\*x^2 + 2\*d\*e\*x)^m, x)

### 3.26 $\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^{-m} dx$

Optimal. Leaf size=73

$$\frac{F^{c\left(a-\frac{bd}{e}\right)}((d+ex)^3)^{-m} \Gamma\left(1-3m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{3m}}{bc \log(F)}$$

[Out]  $F^{c*(a-b*d/e)}*GAMMA(1-3*m, -b*c*(e*x+d)*\ln(F)/e)*(-b*c*(e*x+d)*\ln(F)/e)^{(3*m)}/b/c/(((e*x+d)^3)^m)/\ln(F)$

Rubi [A]

time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ ,

Rules used = {2219, 2212}

$$\frac{((d+ex)^3)^{-m} F^{c\left(a-\frac{bd}{e}\right)} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{3m} \text{Gamma}\left(1-3m, -\frac{bc \log(F)(d+ex)}{e}\right)}{bc \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c*(a + b*x)}/(d^3 + 3*d^2*e*x + 3*d*e^2*x^2 + e^3*x^3)^m, x]$

[Out]  $(F^{c*(a - (b*d)/e)}*Gamma[1 - 3*m, -((b*c*(d + e*x)*Log[F])/e)]*(-((b*c*(d + e*x)*Log[F])/e))^{(3*m)})/(b*c*((d + e*x)^3)^m*Log[F])$

Rule 2212

```
Int[(F_)^((g_.)*(e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d))^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d)^FracPart[m]))*Gamma[m + 1, ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

Rule 2219

```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*(u_)^(m_.), x_Symbol]
> Module[{uu = NormalizePowerOfLinear[u, x], z}, Simp[z = If[PowerQ[uu] && FreeQ[uu[[2]], x], uu[[1]]^(m*uu[[2]]), uu^m]; (uu^m/z)*Int[z*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p, x]] /; FreeQ[{F, a, b, g, m, n, p}, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && !(LinearMatchQ[v, x] && PowerOfLinearMatchQ[u, x]) && !IntegerQ[m]
```

Rubi steps

$$\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^{-m} dx = (d + ex)^{3m} ((d + ex)^3)^{-m} \int F^{c(a+bx)}(d + ex)^{-3m} dx$$

$$= \frac{F^{c\left(a-\frac{bd}{e}\right)}((d + ex)^3)^{-m} \Gamma\left(1 - 3m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{3m}}{bc \log(F)}$$

**Mathematica [A]**

time = 0.03, size = 73, normalized size = 1.00

$$\frac{F^{c\left(a-\frac{bd}{e}\right)}((d + ex)^3)^{-m} \Gamma\left(1 - 3m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{3m}}{bc \log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(d^3 + 3*d^2*e*x + 3*d*e^2*x^2 + e^3*x^3)^m, x]
```

```
[Out] (F^(c*(a - (b*d)/e))*Gamma[1 - 3*m, -((b*c*(d + e*x)*Log[F])/e)]*(-((b*c*(d + e*x)*Log[F])/e))^(3*m))/(b*c*((d + e*x)^3)^m*Log[F])
```

**Maple [F]**

time = 0.02, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)^{-m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))/((e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m), x)
```

```
[Out] int(F^(c*(b*x+a))/((e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m), x)
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/((e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m), x, algorithm="maxima")
```

```
[Out] integrate(F^((b*x + a)*c)/(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3)^m, x)
```

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/((e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3)^m),x, algorithm="fricas")

[Out] integral(F^(b\*c\*x + a\*c)/(x^3\*e^3 + 3\*d\*x^2\*e^2 + 3\*d^2\*x\*e + d^3)^m, x)

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))/((e\*\*3\*x\*\*3+3\*d\*e\*\*2\*x\*\*2+3\*d\*\*2\*e\*x+d\*\*3)\*\*m),x)

[Out] Timed out

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/((e^3\*x^3+3\*d\*e^2\*x^2+3\*d^2\*e\*x+d^3)^m),x, algorithm="giac")

[Out] integrate(F^((b\*x + a)\*c)/(x^3\*e^3 + 3\*d\*x^2\*e^2 + 3\*d^2\*x\*e + d^3)^m, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c(a+bx)}}{(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))/(d^3 + e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x)^m,x)

[Out] int(F^(c\*(a + b\*x))/(d^3 + e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x)^m, x)

### 3.27 $\int F^{2+5x} dx$

Optimal. Leaf size=15

$$\frac{F^{2+5x}}{5 \log(F)}$$

[Out]  $1/5 * F^{(2+5*x)} / \ln(F)$

Rubi [A]

time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2225}

$$\frac{F^{5x+2}}{5 \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(2 + 5\*x), x]

[Out] F^(2 + 5\*x)/(5\*Log[F])

Rule 2225

Int[((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(n\_), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\int F^{2+5x} dx = \frac{F^{2+5x}}{5 \log(F)}$$

Mathematica [A]

time = 0.01, size = 15, normalized size = 1.00

$$\frac{F^{2+5x}}{5 \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(2 + 5\*x), x]

[Out] F^(2 + 5\*x)/(5\*Log[F])

Maple [A]

time = 0.01, size = 14, normalized size = 0.93

method	result	size
gospers	$\frac{F^{2+5x}}{5 \ln(F)}$	14
derivativdivides	$\frac{F^{2+5x}}{5 \ln(F)}$	14
default	$\frac{F^{2+5x}}{5 \ln(F)}$	14
risch	$\frac{F^{2+5x}}{5 \ln(F)}$	14
norman	$\frac{e^{(2+5x) \ln(F)}}{5 \ln(F)}$	16
meijerg	$-\frac{F^2(1-e^{5x \ln(F)})}{5 \ln(F)}$	20

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(2+5*x),x,method=_RETURNVERBOSE)`

[Out]  $1/5 * F^{(2+5*x)} / \ln(F)$

**Maxima** [A]

time = 0.28, size = 13, normalized size = 0.87

$$\frac{F^{5x+2}}{5 \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(2+5*x),x, algorithm="maxima")`

[Out]  $1/5 * F^{(5*x + 2)} / \log(F)$

**Fricas** [A]

time = 0.45, size = 13, normalized size = 0.87

$$\frac{F^{5x+2}}{5 \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(2+5*x),x, algorithm="fricas")`

[Out]  $1/5 * F^{(5*x + 2)} / \log(F)$

**Sympy** [A]

time = 0.02, size = 14, normalized size = 0.93

$$\begin{cases} \frac{F^{5x+2}}{5 \log(F)} & \text{for } \log(F) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(2+5\*x),x)

[Out] Piecewise((F\*\*(5\*x + 2)/(5\*log(F)), Ne(log(F), 0)), (x, True))

**Giac** [A]

time = 2.53, size = 13, normalized size = 0.87

$$\frac{F^{5x+2}}{5 \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(2+5\*x),x, algorithm="giac")

[Out] 1/5\*F^(5\*x + 2)/log(F)

**Mupad** [B]

time = 3.54, size = 13, normalized size = 0.87

$$\frac{F^{5x+2}}{5 \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(5\*x + 2),x)

[Out] F^(5\*x + 2)/(5\*log(F))

### 3.28 $\int F^{a+bx} dx$

Optimal. Leaf size=15

$$\frac{F^{a+bx}}{b \log(F)}$$

[Out]  $F^{(b*x+a)}/b/\ln(F)$

Rubi [A]

time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2225}

$$\frac{F^{a+bx}}{b \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*x), x]

[Out] F^(a + b\*x)/(b\*Log[F])

Rule 2225

Int[((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(n\_), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\int F^{a+bx} dx = \frac{F^{a+bx}}{b \log(F)}$$

Mathematica [A]

time = 0.01, size = 15, normalized size = 1.00

$$\frac{F^{a+bx}}{b \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*x), x]

[Out] F^(a + b\*x)/(b\*Log[F])

Maple [A]

time = 0.01, size = 16, normalized size = 1.07



method	result	size
gospers	$\frac{F^{bx+a}}{b \ln(F)}$	16
derivativdivides	$\frac{F^{bx+a}}{b \ln(F)}$	16
default	$\frac{F^{bx+a}}{b \ln(F)}$	16
risch	$\frac{F^{bx+a}}{b \ln(F)}$	16
norman	$\frac{e^{(bx+a) \ln(F)}}{b \ln(F)}$	18
meijerg	$-\frac{F^a (1 - e^{xb \ln(F)})}{b \ln(F)}$	23

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(b*x+a),x,method=_RETURNVERBOSE)`

[Out]  $F^{(b*x+a)}/b/\ln(F)$

**Maxima** [A]

time = 0.27, size = 15, normalized size = 1.00

$$\frac{F^{bx+a}}{b \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(b*x+a),x, algorithm="maxima")`

[Out]  $F^{(b*x + a)}/(b*\log(F))$

**Fricas** [A]

time = 0.47, size = 15, normalized size = 1.00

$$\frac{F^{bx+a}}{b \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(b*x+a),x, algorithm="fricas")`

[Out]  $F^{(b*x + a)}/(b*\log(F))$

**Sympy** [A]

time = 0.03, size = 15, normalized size = 1.00

$$\begin{cases} \frac{F^{a+bx}}{b \log(F)} & \text{for } b \log(F) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(b\*x+a),x)

[Out] Piecewise((F\*\*(a + b\*x)/(b\*log(F)), Ne(b\*log(F), 0)), (x, True))

**Giac [A]**

time = 2.29, size = 15, normalized size = 1.00

$$\frac{F^{bx+a}}{b \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(b\*x+a),x, algorithm="giac")

[Out] F^(b\*x + a)/(b\*log(F))

**Mupad [B]**

time = 3.44, size = 15, normalized size = 1.00

$$\frac{F^{a+bx}}{b \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a + b\*x),x)

[Out] F^(a + b\*x)/(b\*log(F))

### 3.29 $\int 10^{2+5x} dx$

Optimal. Leaf size=19

$$\frac{2^{2+5x} 5^{1+5x}}{\log(10)}$$

[Out]  $2^{(2+5*x)}*5^{(1+5*x)}/\ln(10)$

Rubi [A]

time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2225}

$$\frac{2^{5x+2} 5^{5x+1}}{\log(10)}$$

Antiderivative was successfully verified.

[In] Int[10^(2 + 5\*x), x]

[Out] (2^(2 + 5\*x)\*5^(1 + 5\*x))/Log[10]

Rule 2225

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\int 10^{2+5x} dx = \frac{2^{2+5x} 5^{1+5x}}{\log(10)}$$

Mathematica [A]

time = 0.01, size = 19, normalized size = 1.00

$$\frac{2^{2+5x} 5^{1+5x}}{\log(10)}$$

Antiderivative was successfully verified.

[In] Integrate[10^(2 + 5\*x), x]

[Out] (2^(2 + 5\*x)\*5^(1 + 5\*x))/Log[10]

Maple [A]

time = 0.03, size = 14, normalized size = 0.74

method	result	size
gospers	$\frac{10^{2+5x}}{5 \ln(10)}$	14
derivativedivides	$\frac{10^{2+5x}}{5 \ln(10)}$	14
default	$\frac{10^{2+5x}}{5 \ln(10)}$	14
norman	$\frac{e^{(2+5x) \ln(10)}}{5 \ln(10)}$	16
risch	$\frac{20 \cdot 3 \cdot 125^x \cdot 32^x}{\ln(2) + \ln(5)}$	16
meijerg	$-\frac{20(1 - e^{5x \ln(10)})}{\ln(10)}$	17

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(10^(2+5*x),x,method=_RETURNVERBOSE)`

[Out]  $1/5/\ln(10)*10^{(2+5*x)}$

**Maxima** [A]

time = 0.27, size = 13, normalized size = 0.68

$$\frac{10^{5x+2}}{5 \log(10)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(10^(2+5*x),x, algorithm="maxima")`

[Out]  $1/5*10^{(5*x + 2)}/\log(10)$

**Fricas** [A]

time = 0.38, size = 13, normalized size = 0.68

$$\frac{10^{5x+2}}{5 \log(10)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(10^(2+5*x),x, algorithm="fricas")`

[Out]  $1/5*10^{(5*x + 2)}/\log(10)$

**Sympy** [A]

time = 0.02, size = 10, normalized size = 0.53

$$\frac{10^{5x+2}}{5 \log(10)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(10\*\*(2+5\*x),x)

[Out] 10\*\*(5\*x + 2)/(5\*log(10))

**Giac [A]**

time = 3.47, size = 13, normalized size = 0.68

$$\frac{10^{5x+2}}{5 \log(10)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(10^(2+5\*x),x, algorithm="giac")

[Out] 1/5\*10^(5\*x + 2)/log(10)

**Mupad [B]**

time = 0.09, size = 11, normalized size = 0.58

$$\frac{20 \cdot 10^{5x}}{\ln(10)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(10^(5\*x + 2),x)

[Out] (20\*10^(5\*x))/log(10)

### 3.30 $\int F^{a+bx} x^{7/2} dx$

**Optimal.** Leaf size=131

$$\frac{105F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right)}{16b^{9/2} \log^{9/2}(F)} - \frac{105F^{a+bx} \sqrt{x}}{8b^4 \log^4(F)} + \frac{35F^{a+bx} x^{3/2}}{4b^3 \log^3(F)} - \frac{7F^{a+bx} x^{5/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{7/2}}{b \log(F)}$$

[Out]  $35/4 * F^{(b*x+a)} * x^{(3/2)} / b^3 / \ln(F)^3 - 7/2 * F^{(b*x+a)} * x^{(5/2)} / b^2 / \ln(F)^2 + F^{(b*x+a)} * x^{(7/2)} / b / \ln(F) + 105/16 * F^a * \operatorname{erfi}(b^{(1/2)} * x^{(1/2)} * \ln(F)^{(1/2)}) * \pi^{(1/2)} / b^{(9/2)} / \ln(F)^{(9/2)} - 105/8 * F^{(b*x+a)} * x^{(1/2)} / b^4 / \ln(F)^4$

**Rubi [A]**

time = 0.11, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2207, 2211, 2235}

$$\frac{105\sqrt{\pi} F^a \operatorname{Erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right)}{16b^{9/2} \log^{9/2}(F)} - \frac{105\sqrt{x} F^{a+bx}}{8b^4 \log^4(F)} + \frac{35x^{3/2} F^{a+bx}}{4b^3 \log^3(F)} - \frac{7x^{5/2} F^{a+bx}}{2b^2 \log^2(F)} + \frac{x^{7/2} F^{a+bx}}{b \log(F)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b*x)} * x^{(7/2)}, x]$

[Out]  $(105 * F^a * \operatorname{Sqrt}[\pi] * \operatorname{Erfi}[\operatorname{Sqrt}[b] * \operatorname{Sqrt}[x] * \operatorname{Sqrt}[\operatorname{Log}[F]]]) / (16 * b^{(9/2)} * \operatorname{Log}[F]^{(9/2)}) - (105 * F^{(a + b*x)} * \operatorname{Sqrt}[x]) / (8 * b^4 * \operatorname{Log}[F]^4) + (35 * F^{(a + b*x)} * x^{(3/2)}) / (4 * b^3 * \operatorname{Log}[F]^3) - (7 * F^{(a + b*x)} * x^{(5/2)}) / (2 * b^2 * \operatorname{Log}[F]^2) + (F^{(a + b*x)} * x^{(7/2)}) / (b * \operatorname{Log}[F])$

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

F, a, b, c, d}, x] && PosQ[b]

Rubi steps

$$\begin{aligned}
 \int F^{a+bx} x^{7/2} dx &= \frac{F^{a+bx} x^{7/2}}{b \log(F)} - \frac{7 \int F^{a+bx} x^{5/2} dx}{2b \log(F)} \\
 &= -\frac{7F^{a+bx} x^{5/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{7/2}}{b \log(F)} + \frac{35 \int F^{a+bx} x^{3/2} dx}{4b^2 \log^2(F)} \\
 &= \frac{35F^{a+bx} x^{3/2}}{4b^3 \log^3(F)} - \frac{7F^{a+bx} x^{5/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{7/2}}{b \log(F)} - \frac{105 \int F^{a+bx} \sqrt{x} dx}{8b^3 \log^3(F)} \\
 &= -\frac{105F^{a+bx} \sqrt{x}}{8b^4 \log^4(F)} + \frac{35F^{a+bx} x^{3/2}}{4b^3 \log^3(F)} - \frac{7F^{a+bx} x^{5/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{7/2}}{b \log(F)} + \frac{105 \int \frac{F^{a+bx}}{\sqrt{x}} dx}{16b^4 \log^4(F)} \\
 &= -\frac{105F^{a+bx} \sqrt{x}}{8b^4 \log^4(F)} + \frac{35F^{a+bx} x^{3/2}}{4b^3 \log^3(F)} - \frac{7F^{a+bx} x^{5/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{7/2}}{b \log(F)} + \frac{105 \text{Subst}\left(\int F^{a+bx^2} dx, x, \sqrt{x}\right)}{8b^4 \log^4(F)} \\
 &= \frac{105F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right)}{16b^{9/2} \log^{9/2}(F)} - \frac{105F^{a+bx} \sqrt{x}}{8b^4 \log^4(F)} + \frac{35F^{a+bx} x^{3/2}}{4b^3 \log^3(F)} - \frac{7F^{a+bx} x^{5/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{7/2}}{b \log(F)}
 \end{aligned}$$

**Mathematica [A]**

time = 0.06, size = 36, normalized size = 0.27

$$\frac{F^a \Gamma\left(\frac{9}{2}, -bx \log(F)\right) \sqrt{-bx \log(F)}}{b^5 \sqrt{x} \log^5(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*x)\*x^(7/2), x]

[Out] (F^a\*Gamma[9/2, -(b\*x\*Log[F])]\*Sqrt[-(b\*x\*Log[F])])/(b^5\*Sqrt[x]\*Log[F]^5)

**Maple [A]**

time = 0.03, size = 99, normalized size = 0.76

method	result
meijerg	$  F^a \left( -\frac{\sqrt{x} (-b)^{\frac{9}{2}} \sqrt{\ln(F)} (-72b^3 x^3 \ln(F)^3 + 252b^2 x^2 \ln(F)^2 - 630xb \ln(F) + 945) e^{xb \ln(F)}}{72b^4} + \frac{105(-b)^{\frac{9}{2}} \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\ln(F)}\right)}{16b^{\frac{9}{2}}} \right)  $

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(b*x+a)*x^(7/2),x,method=_RETURNVERBOSE)`

[Out] 
$$-F^a/(-b)^{(7/2)}/\ln(F)^{(9/2)}/b*(-1/72*x^{(1/2)}*(-b)^{(9/2)}*\ln(F)^{(1/2)}*(-72*b^3*x^3*\ln(F)^3+252*b^2*x^2*\ln(F)^2-630*x*b*\ln(F)+945)/b^4*\exp(x*b*\ln(F))+105/16*(-b)^{(9/2)}/b^{(9/2)}*\Pi^{(1/2)}*\operatorname{erfi}(b^{(1/2)}*x^{(1/2)}*\ln(F)^{(1/2)})$$

**Maxima** [A]

time = 0.32, size = 24, normalized size = 0.18

$$-\frac{F^a x^{\frac{9}{2}} \Gamma\left(\frac{9}{2}, -bx \log(F)\right)}{(-bx \log(F))^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(b*x+a)*x^(7/2),x, algorithm="maxima")`

[Out] 
$$-F^a*x^{(9/2)}*\gamma(9/2, -b*x*\log(F))/(-b*x*\log(F))^{(9/2)}$$

**Fricas** [A]

time = 0.37, size = 89, normalized size = 0.68

$$\frac{105 \sqrt{\pi} \sqrt{-b \log(F)} F^a \operatorname{erf}\left(\sqrt{-b \log(F)} \sqrt{x}\right) - 2(8 b^4 x^3 \log(F)^4 - 28 b^3 x^2 \log(F)^3 + 70 b^2 x \log(F)^2 - 105 b \log(F)) F^{bx+a} \sqrt{x}}{16 b^5 \log(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(b*x+a)*x^(7/2),x, algorithm="fricas")`

[Out] 
$$-1/16*(105*\sqrt{\pi}*\sqrt{-b*\log(F)}*F^a*\operatorname{erf}(\sqrt{-b*\log(F)}*\sqrt{x}) - 2*(8*b^4*x^3*\log(F)^4 - 28*b^3*x^2*\log(F)^3 + 70*b^2*x*\log(F)^2 - 105*b*\log(F))*F^{(b*x + a)}*\sqrt{x})/(b^5*\log(F)^5)$$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(b*x+a)*x**(7/2),x)`

[Out] Exception raised: SystemError >> excessive stack use: stack is 3876 deep

**Giac** [A]

time = 2.93, size = 94, normalized size = 0.72

$$-\frac{105 \sqrt{\pi} F^a \operatorname{erf}\left(-\sqrt{-b \log(F)} \sqrt{x}\right)}{16 \sqrt{-b \log(F)} b^4 \log(F)^4} + \frac{\left(8 b^3 x^{\frac{7}{2}} \log(F)^3 - 28 b^2 x^{\frac{5}{2}} \log(F)^2 + 70 b x^{\frac{3}{2}} \log(F) - 105 \sqrt{x}\right) e^{(bx \log(F)+a \log(F))}}{8 b^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(F^(b\*x+a)\*x^(7/2),x, algorithm="giac")

[Out]  $-105/16*\sqrt{\pi}*F^a*\operatorname{erf}(-\sqrt{-b*\log(F)}*\sqrt{x})/(\sqrt{-b*\log(F)}*b^4*\log(F)^4) + 1/8*(8*b^3*x^{7/2}*\log(F)^3 - 28*b^2*x^{5/2}*\log(F)^2 + 70*b*x^{3/2}*\log(F) - 105*\sqrt{x})*e^{(b*x*\log(F) + a*\log(F))}/(b^4*\log(F)^4)$

**Mupad [B]**

time = 3.43, size = 82, normalized size = 0.63

$$\frac{F^a x^{7/2} \left( \frac{105 \sqrt{\pi} \operatorname{erfc}(\sqrt{-b x \ln(F)})}{16} + F^{b x} \left( \frac{105 \sqrt{-b x \ln(F)}}{8} + \frac{35(-b x \ln(F))^{3/2}}{4} + \frac{7(-b x \ln(F))^{5/2}}{2} + (-b x \ln(F))^{7/2} \right) \right)}{b \ln(F) (-b x \ln(F))^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a + b\*x)\*x^(7/2),x)

[Out]  $(F^a*x^{7/2}*((105*\pi^{1/2}*\operatorname{erfc}((-b*x*\log(F))^{1/2}))/16 + F^{(b*x)}*((105*(-b*x*\log(F))^{1/2})/8 + (35*(-b*x*\log(F))^{3/2})/4 + (7*(-b*x*\log(F))^{5/2})/2 + (-b*x*\log(F))^{7/2}))/((b*\log(F))*(-b*x*\log(F))^{7/2}))$

### 3.31 $\int F^{a+bx} x^{5/2} dx$

**Optimal.** Leaf size=108

$$-\frac{15F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right)}{8b^{7/2} \log^{7/2}(F)} + \frac{15F^{a+bx} \sqrt{x}}{4b^3 \log^3(F)} - \frac{5F^{a+bx} x^{3/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{5/2}}{b \log(F)}$$

[Out]  $-5/2 * F^{(b*x+a)} * x^{(3/2)} / b^2 / \ln(F)^2 + F^{(b*x+a)} * x^{(5/2)} / b / \ln(F) - 15/8 * F^a * \operatorname{erfi}(b^{(1/2)} * x^{(1/2)} * \ln(F)^{(1/2)}) * \pi^{(1/2)} / b^{(7/2)} / \ln(F)^{(7/2)} + 15/4 * F^{(b*x+a)} * x^{(1/2)} / b^3 / \ln(F)^3$

**Rubi [A]**

time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2207, 2211, 2235}

$$-\frac{15\sqrt{\pi} F^a \operatorname{Erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right)}{8b^{7/2} \log^{7/2}(F)} + \frac{15\sqrt{x} F^{a+bx}}{4b^3 \log^3(F)} - \frac{5x^{3/2} F^{a+bx}}{2b^2 \log^2(F)} + \frac{x^{5/2} F^{a+bx}}{b \log(F)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b*x)} * x^{(5/2)}, x]$

[Out]  $(-15 * F^a * \operatorname{Sqrt}[\pi] * \operatorname{Erfi}[\operatorname{Sqrt}[b] * \operatorname{Sqrt}[x] * \operatorname{Sqrt}[\operatorname{Log}[F]]]) / (8 * b^{(7/2)} * \operatorname{Log}[F]^{(7/2)}) + (15 * F^{(a + b*x)} * \operatorname{Sqrt}[x]) / (4 * b^3 * \operatorname{Log}[F]^3) - (5 * F^{(a + b*x)} * x^{(3/2)}) / (2 * b^2 * \operatorname{Log}[F]^2) + (F^{(a + b*x)} * x^{(5/2)}) / (b * \operatorname{Log}[F])$

Rule 2207

$\operatorname{Int}[(b_.)(F_)^{((g_.)((e_.) + (f_.)(x_)))^{(n_.)((c_.) + (d_.)(x_))^{(m_.)}, x\_Symbol] :> \operatorname{Simp}[(c + d*x)^m * ((b * F^{(g*(e + f*x)))^n / (f * g * n * \operatorname{Log}[F]))], x] - \operatorname{Dist}[d * (m / (f * g * n * \operatorname{Log}[F])), \operatorname{Int}[(c + d*x)^{(m-1)} * (b * F^{(g*(e + f*x)))^n, x], x] /; \operatorname{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \operatorname{GtQ}[m, 0] \&\& \operatorname{IntegerQ}[2 * m] \&\& !\operatorname{TrueQ}[\$UseGamma]$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)((e_.) + (f_.)(x_))) / \operatorname{Sqrt}[(c_.) + (d_.)(x_)]}, x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f * g * (x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\operatorname{TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.) + (b_.)((c_.) + (d_.)(x_))^2)}, x\_Symbol] :> \operatorname{Simp}[F^a * \operatorname{Sqrt}[\pi] * (\operatorname{Erfi}[(c + d*x) * \operatorname{Rt}[b * \operatorname{Log}[F], 2]] / (2 * d * \operatorname{Rt}[b * \operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \&\& \operatorname{PosQ}[b]$

## Rubi steps

$$\begin{aligned}
\int F^{a+bx} x^{5/2} dx &= \frac{F^{a+bx} x^{5/2}}{b \log(F)} - \frac{5 \int F^{a+bx} x^{3/2} dx}{2b \log(F)} \\
&= -\frac{5F^{a+bx} x^{3/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{5/2}}{b \log(F)} + \frac{15 \int F^{a+bx} \sqrt{x} dx}{4b^2 \log^2(F)} \\
&= \frac{15F^{a+bx} \sqrt{x}}{4b^3 \log^3(F)} - \frac{5F^{a+bx} x^{3/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{5/2}}{b \log(F)} - \frac{15 \int \frac{F^{a+bx}}{\sqrt{x}} dx}{8b^3 \log^3(F)} \\
&= \frac{15F^{a+bx} \sqrt{x}}{4b^3 \log^3(F)} - \frac{5F^{a+bx} x^{3/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{5/2}}{b \log(F)} - \frac{15 \text{Subst}\left(\int F^{a+bx^2} dx, x, \sqrt{x}\right)}{4b^3 \log^3(F)} \\
&= -\frac{15F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right)}{8b^{7/2} \log^{7/2}(F)} + \frac{15F^{a+bx} \sqrt{x}}{4b^3 \log^3(F)} - \frac{5F^{a+bx} x^{3/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{5/2}}{b \log(F)}
\end{aligned}$$

**Mathematica [A]**

time = 0.06, size = 36, normalized size = 0.33

$$\frac{F^a \sqrt{x} \Gamma\left(\frac{7}{2}, -bx \log(F)\right)}{b^3 \log^3(F) \sqrt{-bx \log(F)}}$$

Antiderivative was successfully verified.

`[In] Integrate[F^(a + b*x)*x^(5/2), x]``[Out] (F^a*sqrt[x]*Gamma[7/2, -(b*x*Log[F])])/(b^3*Log[F]^3*sqrt[-(b*x*Log[F])])`**Maple [A]**

time = 0.01, size = 87, normalized size = 0.81

method	result	size
meijerg	$F^a \left( \frac{\sqrt{x} (-b)^{\frac{7}{2}} \sqrt{\ln(F)} (28b^2 x^2 \ln(F)^2 - 70xb \ln(F) + 105) e^{xb \ln(F)}}{28b^3} - \frac{15(-b)^{\frac{7}{2}} \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\ln(F)}\right)}{8b^{\frac{7}{2}}} \right)$	87

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(F^(b*x+a)*x^(5/2), x, method=_RETURNVERBOSE)`
`[Out] -F^a/(-b)^(5/2)/ln(F)^(7/2)/b*(1/28*x^(1/2)*(-b)^(7/2)*ln(F)^(1/2)*(28*b^2*x^2*ln(F)^2-70*x*b*ln(F)+105)/b^3*exp(x*b*ln(F))-15/8*(-b)^(7/2)/b^(7/2)*Pi^(1/2)*erfi(b^(1/2)*x^(1/2)*ln(F)^(1/2))`

**Maxima [A]**

time = 0.39, size = 24, normalized size = 0.22

$$\frac{F^a x^{\frac{7}{2}} \Gamma\left(\frac{7}{2}, -bx \log(F)\right)}{(-bx \log(F))^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(b*x+a)*x^(5/2),x, algorithm="maxima")``[Out] -F^a*x^(7/2)*gamma(7/2, -b*x*log(F))/(-b*x*log(F))^(7/2)`**Fricas [A]**

time = 0.39, size = 77, normalized size = 0.71

$$\frac{15 \sqrt{\pi} \sqrt{-b \log(F)} F^a \operatorname{erf}\left(\sqrt{-b \log(F)} \sqrt{x}\right) + 2(4b^3 x^2 \log(F)^3 - 10b^2 x \log(F)^2 + 15b \log(F)) F^{bx+a} \sqrt{x}}{8b^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(b*x+a)*x^(5/2),x, algorithm="fricas")`
`[Out] 1/8*(15*sqrt(pi)*sqrt(-b*log(F))*F^a*erf(sqrt(-b*log(F))*sqrt(x)) + 2*(4*b^3*x^2*log(F)^3 - 10*b^2*x*log(F)^2 + 15*b*log(F))*F^(b*x + a)*sqrt(x))/(b^4*log(F)^4)`
**Sympy [A]**

time = 128.85, size = 37, normalized size = 0.34

$$-\frac{4F^a F^{bx} b x^{\frac{9}{2}} \log(F)}{63} + \frac{2F^a F^{bx} x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F**(b*x+a)*x**(5/2),x)``[Out] -4*F**a*F**(b*x)*b*x**(9/2)*log(F)/63 + 2*F**a*F**(b*x)*x**(7/2)/7`**Giac [A]**

time = 2.08, size = 82, normalized size = 0.76

$$\frac{15 \sqrt{\pi} F^a \operatorname{erf}\left(-\sqrt{-b \log(F)} \sqrt{x}\right)}{8 \sqrt{-b \log(F)} b^3 \log(F)^3} + \frac{\left(4b^2 x^{\frac{5}{2}} \log(F)^2 - 10bx^{\frac{3}{2}} \log(F) + 15\sqrt{x}\right) e^{(bx \log(F) + a \log(F))}}{4b^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(b*x+a)*x^(5/2),x, algorithm="giac")`

[Out]  $15/8*\sqrt{\pi}*F^a*\operatorname{erf}(-\sqrt{-b*\log(F)}*\sqrt{x})/(\sqrt{-b*\log(F)}*b^3*\log(F)^3) + 1/4*(4*b^2*x^{5/2}*\log(F)^2 - 10*b*x^{3/2}*\log(F) + 15*\sqrt{x})*e^{(b*x*\log(F) + a*\log(F))/(b^3*\log(F)^3)}$

**Mupad [B]**

time = 3.46, size = 72, normalized size = 0.67

$$\frac{F^a x^{5/2} \left( F^{bx} \left( \frac{15 \sqrt{-bx \ln(F)}}{4} + \frac{5(-bx \ln(F))^{3/2}}{2} + (-bx \ln(F))^{5/2} \right) + \frac{15 \sqrt{\pi} \operatorname{erfc}\left(\sqrt{-bx \ln(F)}\right)}{8} \right)}{b \ln(F) (-bx \ln(F))^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{int}(F^{(a + b*x)}*x^{5/2}, x)$

[Out]  $(F^a*x^{5/2}*(F^{(b*x)}*((15*(-b*x*\log(F))^{1/2})/4 + (5*(-b*x*\log(F))^{3/2})/2 + (-b*x*\log(F))^{5/2})) + (15*\pi^{1/2}*\operatorname{erfc}((-b*x*\log(F))^{1/2}))/8)/(b*\log(F)*(-b*x*\log(F))^{5/2})$

### 3.32 $\int F^{a+bx} x^{3/2} dx$

**Optimal.** Leaf size=85

$$\frac{3F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right)}{4b^{5/2} \log^{5/2}(F)} - \frac{3F^{a+bx} \sqrt{x}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{3/2}}{b \log(F)}$$

[Out]  $F^{(b*x+a)}*x^{(3/2)}/b/\ln(F)+3/4*F^a*\operatorname{erfi}(b^{(1/2)}*x^{(1/2)}*\ln(F)^{(1/2)})*\Pi^{(1/2)}/b^{(5/2)}/\ln(F)^{(5/2)}-3/2*F^{(b*x+a)}*x^{(1/2)}/b^2/\ln(F)^2$

**Rubi [A]**

time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2207, 2211, 2235}

$$\frac{3\sqrt{\pi} F^a \operatorname{Erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right)}{4b^{5/2} \log^{5/2}(F)} - \frac{3\sqrt{x} F^{a+bx}}{2b^2 \log^2(F)} + \frac{x^{3/2} F^{a+bx}}{b \log(F)}$$

Antiderivative was successfully verified.

[In] `Int[F^(a + b*x)*x^(3/2), x]`

[Out]  $(3*F^a*\operatorname{Sqrt}[\Pi]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]*\operatorname{Sqrt}[\operatorname{Log}[F]]])/(4*b^{(5/2)}*\operatorname{Log}[F]^{(5/2)}) - (3*F^{(a + b*x)}*\operatorname{Sqrt}[x])/(2*b^2*\operatorname{Log}[F]^2) + (F^{(a + b*x)}*x^{(3/2)})/(b*\operatorname{Log}[F])$

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int F^{a+bx} x^{3/2} dx &= \frac{F^{a+bx} x^{3/2}}{b \log(F)} - \frac{3 \int F^{a+bx} \sqrt{x} dx}{2b \log(F)} \\
&= -\frac{3F^{a+bx} \sqrt{x}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{3/2}}{b \log(F)} + \frac{3 \int \frac{F^{a+bx}}{\sqrt{x}} dx}{4b^2 \log^2(F)} \\
&= -\frac{3F^{a+bx} \sqrt{x}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{3/2}}{b \log(F)} + \frac{3 \text{Subst}\left(\int F^{a+bx^2} dx, x, \sqrt{x}\right)}{2b^2 \log^2(F)} \\
&= \frac{3F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right)}{4b^{5/2} \log^{5/2}(F)} - \frac{3F^{a+bx} \sqrt{x}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{3/2}}{b \log(F)}
\end{aligned}$$

**Mathematica [A]**

time = 0.09, size = 36, normalized size = 0.42

$$\frac{F^a \Gamma\left(\frac{5}{2}, -bx \log(F)\right) \sqrt{-bx \log(F)}}{b^3 \sqrt{x} \log^3(F)}$$

Antiderivative was successfully verified.

`[In] Integrate[F^(a + b*x)*x^(3/2), x]``[Out] (F^a*Gamma[5/2, -(b*x*Log[F])]*Sqrt[-(b*x*Log[F])])/(b^3*Sqrt[x]*Log[F]^3)`**Maple [A]**

time = 0.01, size = 75, normalized size = 0.88

method	result	size
meijerg	$F^a \left( -\frac{\sqrt{x} (-b)^{\frac{5}{2}} \sqrt{\ln(F)} (-10xb \ln(F) + 15) e^{xb \ln(F)}}{10b^2} + \frac{3(-b)^{\frac{5}{2}} \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\ln(F)}\right)}{4b^{\frac{5}{2}}} \right)$	75

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(F^(b*x+a)*x^(3/2), x, method=_RETURNVERBOSE)`

```
[Out] -F^a/(-b)^(3/2)/ln(F)^(5/2)/b*(-1/10*x^(1/2)*(-b)^(5/2)*ln(F)^(1/2)*(-10*x*
b*ln(F)+15)/b^2*exp(x*b*ln(F))+3/4*(-b)^(5/2)/b^(5/2)*Pi^(1/2)*erfi(b^(1/2)
*x^(1/2)*ln(F)^(1/2))
```

**Maxima [A]**

time = 0.33, size = 24, normalized size = 0.28

$$-\frac{F^a x^{\frac{5}{2}} \Gamma\left(\frac{5}{2}, -bx \log(F)\right)}{(-bx \log(F))^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(b*x+a)*x^(3/2),x, algorithm="maxima")``[Out] -F^a*x^(5/2)*gamma(5/2, -b*x*log(F))/(-b*x*log(F))^(5/2)`**Fricas [A]**

time = 0.38, size = 65, normalized size = 0.76

$$\frac{3\sqrt{\pi}\sqrt{-b\log(F)}F^a\operatorname{erf}\left(\sqrt{-b\log(F)}\sqrt{x}\right)-2(2b^2x\log(F)^2-3b\log(F))F^{bx+a}\sqrt{x}}{4b^3\log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(b*x+a)*x^(3/2),x, algorithm="fricas")``[Out] -1/4*(3*sqrt(pi)*sqrt(-b*log(F))*F^a*erf(sqrt(-b*log(F))*sqrt(x)) - 2*(2*b^2*x*log(F)^2 - 3*b*log(F))*F^(b*x + a)*sqrt(x))/(b^3*log(F)^3)`**Sympy [A]**

time = 8.36, size = 37, normalized size = 0.44

$$-\frac{4F^a F^{bx} b x^{\frac{7}{2}} \log(F)}{35} + \frac{2F^a F^{bx} x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F**(b*x+a)*x**(3/2),x)``[Out] -4*F**a*F**(b*x)*b*x**(7/2)*log(F)/35 + 2*F**a*F**(b*x)*x**(5/2)/5`**Giac [A]**

time = 3.01, size = 70, normalized size = 0.82

$$-\frac{3\sqrt{\pi}F^a\operatorname{erf}\left(-\sqrt{-b\log(F)}\sqrt{x}\right)}{4\sqrt{-b\log(F)}b^2\log(F)^2} + \frac{\left(2bx^{\frac{3}{2}}\log(F)-3\sqrt{x}\right)e^{(bx\log(F)+a\log(F))}}{2b^2\log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(b*x+a)*x^(3/2),x, algorithm="giac")``[Out] -3/4*sqrt(pi)*F^a*erf(-sqrt(-b*log(F))*sqrt(x))/(sqrt(-b*log(F))*b^2*log(F)^2) + 1/2*(2*b*x^(3/2)*log(F) - 3*sqrt(x))*e^(b*x*log(F) + a*log(F))/(b^2*log(F)^2)`



**Mupad [B]**

time = 3.42, size = 75, normalized size = 0.88

$$\frac{F^a F^{bx} x^{3/2}}{b \ln(F)} - \frac{3 F^a F^{bx} \sqrt{x}}{2 b^2 \ln(F)^2} + \frac{3 F^a x^{3/2} \sqrt{\pi} \operatorname{erfc}\left(\sqrt{-bx \ln(F)}\right)}{4 b \ln(F) (-bx \ln(F))^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a + b*x)*x^(3/2),x)`

[Out] `(F^a*F^(b*x)*x^(3/2))/(b*log(F)) - (3*F^a*F^(b*x)*x^(1/2))/(2*b^2*log(F)^2) + (3*F^a*x^(3/2)*pi^(1/2)*erfc((-b*x*log(F))^(1/2)))/(4*b*log(F)*(-b*x*log(F))^(3/2))`

### 3.33 $\int F^{a+bx} \sqrt{x} dx$

Optimal. Leaf size=62

$$-\frac{F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right)}{2b^{3/2} \log^{3/2}(F)} + \frac{F^{a+bx} \sqrt{x}}{b \log(F)}$$

[Out]  $-1/2 * F^a * \operatorname{erfi}(b^{(1/2)} * x^{(1/2)} * \ln(F)^{(1/2)}) * \pi^{(1/2)} / b^{(3/2)} / \ln(F)^{(3/2)} + F^{(b*x+a)} * x^{(1/2)} / b / \ln(F)$

Rubi [A]

time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2207, 2211, 2235}

$$\frac{\sqrt{x} F^{a+bx}}{b \log(F)} - \frac{\sqrt{\pi} F^a \operatorname{Erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right)}{2b^{3/2} \log^{3/2}(F)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b*x)} * \operatorname{Sqrt}[x], x]$

[Out]  $-1/2 * (F^a * \operatorname{Sqrt}[\pi] * \operatorname{Erfi}[\operatorname{Sqrt}[b] * \operatorname{Sqrt}[x] * \operatorname{Sqrt}[\log[F]]]) / (b^{(3/2)} * \log[F]^{(3/2)}) + (F^{(a + b*x)} * \operatorname{Sqrt}[x]) / (b * \log[F])$

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int F^{a+bx} \sqrt{x} \, dx &= \frac{F^{a+bx} \sqrt{x}}{b \log(F)} - \frac{\int \frac{F^{a+bx}}{\sqrt{x}} \, dx}{2b \log(F)} \\
&= \frac{F^{a+bx} \sqrt{x}}{b \log(F)} - \frac{\text{Subst}\left(\int F^{a+bx^2} \, dx, x, \sqrt{x}\right)}{b \log(F)} \\
&= -\frac{F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right)}{2b^{3/2} \log^{3/2}(F)} + \frac{F^{a+bx} \sqrt{x}}{b \log(F)}
\end{aligned}$$

**Mathematica [A]**

time = 0.05, size = 30, normalized size = 0.48

$$-\frac{F^a x^{3/2} \Gamma\left(\frac{3}{2}, -bx \log(F)\right)}{(-bx \log(F))^{3/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[F^(a + b*x)*Sqrt[x], x]``[Out] -((F^a*x^(3/2)*Gamma[3/2, -(b*x*Log[F])])/(-(b*x*Log[F]))^(3/2))`**Maple [A]**

time = 0.01, size = 66, normalized size = 1.06

method	result	size
meijerg	$-\frac{F^a \left( \frac{\sqrt{x} (-b)^{\frac{3}{2}} \sqrt{\ln(F)} e^{xb \ln(F)} (-b)^{\frac{3}{2}} \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\ln(F)}\right)}{b \cdot 2b^{\frac{3}{2}}} \right)}{\sqrt{-b} \ln(F)^{\frac{3}{2}} b}$	66

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(F^(b*x+a)*x^(1/2), x, method=_RETURNVERBOSE)`
`[Out] -F^a/(-b)^(1/2)/ln(F)^(3/2)/b*(x^(1/2)*(-b)^(3/2)*ln(F)^(1/2)/b*exp(x*b*ln(F))-1/2*(-b)^(3/2)/b^(3/2)*Pi^(1/2)*erfi(b^(1/2)*x^(1/2)*ln(F)^(1/2))`
**Maxima [A]**

time = 0.32, size = 24, normalized size = 0.39

$$-\frac{F^a x^{\frac{3}{2}} \Gamma\left(\frac{3}{2}, -bx \log(F)\right)}{(-bx \log(F))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(b\*x+a)\*x^(1/2),x, algorithm="maxima")

[Out]  $-F^a x^{3/2} \gamma(3/2, -b x \log(F)) / (-b x \log(F))^{3/2}$

**Fricas** [A]

time = 0.39, size = 51, normalized size = 0.82

$$\frac{2 F^{bx+a} b \sqrt{x} \log(F) + \sqrt{\pi} \sqrt{-b \log(F)} F^a \operatorname{erf}\left(\sqrt{-b \log(F)} \sqrt{x}\right)}{2 b^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(b\*x+a)\*x^(1/2),x, algorithm="fricas")

[Out]  $1/2 * (2 * F^{(b*x + a)} * b * \sqrt{x} * \log(F) + \sqrt{\pi} * \sqrt{-b * \log(F)} * F^a * \operatorname{erf}(\sqrt{-b * \log(F)} * \sqrt{x})) / (b^2 * \log(F)^2)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int F^{a+bx} \sqrt{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(b\*x+a)\*x\*\*(1/2),x)

[Out] Integral(F\*\*(a + b\*x)\*sqrt(x), x)

**Giac** [A]

time = 3.13, size = 58, normalized size = 0.94

$$\frac{\sqrt{\pi} F^a \operatorname{erf}\left(-\sqrt{-b \log(F)} \sqrt{x}\right)}{2 \sqrt{-b \log(F)} b \log(F)} + \frac{\sqrt{x} e^{(bx \log(F) + a \log(F))}}{b \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(b\*x+a)\*x^(1/2),x, algorithm="giac")

[Out]  $1/2 * \sqrt{\pi} * F^a * \operatorname{erf}(-\sqrt{-b * \log(F)} * \sqrt{x}) / (\sqrt{-b * \log(F)} * b * \log(F)) + \sqrt{x} * e^{(b * x * \log(F) + a * \log(F))} / (b * \log(F))$

**Mupad** [B]

time = 3.41, size = 55, normalized size = 0.89

$$\frac{F^a F^{bx} \sqrt{x}}{b \ln(F)} + \frac{F^a \sqrt{x} \sqrt{\pi} \operatorname{erfc}\left(\sqrt{-bx \ln(F)}\right)}{2 b \ln(F) \sqrt{-bx \ln(F)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a + b\*x)\*x^(1/2),x)

[Out]  $(F^a * F^{(b*x)} * x^{1/2}) / (b * \log(F)) + (F^a * x^{1/2} * \pi^{1/2} * \operatorname{erfc}((-b * x * \log(F))^{1/2})) / (2 * b * \log(F) * (-b * x * \log(F))^{1/2})$

$$3.34 \quad \int \frac{F^{a+bx}}{\sqrt{x}} dx$$

Optimal. Leaf size=38

$$\frac{F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right)}{\sqrt{b} \sqrt{\log(F)}}$$

[Out]  $F^a \operatorname{erfi}(b^{(1/2)} x^{(1/2)} \ln(F)^{(1/2)}) \pi^{(1/2)} / b^{(1/2)} / \ln(F)^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2211, 2235}

$$\frac{\sqrt{\pi} F^a \operatorname{Erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right)}{\sqrt{b} \sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*x)/Sqrt[x], x]

[Out] (F^a\*Sqrt[Pi]\*Erfi[Sqrt[b]\*Sqrt[x]\*Sqrt[Log[F]]])/(Sqrt[b]\*Sqrt[Log[F]])

Rule 2211

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/Sqrt[(c\_) + (d\_)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - c\*(f/d)) + f\*g\*(x^2/d)), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

Rule 2235

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^2), x\_Symbol] :> Simp[F^a\*Sqrt[Pi]\*(Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]]/(2\*d\*Rt[b\*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rubi steps

$$\begin{aligned} \int \frac{F^{a+bx}}{\sqrt{x}} dx &= 2 \operatorname{Subst}\left(\int F^{a+bx^2} dx, x, \sqrt{x}\right) \\ &= \frac{F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right)}{\sqrt{b} \sqrt{\log(F)}} \end{aligned}$$

**Mathematica [A]**

time = 0.06, size = 30, normalized size = 0.79

$$-\frac{F^a \sqrt{x} \Gamma\left(\frac{1}{2}, -bx \log(F)\right)}{\sqrt{-bx \log(F)}}$$

Antiderivative was successfully verified.

`[In] Integrate[F^(a + b*x)/Sqrt[x], x]``[Out] -((F^a*Sqrt[x]*Gamma[1/2, -(b*x*Log[F])])/Sqrt[-(b*x*Log[F])])`**Maple [A]**

time = 0.02, size = 27, normalized size = 0.71

method	result	size
meijerg	$\frac{F^a \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\ln(F)}\right) \sqrt{\pi}}{\sqrt{b} \sqrt{\ln(F)}}$	27

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(F^(b*x+a)/x^(1/2), x, method=_RETURNVERBOSE)``[Out] F^a*erfi(b^(1/2)*x^(1/2)*ln(F)^(1/2))*Pi^(1/2)/b^(1/2)/ln(F)^(1/2)`**Maxima [A]**

time = 0.32, size = 29, normalized size = 0.76

$$\frac{\sqrt{\pi} F^a \sqrt{x} \left( \operatorname{erf}\left(\sqrt{-bx \log(F)}\right) - 1 \right)}{\sqrt{-bx \log(F)}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(b*x+a)/x^(1/2), x, algorithm="maxima")``[Out] sqrt(pi)*F^a*sqrt(x)*(erf(sqrt(-b*x*log(F))) - 1)/sqrt(-b*x*log(F))`**Fricas [A]**

time = 0.40, size = 34, normalized size = 0.89

$$-\frac{\sqrt{\pi} \sqrt{-b \log(F)} F^a \operatorname{erf}\left(\sqrt{-b \log(F)} \sqrt{x}\right)}{b \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(b*x+a)/x^(1/2), x, algorithm="fricas")`

[Out]  $-\sqrt{\pi} \sqrt{-b \log(F)} F^a \operatorname{erf}(\sqrt{-b \log(F)} \sqrt{x}) / (b \log(F))$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{a+bx}}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(b*x+a)/x**(1/2),x)`

[Out] `Integral(F**(a + b*x)/sqrt(x), x)`

**Giac [A]**

time = 3.09, size = 28, normalized size = 0.74

$$-\frac{\sqrt{\pi} F^a \operatorname{erf}\left(-\sqrt{-b \log(F)} \sqrt{x}\right)}{\sqrt{-b \log(F)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(b*x+a)/x^(1/2),x, algorithm="giac")`

[Out]  $-\sqrt{\pi} F^a \operatorname{erf}(-\sqrt{-b \log(F)} \sqrt{x}) / \sqrt{-b \log(F)}$

**Mupad [B]**

time = 3.50, size = 32, normalized size = 0.84

$$\frac{F^a \operatorname{erfc}\left(\sqrt{-b x \ln(F)}\right) \sqrt{-\pi b x \ln(F)}}{b \sqrt{x} \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a + b*x)/x^(1/2),x)`

[Out]  $(F^a \operatorname{erfc}((-b*x*\log(F))^{1/2})) * (-b*x*\pi*\log(F))^{1/2} / (b*x^{1/2}*\log(F))$

### 3.35 $\int \frac{F^{a+bx}}{x^{3/2}} dx$

Optimal. Leaf size=54

$$-\frac{2F^{a+bx}}{\sqrt{x}} + 2\sqrt{b} F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right) \sqrt{\log(F)}$$

[Out]  $-2F^{(b*x+a)}/x^{(1/2)}+2F^a*\operatorname{erfi}(b^{(1/2)}*x^{(1/2)}*\ln(F)^{(1/2)})*b^{(1/2)}*\pi^{(1/2)}*\ln(F)^{(1/2)}$

Rubi [A]

time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2208, 2211, 2235}

$$2\sqrt{\pi} \sqrt{b} F^a \sqrt{\log(F)} \operatorname{Erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right) - \frac{2F^{a+bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] `Int[F^(a + b*x)/x^(3/2), x]`

[Out] `(-2F^(a + b*x))/Sqrt[x] + 2*Sqrt[b]*F^a*Sqrt[Pi]*Erfi[Sqrt[b]*Sqrt[x]*Sqrt[Log[F]]]*Sqrt[Log[F]]`

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1))), x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol]
:> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol]
:> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps



$$\begin{aligned}
\int \frac{F^{a+bx}}{x^{3/2}} dx &= -\frac{2F^{a+bx}}{\sqrt{x}} + (2b \log(F)) \int \frac{F^{a+bx}}{\sqrt{x}} dx \\
&= -\frac{2F^{a+bx}}{\sqrt{x}} + (4b \log(F)) \text{Subst} \left( \int F^{a+bx^2} dx, x, \sqrt{x} \right) \\
&= -\frac{2F^{a+bx}}{\sqrt{x}} + 2\sqrt{b} F^a \sqrt{\pi} \operatorname{erfi} \left( \sqrt{b} \sqrt{x} \sqrt{\log(F)} \right) \sqrt{\log(F)}
\end{aligned}$$

**Mathematica [A]**

time = 0.11, size = 38, normalized size = 0.70

$$-\frac{2F^a \left( F^{bx} - \Gamma\left(\frac{1}{2}, -bx \log(F)\right) \sqrt{-bx \log(F)} \right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

`[In] Integrate[F^(a + b*x)/x^(3/2), x]``[Out] (-2*F^a*(F^(b*x) - Gamma[1/2, -(b*x*Log[F])])*Sqrt[-(b*x*Log[F])])/Sqrt[x]`**Maple [A]**

time = 0.01, size = 64, normalized size = 1.19

method	result	size
meijerg	$-\frac{F^{a(-b)^{\frac{3}{2}} \sqrt{\ln(F)} \left( -\frac{2e^{xb \ln(F)}}{\sqrt{x} \sqrt{-b} \sqrt{\ln(F)}} + \frac{2\sqrt{b} \sqrt{\pi} \operatorname{erfi}(\sqrt{b} \sqrt{x} \sqrt{\ln(F)})}{\sqrt{-b}} \right)}{b}$	64

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(F^(b*x+a)/x^(3/2), x, method=_RETURNVERBOSE)``[Out] -F^a*(-b)^(3/2)*ln(F)^(1/2)/b*(-2/x^(1/2)/(-b)^(1/2)/ln(F)^(1/2)*exp(x*b*ln(F))+2/(-b)^(1/2)*b^(1/2)*Pi^(1/2)*erfi(b^(1/2)*x^(1/2)*ln(F)^(1/2))`**Maxima [A]**

time = 0.32, size = 24, normalized size = 0.44

$$-\frac{\sqrt{-bx \log(F)} F^a \Gamma\left(-\frac{1}{2}, -bx \log(F)\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(b*x+a)/x^(3/2), x, algorithm="maxima")`

[Out]  $-\sqrt{-b*x*\log(F)}*F^a*\gamma(-1/2, -b*x*\log(F))/\sqrt{x}$

**Fricas** [A]

time = 0.45, size = 44, normalized size = 0.81

$$\frac{2 \left( \sqrt{\pi} \sqrt{-b \log(F)} F^a x \operatorname{erf} \left( \sqrt{-b \log(F)} \sqrt{x} \right) + F^{bx+a} \sqrt{x} \right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(b*x+a)/x^(3/2),x, algorithm="fricas")`

[Out]  $-2*(\sqrt{\pi}*\sqrt{-b*\log(F)}*F^a*x*\operatorname{erf}(\sqrt{-b*\log(F)}*\sqrt{x}) + F^{(b*x + a)*\sqrt{x}})/x$

**Sympy** [A]

time = 0.93, size = 34, normalized size = 0.63

$$4F^a F^{bx} b \sqrt{x} \log(F) - \frac{2F^a F^{bx}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(b*x+a)/x**(3/2),x)`

[Out]  $4*F^{a+b*x}*b*\sqrt{x}*\log(F) - 2*F^{a+b*x}/\sqrt{x}$

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(b*x+a)/x^(3/2),x, algorithm="giac")`

[Out] `integrate(F^(b*x + a)/x^(3/2), x)`

**Mupad** [B]

time = 3.49, size = 42, normalized size = 0.78

$$\frac{2 F^a \sqrt{\pi} \operatorname{erfc} \left( \sqrt{-b x \ln(F)} \right) \sqrt{-b x \ln(F)}}{\sqrt{x}} - \frac{2 F^a F^{b x}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a + b*x)/x^(3/2),x)`

[Out]  $(2*F^a*\pi^{(1/2)}*\operatorname{erfc}((-b*x*\log(F))^{(1/2)})*(-b*x*\log(F))^{(1/2)})/x^{(1/2)} - (2*F^a*F^{(b*x)})/x^{(1/2)}$

### 3.36 $\int \frac{F^{a+bx}}{x^{5/2}} dx$

Optimal. Leaf size=77

$$-\frac{2F^{a+bx}}{3x^{3/2}} - \frac{4bF^{a+bx} \log(F)}{3\sqrt{x}} + \frac{4}{3}b^{3/2}F^a\sqrt{\pi} \operatorname{erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right) \log^{3/2}(F)$$

[Out]  $-2/3F^{(b*x+a)}/x^{(3/2)}+4/3b^{(3/2)}*F^a*\operatorname{erfi}(b^{(1/2)}*x^{(1/2)}*\ln(F)^{(1/2)})*\ln(F)^{(3/2)}*\pi^{(1/2)}-4/3*b*F^{(b*x+a)}*\ln(F)/x^{(1/2)}$

Rubi [A]

time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2208, 2211, 2235}

$$\frac{4}{3}\sqrt{\pi}b^{3/2}F^a\log^{3/2}(F)\operatorname{Erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right) - \frac{2F^{a+bx}}{3x^{3/2}} - \frac{4b\log(F)F^{a+bx}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b*x)}/x^{(5/2)}, x]$

[Out]  $(-2*F^{(a + b*x)})/(3*x^{(3/2)}) - (4*b*F^{(a + b*x)}*\operatorname{Log}[F])/(3*\operatorname{Sqrt}[x]) + (4*b^{(3/2)}*F^a*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]*\operatorname{Sqrt}[\operatorname{Log}[F]]]*\operatorname{Log}[F]^{(3/2)})/3$

Rule 2208

$\operatorname{Int}[(b_*)*(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))^{(n_*)*((c_*) + (d_*)*(x_*))^{(m_*)}}, x\_Symbol] :> \operatorname{Simp}[(c + d*x)^{(m + 1)}*((b*F^{(g*(e + f*x)))^n/(d*(m + 1)))], x] - \operatorname{Dist}[f*g*n*(\operatorname{Log}[F]/(d*(m + 1))), \operatorname{Int}[(c + d*x)^{(m + 1)}*(b*F^{(g*(e + f*x)))^n, x], x] /; \operatorname{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{IntegerQ}[2*m] \&\& !\operatorname{TrueQ}[\$UseGamma]$

Rule 2211

$\operatorname{Int}[(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))}/\operatorname{Sqrt}[(c_*) + (d_*)*(x_*)], x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\operatorname{TrueQ}[\$UseGamma]$

Rule 2235

$\operatorname{Int}[(F_*)^{((a_*) + (b_*)*((c_*) + (d_*)*(x_*))^{(2)}), x\_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\pi]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \&\& \operatorname{PosQ}[b]$

Rubi steps

$$\begin{aligned}
\int \frac{F^{a+bx}}{x^{5/2}} dx &= -\frac{2F^{a+bx}}{3x^{3/2}} + \frac{1}{3}(2b \log(F)) \int \frac{F^{a+bx}}{x^{3/2}} dx \\
&= -\frac{2F^{a+bx}}{3x^{3/2}} - \frac{4bF^{a+bx} \log(F)}{3\sqrt{x}} + \frac{1}{3}(4b^2 \log^2(F)) \int \frac{F^{a+bx}}{\sqrt{x}} dx \\
&= -\frac{2F^{a+bx}}{3x^{3/2}} - \frac{4bF^{a+bx} \log(F)}{3\sqrt{x}} + \frac{1}{3}(8b^2 \log^2(F)) \text{Subst}\left(\int F^{a+bx^2} dx, x, \sqrt{x}\right) \\
&= -\frac{2F^{a+bx}}{3x^{3/2}} - \frac{4bF^{a+bx} \log(F)}{3\sqrt{x}} + \frac{4}{3}b^{3/2}F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right) \log^{\frac{3}{2}}(F)
\end{aligned}$$

**Mathematica [A]**

time = 0.10, size = 49, normalized size = 0.64

$$-\frac{2F^a(2\Gamma(\frac{1}{2}, -bx \log(F))(-bx \log(F))^{3/2} + F^{bx}(1 + 2bx \log(F)))}{3x^{3/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[F^(a + b*x)/x^(5/2), x]`

```
[Out] (-2*F^a*(2*Gamma[1/2, -(b*x*Log[F])]*(-(b*x*Log[F]))^(3/2) + F^(b*x)*(1 + 2
*b*x*Log[F]))) / (3*x^(3/2))
```

**Maple [A]**

time = 0.01, size = 72, normalized size = 0.94

method	result	size
meijerg	$-\frac{F^a(-b)^{\frac{5}{2}} \ln(F)^{\frac{3}{2}} \left( -\frac{2(2xb \ln(F)+1)e^{xb \ln(F)}}{3x^{\frac{3}{2}}(-b)^{\frac{3}{2}} \ln(F)^{\frac{3}{2}}} + \frac{4b^{\frac{3}{2}} \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\ln(F)}\right)}{3(-b)^{\frac{3}{2}}} \right)}{b}$	72

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(F^(b*x+a)/x^(5/2), x, method=_RETURNVERBOSE)`

```
[Out] -F^a*(-b)^(5/2)*ln(F)^(3/2)/b*(-2/3/x^(3/2)/(-b)^(3/2)/ln(F)^(3/2)*(2*x*b*ln(F)+1)*exp(x*b*ln(F))+4/3/(-b)^(3/2)*b^(3/2)*Pi^(1/2)*erfi(b^(1/2)*x^(1/2)*ln(F)^(1/2))
```

**Maxima [A]**

time = 0.32, size = 24, normalized size = 0.31

$$-\frac{(-bx \log(F))^{\frac{3}{2}} F^a \Gamma(-\frac{3}{2}, -bx \log(F))}{x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(b\*x+a)/x^(5/2),x, algorithm="maxima")

[Out]  $-( -b*x*\log(F) )^{3/2} * F^a * \text{gamma}(-3/2, -b*x*\log(F)) / x^{3/2}$

**Fricas** [A]

time = 0.45, size = 58, normalized size = 0.75

$$\frac{2 \left( 2 \sqrt{\pi} \sqrt{-b \log(F)} F^a b x^2 \operatorname{erf} \left( \sqrt{-b \log(F)} \sqrt{x} \right) \log(F) + (2 b x \log(F) + 1) F^{b x + a} \sqrt{x} \right)}{3 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(b\*x+a)/x^(5/2),x, algorithm="fricas")

[Out]  $-2/3 * (2 * \sqrt{\pi} * \sqrt{-b * \log(F)} * F^a * b * x^2 * \operatorname{erf}(\sqrt{-b * \log(F)} * \sqrt{x}) * \log(F) + (2 * b * x * \log(F) + 1) * F^{(b * x + a)} * \sqrt{x}) / x^2$

**Sympy** [A]

time = 9.66, size = 39, normalized size = 0.51

$$-\frac{4 F^a F^{b x} b \log(F)}{3 \sqrt{x}} - \frac{2 F^a F^{b x}}{3 x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(b\*x+a)/x\*\*(5/2),x)

[Out]  $-4 * F^{**a} * F^{**}(b * x) * b * \log(F) / (3 * \sqrt{x}) - 2 * F^{**a} * F^{**}(b * x) / (3 * x^{3/2})$

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(b\*x+a)/x^(5/2),x, algorithm="giac")

[Out] integrate(F^(b\*x + a)/x^(5/2), x)

**Mupad** [B]

time = 3.51, size = 61, normalized size = 0.79

$$\frac{4 F^a b \sqrt{\pi} \operatorname{erfc} \left( \sqrt{-b x \ln(F)} \right) \ln(F) \sqrt{-b x \ln(F)}}{3 \sqrt{x}} - \frac{4 F^a F^{b x} b \ln(F)}{3 \sqrt{x}} - \frac{2 F^a F^{b x}}{3 x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a + b\*x)/x^(5/2),x)

[Out]  $(4 * F^a * b * \pi^{1/2} * \operatorname{erfc}((-b * x * \log(F))^{1/2}) * \log(F) * (-b * x * \log(F))^{1/2}) / (3 * x^{1/2}) - (4 * F^a * F^{(b * x)} * b * \log(F)) / (3 * x^{1/2}) - (2 * F^a * F^{(b * x)}) / (3 * x^{3/2})$

### 3.37 $\int \frac{F^{a+bx}}{x^{7/2}} dx$

**Optimal.** Leaf size=100

$$-\frac{2F^{a+bx}}{5x^{5/2}} - \frac{4bF^{a+bx} \log(F)}{15x^{3/2}} - \frac{8b^2 F^{a+bx} \log^2(F)}{15\sqrt{x}} + \frac{8}{15} b^{5/2} F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right) \log^{5/2}(F)$$

[Out]  $-2/5 * F^{(b*x+a)}/x^{(5/2)} - 4/15 * b * F^{(b*x+a)} * \ln(F) / x^{(3/2)} + 8/15 * b^{(5/2)} * F^a * \operatorname{erfi}(b^{(1/2)} * x^{(1/2)} * \ln(F)^{(1/2)}) * \ln(F)^{(5/2)} * \pi^{(1/2)} - 8/15 * b^2 * F^{(b*x+a)} * \ln(F)^2 / x^{(1/2)}$

**Rubi [A]**

time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2208, 2211, 2235}

$$\frac{8}{15} \sqrt{\pi} b^{5/2} F^a \log^{5/2}(F) \operatorname{Erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right) - \frac{8b^2 \log^2(F) F^{a+bx}}{15\sqrt{x}} - \frac{2F^{a+bx}}{5x^{5/2}} - \frac{4b \log(F) F^{a+bx}}{15x^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b*x)}/x^{(7/2)}, x]$

[Out]  $(-2 * F^{(a + b*x)}) / (5 * x^{(5/2)}) - (4 * b * F^{(a + b*x)} * \operatorname{Log}[F]) / (15 * x^{(3/2)}) - (8 * b^2 * F^{(a + b*x)} * \operatorname{Log}[F]^2) / (15 * \operatorname{Sqrt}[x]) + (8 * b^{(5/2)} * F^a * \operatorname{Sqrt}[\pi] * \operatorname{Erfi}[\operatorname{Sqrt}[b] * \operatorname{Sqrt}[x] * \operatorname{Sqrt}[\operatorname{Log}[F]]] * \operatorname{Log}[F]^{(5/2)}) / 15$

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{F^{a+bx}}{x^{7/2}} dx &= -\frac{2F^{a+bx}}{5x^{5/2}} + \frac{1}{5}(2b \log(F)) \int \frac{F^{a+bx}}{x^{5/2}} dx \\
&= -\frac{2F^{a+bx}}{5x^{5/2}} - \frac{4bF^{a+bx} \log(F)}{15x^{3/2}} + \frac{1}{15}(4b^2 \log^2(F)) \int \frac{F^{a+bx}}{x^{3/2}} dx \\
&= -\frac{2F^{a+bx}}{5x^{5/2}} - \frac{4bF^{a+bx} \log(F)}{15x^{3/2}} - \frac{8b^2 F^{a+bx} \log^2(F)}{15\sqrt{x}} + \frac{1}{15}(8b^3 \log^3(F)) \int \frac{F^{a+bx}}{\sqrt{x}} dx \\
&= -\frac{2F^{a+bx}}{5x^{5/2}} - \frac{4bF^{a+bx} \log(F)}{15x^{3/2}} - \frac{8b^2 F^{a+bx} \log^2(F)}{15\sqrt{x}} + \frac{1}{15}(16b^3 \log^3(F)) \text{Subst}\left(\int F^{a+bx^2} dx, x, \sqrt{x}\right) \\
&= -\frac{2F^{a+bx}}{5x^{5/2}} - \frac{4bF^{a+bx} \log(F)}{15x^{3/2}} - \frac{8b^2 F^{a+bx} \log^2(F)}{15\sqrt{x}} + \frac{8}{15}b^{5/2}F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right) \log(F)
\end{aligned}$$

**Mathematica [A]**

time = 0.11, size = 61, normalized size = 0.61

$$-\frac{2F^a \left(-4\Gamma\left(\frac{1}{2}, -bx \log(F)\right) (-bx \log(F))^{5/2} + F^{bx} (3 + 2bx \log(F) + 4b^2 x^2 \log^2(F))\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

**[In]** Integrate[F^(a + b\*x)/x^(7/2), x]**[Out]**  $(-2F^a(-4\Gamma[1/2, -(b*x*\text{Log}[F])])*(-(b*x*\text{Log}[F]))^{(5/2)} + F^{(b*x)}*(3 + 2*b*x*\text{Log}[F] + 4*b^2*x^2*\text{Log}[F]^2))/(15*x^{(5/2)})$ **Maple [A]**

time = 0.01, size = 84, normalized size = 0.84

method	result	size
meijerg	$ \frac{F^a(-b)^{\frac{7}{2}} \ln(F)^{\frac{5}{2}} \left( -\frac{2 \left( \frac{4b^2 x^2 \ln(F)^2}{3} + \frac{2xb \ln(F)}{3} + 1 \right) e^{xb \ln(F)} + 8b^{\frac{5}{2}} \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\ln(F)}\right)}{5x^{\frac{5}{2}} (-b)^{\frac{5}{2}} \ln(F)^{\frac{5}{2}}} + \frac{1}{15(-b)^{\frac{5}{2}}} \right)}{b} $	84

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(F^(b\*x+a)/x^(7/2), x, method=\_RETURNVERBOSE)**[Out]**  $-F^a(-b)^{(7/2)}*\ln(F)^{(5/2)}/b*(-2/5/x^{(5/2)}/(-b)^{(5/2)}/\ln(F)^{(5/2)}*(4/3*b^2*x^2*\ln(F)^2+2/3*x*b*\ln(F)+1)*\exp(x*b*\ln(F))+8/15/(-b)^{(5/2)}*b^{(5/2)}*\text{Pi}^{(1/2)}*\operatorname{erfi}(b^{(1/2)}*x^{(1/2)}*\ln(F)^{(1/2)}))$

**Maxima [A]**

time = 0.33, size = 24, normalized size = 0.24

$$-\frac{(-bx \log(F))^{\frac{5}{2}} F^a \Gamma(-\frac{5}{2}, -bx \log(F))}{x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(b*x+a)/x^(7/2),x, algorithm="maxima")``[Out] -(-b*x*log(F))^(5/2)*F^a*gamma(-5/2, -b*x*log(F))/x^(5/2)`**Fricas [A]**

time = 0.42, size = 74, normalized size = 0.74

$$-\frac{2\left(4\sqrt{\pi}\sqrt{-b\log(F)}F^ab^2x^3\operatorname{erf}\left(\sqrt{-b\log(F)}\sqrt{x}\right)\log(F)^2+(4b^2x^2\log(F)^2+2bx\log(F)+3)F^{bx+a}\sqrt{x}\right)}{15x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(b*x+a)/x^(7/2),x, algorithm="fricas")``[Out] -2/15*(4*sqrt(pi)*sqrt(-b*log(F))*F^a*b^2*x^3*erf(sqrt(-b*log(F))*sqrt(x))*log(F)^2+(4*b^2*x^2*log(F)^2+2*b*x*log(F)+3)*F^(b*x+a)*sqrt(x))/x^3`**Sympy [A]**

time = 144.63, size = 39, normalized size = 0.39

$$\frac{4F^a F^{bx} b \log(F)}{15x^{\frac{3}{2}}} - \frac{2F^a F^{bx}}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F**(b*x+a)/x**(7/2),x)``[Out] -4*F**a*F**(b*x)*b*log(F)/(15*x**(3/2)) - 2*F**a*F**(b*x)/(5*x**(5/2))`**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(b*x+a)/x^(7/2),x, algorithm="giac")``[Out] integrate(F^(b*x+a)/x^(7/2), x)`**Mupad [B]**

time = 3.45, size = 80, normalized size = 0.80

$$-\frac{\frac{2F^{a+bx}}{5} + \frac{4F^{a+bx}bx\ln(F)}{15} + \frac{8F^{a+bx}b^2x^2\ln(F)^2}{15} - \frac{8F^ab^2x^2\operatorname{erfc}\left(\sqrt{-bx\ln(F)}\right)\ln(F)^2\sqrt{-\pi bx\ln(F)}}{15}}{x^{5/2}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a + b*x)/x^(7/2),x)`

[Out]  $-\left(\frac{2F^{(a + b*x)}}{5} + \frac{4F^{(a + b*x)}*b*x*\log(F)}{15} + \frac{8F^{(a + b*x)}*b^2*x^2*\log(F)^2}{15} - \frac{8F^{a*b^2*x^2}*erfc((-b*x*\log(F))^{1/2})*\log(F)^2*(-b*x*\pi*\log(F))^{1/2}}{15}\right)/x^{5/2}$

### 3.38 $\int \frac{F^{a+bx}}{x^{9/2}} dx$

**Optimal.** Leaf size=123

$$-\frac{2F^{a+bx}}{7x^{7/2}} - \frac{4bF^{a+bx} \log(F)}{35x^{5/2}} - \frac{8b^2 F^{a+bx} \log^2(F)}{105x^{3/2}} - \frac{16b^3 F^{a+bx} \log^3(F)}{105\sqrt{x}} + \frac{16}{105} b^{7/2} F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right)$$

[Out]  $-2/7 * F^{(b*x+a)}/x^{(7/2)} - 4/35 * b * F^{(b*x+a)} * \ln(F)/x^{(5/2)} - 8/105 * b^2 * F^{(b*x+a)} * \ln(F)^2/x^{(3/2)} + 16/105 * b^{(7/2)} * F^a * \operatorname{erfi}(b^{(1/2)} * x^{(1/2)} * \ln(F)^{(1/2)}) * \ln(F)^{(7/2)} * \pi^{(1/2)} - 16/105 * b^3 * F^{(b*x+a)} * \ln(F)^3/x^{(1/2)}$

**Rubi [A]**

time = 0.08, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2208, 2211, 2235}

$$\frac{16}{105} \sqrt{\pi} b^{7/2} F^a \log^{7/2}(F) \operatorname{Erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right) - \frac{16b^3 \log^3(F) F^{a+bx}}{105\sqrt{x}} - \frac{8b^2 \log^2(F) F^{a+bx}}{105x^{3/2}} - \frac{2F^{a+bx}}{7x^{7/2}} - \frac{4b \log(F) F^{a+bx}}{35x^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(a + b*x)}/x^{(9/2)}, x]$

[Out]  $(-2 * F^{(a + b*x)})/(7 * x^{(7/2)}) - (4 * b * F^{(a + b*x)} * \operatorname{Log}[F])/(35 * x^{(5/2)}) - (8 * b^2 * F^{(a + b*x)} * \operatorname{Log}[F]^2)/(105 * x^{(3/2)}) - (16 * b^3 * F^{(a + b*x)} * \operatorname{Log}[F]^3)/(105 * \operatorname{Sqrt}[x]) + (16 * b^{(7/2)} * F^a * \operatorname{Sqrt}[\pi] * \operatorname{Erfi}[\operatorname{Sqrt}[b] * \operatorname{Sqrt}[x] * \operatorname{Sqrt}[\operatorname{Log}[F]]] * \operatorname{Log}[F]^{(7/2)})/105$

**Rule 2208**

$\operatorname{Int}[(b_*) * (F_*)^{((g_*) * ((e_*) + (f_*) * (x_*)))^{(n_*) * ((c_*) + (d_*) * (x_*))^{(m_*)}}, x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m+1)} * ((b * F^{(g*(e + f*x))})^n / (d * (m+1))), x] - \operatorname{Dist}[f * g * n * (\operatorname{Log}[F] / (d * (m+1))), \operatorname{Int}[(c + d*x)^{(m+1)} * (b * F^{(g*(e + f*x))})^n, x], x] /;$   $\operatorname{FreeQ}\{F, b, c, d, e, f, g, n\}, x \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{IntegerQ}[2*m] \&\& \operatorname{!TrueQ}[\$UseGamma]$

**Rule 2211**

$\operatorname{Int}[(F_*)^{((g_*) * ((e_*) + (f_*) * (x_*))) / \operatorname{Sqrt}[(c_*) + (d_*) * (x_*)]}, x\_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x \&\& \operatorname{!TrueQ}[\$UseGamma]$

**Rule 2235**

$\operatorname{Int}[(F_*)^{((a_*) + (b_*) * ((c_*) + (d_*) * (x_*))^2)}, x\_Symbol] \rightarrow \operatorname{Simp}[F^a * \operatorname{Sqrt}[\pi] * (\operatorname{Erfi}[(c + d*x) * \operatorname{Rt}[b * \operatorname{Log}[F], 2]] / (2 * d * \operatorname{Rt}[b * \operatorname{Log}[F], 2])), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \&\& \operatorname{PosQ}[b]$

Rubi steps

$$\begin{aligned}
\int \frac{F^{a+bx}}{x^{9/2}} dx &= -\frac{2F^{a+bx}}{7x^{7/2}} + \frac{1}{7}(2b \log(F)) \int \frac{F^{a+bx}}{x^{7/2}} dx \\
&= -\frac{2F^{a+bx}}{7x^{7/2}} - \frac{4bF^{a+bx} \log(F)}{35x^{5/2}} + \frac{1}{35}(4b^2 \log^2(F)) \int \frac{F^{a+bx}}{x^{5/2}} dx \\
&= -\frac{2F^{a+bx}}{7x^{7/2}} - \frac{4bF^{a+bx} \log(F)}{35x^{5/2}} - \frac{8b^2 F^{a+bx} \log^2(F)}{105x^{3/2}} + \frac{1}{105}(8b^3 \log^3(F)) \int \frac{F^{a+bx}}{x^{3/2}} dx \\
&= -\frac{2F^{a+bx}}{7x^{7/2}} - \frac{4bF^{a+bx} \log(F)}{35x^{5/2}} - \frac{8b^2 F^{a+bx} \log^2(F)}{105x^{3/2}} - \frac{16b^3 F^{a+bx} \log^3(F)}{105\sqrt{x}} + \frac{1}{105}(16b^4 \log^4(F)) \\
&= -\frac{2F^{a+bx}}{7x^{7/2}} - \frac{4bF^{a+bx} \log(F)}{35x^{5/2}} - \frac{8b^2 F^{a+bx} \log^2(F)}{105x^{3/2}} - \frac{16b^3 F^{a+bx} \log^3(F)}{105\sqrt{x}} + \frac{1}{105}(32b^4 \log^4(F)) \\
&= -\frac{2F^{a+bx}}{7x^{7/2}} - \frac{4bF^{a+bx} \log(F)}{35x^{5/2}} - \frac{8b^2 F^{a+bx} \log^2(F)}{105x^{3/2}} - \frac{16b^3 F^{a+bx} \log^3(F)}{105\sqrt{x}} + \frac{16}{105}b^{7/2}F^a\sqrt{\pi} \operatorname{erfi}
\end{aligned}$$

**Mathematica [A]**

time = 0.12, size = 73, normalized size = 0.59

$$-\frac{2F^a(8\Gamma(\frac{1}{2}, -bx \log(F))(-bx \log(F))^{7/2} + F^{bx}(15 + 6bx \log(F) + 4b^2x^2 \log^2(F) + 8b^3x^3 \log^3(F)))}{105x^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*x)/x^(9/2), x]

[Out]  $(-2F^a(8\Gamma[1/2, -(b*x*\text{Log}[F])])*(-(b*x*\text{Log}[F]))^{7/2} + F^{b*x}(15 + 6*b*x*\text{Log}[F] + 4*b^2*x^2*\text{Log}[F]^2 + 8*b^3*x^3*\text{Log}[F]^3))/(105*x^{7/2})$

**Maple [A]**

time = 0.01, size = 96, normalized size = 0.78

method	result	size
meijerg	$ -\frac{F^a(-b)^{\frac{9}{2}} \ln(F)^{\frac{7}{2}} \left( -\frac{2 \left( \frac{8b^3x^3 \ln(F)^3}{15} + \frac{4b^2x^2 \ln(F)^2}{15} + \frac{2xb \ln(F)}{5} + 1 \right) e^{xb \ln(F)} + \frac{16b^{\frac{7}{2}} \sqrt{\pi} \operatorname{erfi}(\sqrt{b} \sqrt{x} \sqrt{\ln(F)})}{105(-b)^{\frac{7}{2}}} \right)}{b} $	96

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(b\*x+a)/x^(9/2), x, method=\_RETURNVERBOSE)

[Out]  $-F^a(-b)^{(9/2)}*\ln(F)^{(7/2)}/b*(-2/7/x^{(7/2)}/(-b)^{(7/2)}/\ln(F)^{(7/2)}*(8/15*b^3*x^3*\ln(F)^3+4/15*b^2*x^2*\ln(F)^2+2/5*x*b*\ln(F)+1)*\exp(x*b*\ln(F))+16/105/((-b)^{(7/2)}*b^{(7/2)}*\text{Pi}^{(1/2)}*\operatorname{erfi}(b^{(1/2)}*x^{(1/2)}*\ln(F)^{(1/2)}))$

**Maxima [A]**

time = 0.32, size = 24, normalized size = 0.20

$$-\frac{(-bx \log(F))^{\frac{7}{2}} F^a \Gamma\left(-\frac{7}{2}, -bx \log(F)\right)}{x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(b*x+a)/x^(9/2),x, algorithm="maxima")``[Out] -(b*x*log(F))^(7/2)*F^a*gamma(-7/2, -b*x*log(F))/x^(7/2)`**Fricas [A]**

time = 0.39, size = 86, normalized size = 0.70

$$\frac{2\left(8\sqrt{\pi}\sqrt{-b\log(F)}F^ab^3x^4\operatorname{erf}\left(\sqrt{-b\log(F)}\sqrt{x}\right)\log(F)^3+(8b^3x^3\log(F)^3+4b^2x^2\log(F)^2+6bx\log(F)+15)F^{bx+a}\sqrt{x}\right)}{105x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(b*x+a)/x^(9/2),x, algorithm="fricas")`

```
[Out] -2/105*(8*sqrt(pi)*sqrt(-b*log(F))*F^a*b^3*x^4*erf(sqrt(-b*log(F))*sqrt(x))
*log(F)^3 + (8*b^3*x^3*log(F)^3 + 4*b^2*x^2*log(F)^2 + 6*b*x*log(F) + 15)*F
^(b*x + a)*sqrt(x))/x^4
```

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F**(b*x+a)/x**(9/2),x)``[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep`**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(b*x+a)/x^(9/2),x, algorithm="giac")``[Out] integrate(F^(b*x + a)/x^(9/2), x)`**Mupad [B]**

time = 3.53, size = 99, normalized size = 0.80

$$-\frac{\frac{2F^{a+bx}}{7} + \frac{4F^{a+bx}bx\ln(F)}{35} + \frac{8F^{a+bx}b^2x^2\ln(F)^2}{105} + \frac{16F^{a+bx}b^3x^3\ln(F)^3}{105} - \frac{16F^ab^3x^3\operatorname{erfc}\left(\sqrt{-bx\ln(F)}\right)\ln(F)^3\sqrt{-\pi bx\ln(F)}}{105}}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(a + b*x)/x^(9/2),x)
```

```
[Out] -((2*F^(a + b*x))/7 + (4*F^(a + b*x)*b*x*log(F))/35 + (8*F^(a + b*x)*b^2*x^2*log(F)^2)/105 + (16*F^(a + b*x)*b^3*x^3*log(F)^3)/105 - (16*F^a*b^3*x^3*erfc((-b*x*log(F))^(1/2))*log(F)^3*(-b*x*pi*log(F))^(1/2))/105)/x^(7/2)
```

### 3.39 $\int F^{c(a+bx)}(d+ex)^{7/2} dx$

**Optimal.** Leaf size=208

$$\frac{105e^{7/2}F^{c(a-\frac{bd}{e})}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{\log(F)}}{\sqrt{e}}\right)}{16b^{9/2}c^{9/2}\log^{9/2}(F)} - \frac{105e^3F^{c(a+bx)}\sqrt{d+ex}}{8b^4c^4\log^4(F)} + \frac{35e^2F^{c(a+bx)}(d+ex)^{3/2}}{4b^3c^3\log^3(F)}$$

[Out]  $35/4*e^{7/2}*F^{(c*(b*x+a))}*(e*x+d)^{(3/2)}/b^3/c^3/\ln(F)^3-7/2*e*F^{(c*(b*x+a))}*(e*x+d)^{(5/2)}/b^2/c^2/\ln(F)^2+F^{(c*(b*x+a))}*(e*x+d)^{(7/2)}/b/c/\ln(F)+105/16*e^{(7/2)}*F^{(c*(a-b*d/e))}*\operatorname{erfi}(b^{(1/2)}*c^{(1/2)}*(e*x+d)^{(1/2)}*\ln(F)^{(1/2)}/e^{(1/2)})*\pi^{(1/2)}/b^{(9/2)}/c^{(9/2)}/\ln(F)^{(9/2)}-105/8*e^3*F^{(c*(b*x+a))}*(e*x+d)^{(1/2)}/b^4/c^4/\ln(F)^4$

**Rubi [A]**

time = 0.20, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2207, 2211, 2235}

$$\frac{105\sqrt{\pi}e^{7/2}F^{c(a-\frac{bd}{e})}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{16b^{9/2}c^{9/2}\log^{9/2}(F)} - \frac{105e^3\sqrt{d+ex}F^{c(a+bx)}}{8b^4c^4\log^4(F)} + \frac{35e^2(d+ex)^{3/2}F^{c(a+bx)}}{4b^3c^3\log^3(F)} - \frac{7e(d+ex)^{5/2}F^{c(a+bx)}}{2b^2c^2\log^2(F)} + \frac{(d+ex)^{7/2}F^{c(a+bx)}}{bc\log(F)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(c*(a+b*x))}*(d+e*x)^{(7/2)},x]$

[Out]  $(105*e^{(7/2)}*F^{(c*(a-(b*d)/e))}*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[d+e*x]*\operatorname{Sqrt}[\operatorname{Log}[F]])/\operatorname{Sqrt}[e]])/\operatorname{Sqrt}[e]/(16*b^{(9/2)}*c^{(9/2)}*\operatorname{Log}[F]^{(9/2)}) - (105*e^3*F^{(c*(a+b*x))}*\operatorname{Sqrt}[d+e*x])/ (8*b^4*c^4*\operatorname{Log}[F]^4) + (35*e^2*F^{(c*(a+b*x))}*(d+e*x)^{(3/2)})/(4*b^3*c^3*\operatorname{Log}[F]^3) - (7*e*F^{(c*(a+b*x))}*(d+e*x)^{(5/2)})/(2*b^2*c^2*\operatorname{Log}[F]^2) + (F^{(c*(a+b*x))}*(d+e*x)^{(7/2)})/(b*c*\operatorname{Log}[F])$

Rule 2207

$\operatorname{Int}[(b_.)*(F_)^{((g_.)*((e_.)+(f_.)*(x_)))^{(n_.)*((c_.)+(d_.)*(x_))^{(m_.)}, x\_Symbol] :> \operatorname{Simp}[(c+d*x)^m*((b*F^{(g*(e+f*x)))^n/(f*g*n*\operatorname{Log}[F]))], x] - \operatorname{Dist}[d*(m/(f*g*n*\operatorname{Log}[F])), \operatorname{Int}[(c+d*x)^{(m-1)}*(b*F^{(g*(e+f*x)))^n], x], x] /; \operatorname{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \operatorname{GtQ}[m, 0] \&\& \operatorname{IntegerQ}[2*m] \&\& \operatorname{!TrueQ}[\$UseGamma]$

Rule 2211

$\operatorname{Int}[(F_)^{((g_.)*((e_.)+(f_.)*(x_)))/\operatorname{Sqrt}[(c_.)+(d_.)*(x_)]}, x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e-c*(f/d))+f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c+d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& \operatorname{!TrueQ}[\$UseGamma]$

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int F^{c(a+bx)}(d+ex)^{7/2} dx &= \frac{F^{c(a+bx)}(d+ex)^{7/2}}{bc \log(F)} - \frac{(7e) \int F^{c(a+bx)}(d+ex)^{5/2} dx}{2bc \log(F)} \\
&= -\frac{7eF^{c(a+bx)}(d+ex)^{5/2}}{2b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^{7/2}}{bc \log(F)} + \frac{(35e^2) \int F^{c(a+bx)}(d+ex)^{3/2} dx}{4b^2c^2 \log^2(F)} \\
&= \frac{35e^2F^{c(a+bx)}(d+ex)^{3/2}}{4b^3c^3 \log^3(F)} - \frac{7eF^{c(a+bx)}(d+ex)^{5/2}}{2b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^{7/2}}{bc \log(F)} - \frac{(105e^3) \int F^{c(a+bx)}(d+ex)^{1/2} dx}{8b^4c^4 \log^4(F)} \\
&= -\frac{105e^3F^{c(a+bx)}\sqrt{d+ex}}{8b^4c^4 \log^4(F)} + \frac{35e^2F^{c(a+bx)}(d+ex)^{3/2}}{4b^3c^3 \log^3(F)} - \frac{7eF^{c(a+bx)}(d+ex)^{5/2}}{2b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^{7/2}}{bc \log(F)} \\
&= -\frac{105e^3F^{c(a+bx)}\sqrt{d+ex}}{8b^4c^4 \log^4(F)} + \frac{35e^2F^{c(a+bx)}(d+ex)^{3/2}}{4b^3c^3 \log^3(F)} - \frac{7eF^{c(a+bx)}(d+ex)^{5/2}}{2b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^{7/2}}{bc \log(F)} \\
&= \frac{105e^{7/2}F^{c(a-\frac{bd}{e})}\sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{\log(F)}}{\sqrt{e}}\right)}{16b^{9/2}c^{9/2}\log^{9/2}(F)} - \frac{105e^3F^{c(a+bx)}\sqrt{d+ex}}{8b^4c^4 \log^4(F)}
\end{aligned}$$

**Mathematica [A]**

time = 0.15, size = 72, normalized size = 0.35

$$\frac{e^4 F^{c\left(a-\frac{bd}{e}\right)} \Gamma\left(\frac{9}{2}, -\frac{bc(d+ex)\log(F)}{e}\right) \sqrt{-\frac{bc(d+ex)\log(F)}{e}}}{b^5 c^5 \sqrt{d+ex} \log^5(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d + e\*x)^(7/2), x]

[Out] (e^4 \* F^(c\*(a - (b\*d)/e)) \* Gamma[9/2, -((b\*c\*(d + e\*x)\*Log[F])/e)] \* Sqrt[-((b\*c\*(d + e\*x)\*Log[F])/e)]) / (b^5 \* c^5 \* Sqrt[d + e\*x] \* Log[F]^5)

**Maple [F]**

time = 0.01, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}(ex+d)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a))*(e*x+d)^(7/2),x)`

[Out] `int(F^(c*(b*x+a))*(e*x+d)^(7/2),x)`

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e*x+d)^(7/2),x, algorithm="maxima")`

[Out] `integrate((x*e + d)^(7/2)*F^((b*x + a)*c), x)`

**Fricas** [A]

time = 0.36, size = 224, normalized size = 1.08

$$\frac{2(8(b^4c^2x^3e^3 + 3b^4cd^2x^2e^2 + 3b^4c^2d^2xe + b^4c^3d^2)\log(F)^4 - 105bce^3\log(F) - 28(b^3c^2x^2e^3 + 2b^3c^2dxe^2 + b^3c^2d^2e)\log(F)^3 + 70(b^2c^2xe^3 + b^2c^2d^2e)\log(F)^2)\sqrt{xe+d}F^{bcx+ac} - \frac{105\sqrt{\pi}\sqrt{-bce^{(-1)}\log(F)}\operatorname{erf}\left(\sqrt{-bce^{(-1)}\log(F)}\sqrt{xe+d}\right)e^c}{\sqrt{b^2d-ace^{(-1)}}}}{16b^5c^5\log(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e*x+d)^(7/2),x, algorithm="fricas")`

[Out] `1/16*(2*(8*(b^4*c^4*x^3*e^3 + 3*b^4*c^4*d*x^2*e^2 + 3*b^4*c^4*d^2*x*e + b^4*c^4*d^3)*log(F)^4 - 105*b*c*e^3*log(F) - 28*(b^3*c^3*x^2*e^3 + 2*b^3*c^3*d*x*e^2 + b^3*c^3*d^2*e)*log(F)^3 + 70*(b^2*c^2*x*e^3 + b^2*c^2*d*e^2)*log(F)^2)*sqrt(x*e + d)*F^(b*c*x + a*c) - 105*sqrt(pi)*sqrt(-b*c*e^(-1)*log(F))*erf(sqrt(-b*c*e^(-1)*log(F))*sqrt(x*e + d))*e^4/F^((b*c*d - a*c*e)*e^(-1)))/(b^5*c^5*log(F)^5)`

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(c*(b*x+a))*(e*x+d)**(7/2),x)`

[Out] Exception raised: SystemError >> excessive stack use: stack is 3876 deep

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 1091 vs. 2(176) = 352.

time = 2.59, size = 1091, normalized size = 5.25



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^(7/2),x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/16*(16*\sqrt{\pi}*d^4*\operatorname{erf}(-\sqrt{-b*c*e*\log(F)})*\sqrt{x*e+d}*e^{-1})*e^{-(-} \\ & b*c*d*\log(F) - a*c*e*\log(F))*e^{-1} + 1)/\sqrt{-b*c*e*\log(F)} - 32*d^3*(\sqrt{\pi} \\ & *(2*b*c*d*\log(F) + e)*\operatorname{erf}(-\sqrt{-b*c*e*\log(F)})*\sqrt{x*e+d}*e^{-1})*e^{-} \\ & (-b*c*d*\log(F) - a*c*e*\log(F))*e^{-1} + 1)/(\sqrt{-b*c*e*\log(F)}*b*c*\log(F) \\ & ) + 2*\sqrt{x*e+d}*e^{((x*e+d)*b*c*\log(F) - b*c*d*\log(F) + a*c*e*\log(F))} \\ & *e^{-1} + 1)/(b*c*\log(F)) + 24*d^2*(\sqrt{\pi}*(4*b^2*c^2*d^2*\log(F)^2 + 4*b \\ & *c*d*e*\log(F) + 3*e^2)*\operatorname{erf}(-\sqrt{-b*c*e*\log(F)})*\sqrt{x*e+d}*e^{-1})*e^{-(-} \\ & b*c*d*\log(F) - a*c*e*\log(F) + 2*e)*e^{-1} + 1)/(\sqrt{-b*c*e*\log(F)}*b^2*c^2 \\ & *\log(F)^2) - 2*(2*(x*e+d)^{(3/2)}*b*c*e*\log(F) - 4*\sqrt{x*e+d}*b*c*d*e*\log \\ & (F) - 3*\sqrt{x*e+d}*e^2)*e^{((x*e+d)*b*c*\log(F) - b*c*d*\log(F) + a*c*e \\ & *\log(F) - 2*e)*e^{-1}}/(b^2*c^2*\log(F)^2))*e^2 - 8*d*(\sqrt{\pi}*(8*b^3*c^3*d \\ & ^3*\log(F)^3 + 12*b^2*c^2*d^2*e*\log(F)^2 + 18*b*c*d*e^2*\log(F) + 15*e^3)*\operatorname{erf} \\ & (-\sqrt{-b*c*e*\log(F)})*\sqrt{x*e+d}*e^{-1})*e^{-(b*c*d*\log(F) - a*c*e*\log(F) \\ & ) + 3*e)*e^{-1} + 1)/(\sqrt{-b*c*e*\log(F)}*b^3*c^3*\log(F)^3) + 2*(4*(x*e+d) \\ & )^{(5/2)}*b^2*c^2*e*\log(F)^2 - 12*(x*e+d)^{(3/2)}*b^2*c^2*d*e*\log(F)^2 + 12*s \\ & \sqrt{x*e+d}*b^2*c^2*d^2*e*\log(F)^2 - 10*(x*e+d)^{(3/2)}*b*c*e^2*\log(F) + 1 \\ & 8*\sqrt{x*e+d}*b*c*d*e^2*\log(F) + 15*\sqrt{x*e+d}*e^3)*e^{((x*e+d)*b*c* \\ & \log(F) - b*c*d*\log(F) + a*c*e*\log(F) - 3*e)*e^{-1}}/(b^3*c^3*\log(F)^3))*e^3 \\ & + (\sqrt{\pi}*(16*b^4*c^4*d^4*\log(F)^4 + 32*b^3*c^3*d^3*e*\log(F)^3 + 72*b^2*c^2 \\ & *d^2*e^2*\log(F)^2 + 120*b*c*d*e^3*\log(F) + 105*e^4)*\operatorname{erf}(-\sqrt{-b*c*e*\log \\ & (F)})*\sqrt{x*e+d}*e^{-1})*e^{-(b*c*d*\log(F) - a*c*e*\log(F) + 4*e)*e^{-1} + \\ & 1)/(\sqrt{-b*c*e*\log(F)}*b^4*c^4*\log(F)^4) - 2*(8*(x*e+d)^{(7/2)}*b^3*c^3*e \\ & *\log(F)^3 - 32*(x*e+d)^{(5/2)}*b^3*c^3*d*e*\log(F)^3 + 48*(x*e+d)^{(3/2)}*b^ \\ & 3*c^3*d^2*e*\log(F)^3 - 32*\sqrt{x*e+d}*b^3*c^3*d^3*e*\log(F)^3 - 28*(x*e+d) \\ & )^{(5/2)}*b^2*c^2*e^2*\log(F)^2 + 80*(x*e+d)^{(3/2)}*b^2*c^2*d*e^2*\log(F)^2 - \\ & 72*\sqrt{x*e+d}*b^2*c^2*d^2*e^2*\log(F)^2 + 70*(x*e+d)^{(3/2)}*b*c*e^3*\log \\ & (F) - 120*\sqrt{x*e+d}*b*c*d*e^3*\log(F) - 105*\sqrt{x*e+d}*e^4)*e^{((x*e \\ & +d)*b*c*\log(F) - b*c*d*\log(F) + a*c*e*\log(F) - 4*e)*e^{-1}}/(b^4*c^4*\log(F) \\ & )^4))*e^4)*e^{-1} \end{aligned}$$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int F^{c(a+bx)}(d+ex)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))\*(d + e\*x)^(7/2),x)

[Out] int(F^(c\*(a + b\*x))\*(d + e\*x)^(7/2), x)

### 3.40 $\int F^{c(a+bx)}(d+ex)^{5/2} dx$

**Optimal.** Leaf size=173

$$\frac{15e^{5/2}F^{c(a-\frac{bd}{e})}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{\log(F)}}{\sqrt{e}}\right)}{8b^{7/2}c^{7/2}\log^{\frac{7}{2}}(F)} + \frac{15e^2F^{c(a+bx)}\sqrt{d+ex}}{4b^3c^3\log^3(F)} - \frac{5eF^{c(a+bx)}(d+ex)^{3/2}}{2b^2c^2\log^2(F)} + \frac{F^c}{bc\log(F)}$$

[Out]  $-5/2*e*F^{(c*(b*x+a))*(e*x+d)^{(3/2)}/b^2/c^2/\ln(F)^2+F^{(c*(b*x+a))*(e*x+d)^{(5/2)}/b/c/\ln(F)-15/8*e^{(5/2)*F^{(c*(a-b*d/e))*\operatorname{erfi}(b^{(1/2)*c^{(1/2)*(e*x+d)^{(1/2)*\ln(F)^{(1/2)/e^{(1/2)}}*\Pi^{(1/2)/b^{(7/2)/c^{(7/2)/\ln(F)^{(7/2)+15/4*e^2*F^{(c*(b*x+a))*(e*x+d)^{(1/2)/b^3/c^3/\ln(F)^3}}$

**Rubi [A]**

time = 0.13, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2207, 2211, 2235}

$$\frac{15\sqrt{\pi}e^{5/2}F^{c(a-\frac{bd}{e})}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{8b^{7/2}c^{7/2}\log^{\frac{7}{2}}(F)} + \frac{15e^2\sqrt{d+ex}F^{c(a+bx)}}{4b^3c^3\log^3(F)} - \frac{5e(d+ex)^{3/2}F^{c(a+bx)}}{2b^2c^2\log^2(F)} + \frac{(d+ex)^{5/2}F^{c(a+bx)}}{bc\log(F)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{c(a+bx)}(d+ex)^{5/2}, x]$

[Out]  $(-15*e^{(5/2)*F^{(c*(a-(b*d)/e))*\operatorname{Sqrt}[\Pi]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[d+e*x]*\operatorname{Sqrt}[\operatorname{Log}[F]])/\operatorname{Sqrt}[e]]]/(8*b^{(7/2)*c^{(7/2)*\operatorname{Log}[F]^{(7/2)}}+(15*e^2*F^{(c*(a+bx))*\operatorname{Sqrt}[d+e*x]}/(4*b^3*c^3*\operatorname{Log}[F]^3)-(5*e*F^{(c*(a+bx))*(d+e*x)^{(3/2)}}/(2*b^2*c^2*\operatorname{Log}[F]^2)+(F^{(c*(a+bx))*(d+e*x)^{(5/2)}}/(b*c*\operatorname{Log}[F]))$

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.)+(f_.)*(x_))))^(n_.)*((c_.)+(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(c+d*x)^m*((b*F^(g*(e+f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c+d*x)^(m-1)*(b*F^(g*(e+f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.)+(f_.)*(x_)))/Sqrt[(c_.)+(d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e-c*(f/d))+f*g*(x^2/d)), x], x, Sqrt[c+d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int F^{c(a+bx)}(d+ex)^{5/2} dx &= \frac{F^{c(a+bx)}(d+ex)^{5/2}}{bc \log(F)} - \frac{(5e) \int F^{c(a+bx)}(d+ex)^{3/2} dx}{2bc \log(F)} \\
&= -\frac{5eF^{c(a+bx)}(d+ex)^{3/2}}{2b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^{5/2}}{bc \log(F)} + \frac{(15e^2) \int F^{c(a+bx)}\sqrt{d+ex} dx}{4b^2c^2 \log^2(F)} \\
&= \frac{15e^2 F^{c(a+bx)}\sqrt{d+ex}}{4b^3c^3 \log^3(F)} - \frac{5eF^{c(a+bx)}(d+ex)^{3/2}}{2b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^{5/2}}{bc \log(F)} - \frac{(15e^3)}{8b^3c^3} \\
&= \frac{15e^2 F^{c(a+bx)}\sqrt{d+ex}}{4b^3c^3 \log^3(F)} - \frac{5eF^{c(a+bx)}(d+ex)^{3/2}}{2b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^{5/2}}{bc \log(F)} - \frac{(15e^2)}{8b^3c^3} \\
&= -\frac{15e^{5/2} F^{c(a-\frac{bd}{e})} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c} \sqrt{d+ex} \sqrt{\log(F)}}{\sqrt{e}}\right)}{8b^{7/2}c^{7/2} \log^{7/2}(F)} + \frac{15e^2 F^{c(a+bx)}\sqrt{d+ex}}{4b^3c^3 \log^3(F)}
\end{aligned}$$

**Mathematica [A]**

time = 0.12, size = 72, normalized size = 0.42

$$\frac{e^2 F^{c(a-\frac{bd}{e})} \sqrt{d+ex} \Gamma\left(\frac{7}{2}, -\frac{bc(d+ex)\log(F)}{e}\right)}{b^3c^3 \log^3(F) \sqrt{-\frac{bc(d+ex)\log(F)}{e}}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d + e\*x)^(5/2), x]

[Out] (e^2 \* F^(c\*(a - (b\*d)/e)) \* Sqrt[d + e\*x] \* Gamma[7/2, -((b\*c\*(d + e\*x)\*Log[F])/e)]) / (b^3 \* c^3 \* Log[F]^3 \* Sqrt[-((b\*c\*(d + e\*x)\*Log[F])/e)])

**Maple [F]**

time = 0.01, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}(ex+d)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a))*(e*x+d)^(5/2),x)`

[Out] `int(F^(c*(b*x+a))*(e*x+d)^(5/2),x)`

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e*x+d)^(5/2),x, algorithm="maxima")`

[Out] `integrate((x*e + d)^(5/2)*F^((b*x + a)*c), x)`

**Fricas** [A]

time = 0.37, size = 165, normalized size = 0.95

$$\frac{2(15bce^2 \log(F) + 4(b^3c^3x^2e^2 + 2b^3c^3dxe + b^3c^3d^2) \log(F)^3 - 10(b^2c^2xe^2 + b^2c^2de) \log(F)^2) \sqrt{xe+d} F^{bcx+ac} + \frac{15\sqrt{\pi} \sqrt{-bce^{(-1)} \log(F)} \operatorname{erf}\left(\sqrt{-bce^{(-1)} \log(F)} \sqrt{xe+d}\right) e^3}{F^{(bcd-ace)e^{(-1)}}}}{8b^4c^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e*x+d)^(5/2),x, algorithm="fricas")`

[Out] `1/8*(2*(15*b*c*e^2*log(F) + 4*(b^3*c^3*x^2*e^2 + 2*b^3*c^3*d*x*e + b^3*c^3*d^2)*log(F)^3 - 10*(b^2*c^2*x*e^2 + b^2*c^2*d*e)*log(F)^2)*sqrt(x*e + d)*F^(b*c*x + a*c) + 15*sqrt(pi)*sqrt(-b*c*e^(-1)*log(F))*erf(sqrt(-b*c*e^(-1)*log(F))*sqrt(x*e + d))*e^3/F^((b*c*d - a*c*e)*e^(-1)))/(b^4*c^4*log(F)^4)`

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(c*(b*x+a))*(e*x+d)**(5/2),x)`

[Out] Timed out

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 691 vs. 2(145) = 290.

time = 2.60, size = 691, normalized size = 3.99

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(e*x+d)^(5/2),x, algorithm="giac")`

```
[Out] -1/8*(8*sqrt(pi)*d^3*erf(-sqrt(-b*c*e*log(F))*sqrt(x*e + d)*e^(-1))*e^(-(b*
c*d*log(F) - a*c*e*log(F))*e^(-1) + 1)/sqrt(-b*c*e*log(F)) - 12*d^2*(sqrt(p
i)*(2*b*c*d*log(F) + e)*erf(-sqrt(-b*c*e*log(F))*sqrt(x*e + d)*e^(-1))*e^(-
(b*c*d*log(F) - a*c*e*log(F))*e^(-1) + 1)/(sqrt(-b*c*e*log(F))*b*c*log(F))
+ 2*sqrt(x*e + d)*e^(((x*e + d)*b*c*log(F) - b*c*d*log(F) + a*c*e*log(F))*e
^(-1) + 1)/(b*c*log(F))) + 6*d*(sqrt(pi)*(4*b^2*c^2*d^2*log(F)^2 + 4*b*c*d*
e*log(F) + 3*e^2)*erf(-sqrt(-b*c*e*log(F))*sqrt(x*e + d)*e^(-1))*e^(-(b*c*d
*log(F) - a*c*e*log(F) + 2*e)*e^(-1) + 1)/(sqrt(-b*c*e*log(F))*b^2*c^2*log(
F)^2) - 2*(2*(x*e + d)^(3/2)*b*c*e*log(F) - 4*sqrt(x*e + d)*b*c*d*e*log(F)
- 3*sqrt(x*e + d)*e^2)*e^(((x*e + d)*b*c*log(F) - b*c*d*log(F) + a*c*e*log(
F) - 2*e)*e^(-1))/(b^2*c^2*log(F)^2))*e^2 - (sqrt(pi)*(8*b^3*c^3*d^3*log(F)
^3 + 12*b^2*c^2*d^2*e*log(F)^2 + 18*b*c*d*e^2*log(F) + 15*e^3)*erf(-sqrt(-b
*c*e*log(F))*sqrt(x*e + d)*e^(-1))*e^(-(b*c*d*log(F) - a*c*e*log(F) + 3*e)*
e^(-1) + 1)/(sqrt(-b*c*e*log(F))*b^3*c^3*log(F)^3) + 2*(4*(x*e + d)^(5/2)*b
^2*c^2*e*log(F)^2 - 12*(x*e + d)^(3/2)*b^2*c^2*d*e*log(F)^2 + 12*sqrt(x*e +
d)*b^2*c^2*d^2*e*log(F)^2 - 10*(x*e + d)^(3/2)*b*c*e^2*log(F) + 18*sqrt(x*
e + d)*b*c*d*e^2*log(F) + 15*sqrt(x*e + d)*e^3)*e^(((x*e + d)*b*c*log(F) -
b*c*d*log(F) + a*c*e*log(F) - 3*e)*e^(-1))/(b^3*c^3*log(F)^3))*e^3)*e^(-1)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int F^{c(a+bx)} (d+ex)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*(d + e*x)^(5/2),x)
```

```
[Out] int(F^(c*(a + b*x))*(d + e*x)^(5/2), x)
```

### 3.41 $\int F^{c(a+bx)}(d+ex)^{3/2} dx$

**Optimal.** Leaf size=138

$$\frac{3e^{3/2}F^{c(a-\frac{bd}{e})}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{\log(F)}}{\sqrt{e}}\right)}{4b^{5/2}c^{5/2}\log^{5/2}(F)} - \frac{3eF^{c(a+bx)}\sqrt{d+ex}}{2b^2c^2\log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^{3/2}}{bc\log(F)}$$

[Out]  $F^{c(bx+a)}(ex+d)^{3/2}/b/c/\ln(F)+3/4e^{3/2}F^{c(a-bd/e)}\operatorname{erfi}(b^{1/2}c^{1/2}(ex+d)^{1/2}\ln(F)^{1/2}/e^{1/2})\pi^{1/2}/b^{5/2}/c^{5/2}/\ln(F)^{5/2}-3/2eF^{c(bx+a)}(ex+d)^{1/2}/b^2/c^2/\ln(F)^2$

**Rubi [A]**

time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2207, 2211, 2235}

$$\frac{3\sqrt{\pi}e^{3/2}F^{c(a-\frac{bd}{e})}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{4b^{5/2}c^{5/2}\log^{5/2}(F)} - \frac{3e\sqrt{d+ex}F^{c(a+bx)}}{2b^2c^2\log^2(F)} + \frac{(d+ex)^{3/2}F^{c(a+bx)}}{bc\log(F)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{c(a+bx)}(d+ex)^{3/2}, x]$

[Out]  $(3e^{3/2}F^{c(a-bd/e)}\sqrt{\pi}\operatorname{Erfi}[(\sqrt{b}\sqrt{c}\sqrt{d+ex})\sqrt{\log(F)}])/\sqrt{e}/(4b^{5/2}c^{5/2}\log^{5/2}(F)) - (3eF^{c(a+bx)}\sqrt{d+ex})/(2b^2c^2\log^2(F)) + (F^{c(a+bx)}(d+ex)^{3/2})/(bc\log(F))$

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.)+(f_.)*(x_)))^(n_.)*((c_.)+(d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(c+d*x)^m*((bF^(g*(e+f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c+d*x)^(m-1)*(bF^(g*(e+f*x)))^n, x], x]
;/; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.)+(f_.)*(x_)))/Sqrt[(c_.)+(d_.)*(x_)], x_Symbol]
:> Dist[2/d, Subst[Int[F^(g*(e-c*(f/d))+f*g*(x^2/d)), x], x, Sqrt[c+d*x]], x]
;/; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.)+(b_.)*((c_.)+(d_.)*(x_))^2), x_Symbol]
:> Simp[F^a*Sqrt[Pi]*(Erfi[(c+d*x)*Rt[b*Log[F], 2])/(2*d*Rt[b*Log[F], 2])], x]
;/; FreeQ[{
```

F, a, b, c, d}, x] && PosQ[b]

Rubi steps

$$\begin{aligned}
 \int F^{c(a+bx)}(d+ex)^{3/2} dx &= \frac{F^{c(a+bx)}(d+ex)^{3/2}}{bc \log(F)} - \frac{(3e) \int F^{c(a+bx)} \sqrt{d+ex} dx}{2bc \log(F)} \\
 &= -\frac{3e F^{c(a+bx)} \sqrt{d+ex}}{2b^2 c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^{3/2}}{bc \log(F)} + \frac{(3e^2) \int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx}{4b^2 c^2 \log^2(F)} \\
 &= -\frac{3e F^{c(a+bx)} \sqrt{d+ex}}{2b^2 c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^{3/2}}{bc \log(F)} + \frac{(3e) \text{Subst}\left(\int F^{c\left(a-\frac{bd}{e}\right)+\frac{bcx^2}{e}} dx, x\right)}{2b^2 c^2 \log^2(F)} \\
 &= \frac{3e^{3/2} F^{c\left(a-\frac{bd}{e}\right)} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c} \sqrt{d+ex} \sqrt{\log(F)}}{\sqrt{e}}\right)}{4b^{5/2} c^{5/2} \log^{5/2}(F)} - \frac{3e F^{c(a+bx)} \sqrt{d+ex}}{2b^2 c^2 \log^2(F)} +
 \end{aligned}$$

Mathematica [A]

time = 0.15, size = 63, normalized size = 0.46

$$\frac{F^{c\left(a-\frac{bd}{e}\right)}(d+ex)^{5/2} \Gamma\left(\frac{5}{2}, -\frac{bc(d+ex) \log(F)}{e}\right)}{e \left(-\frac{bc(d+ex) \log(F)}{e}\right)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d + e\*x)^(3/2), x]

[Out] -((F^(c\*(a - (b\*d)/e))\*(d + e\*x)^(5/2)\*Gamma[5/2, -((b\*c\*(d + e\*x)\*Log[F])/e]])/e)/((e\*(-((b\*c\*(d + e\*x)\*Log[F])/e)))^(5/2))

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}(ex+d)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*(e\*x+d)^(3/2), x)

[Out] int(F^(c\*(b\*x+a))\*(e\*x+d)^(3/2), x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^(3/2),x, algorithm="maxima")

[Out] integrate((x\*e + d)^(3/2)\*F^((b\*x + a)\*c), x)

**Fricas** [A]

time = 0.37, size = 122, normalized size = 0.88

$$\frac{2(3bce \log(F) - 2(b^2c^2xe + b^2c^2d) \log(F)^2) \sqrt{xe+d} F^{bcx+ac} + \frac{3\sqrt{\pi} \sqrt{-bce^{(-1)} \log(F)} \operatorname{erf}\left(\sqrt{-bce^{(-1)} \log(F)} \sqrt{xe+d}\right) e^2}{F^{(bcd-ace)e^{(-1)}}}}{4b^3c^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^(3/2),x, algorithm="fricas")

[Out] -1/4\*(2\*(3\*b\*c\*e\*log(F) - 2\*(b^2\*c^2\*x\*e + b^2\*c^2\*d)\*log(F)^2)\*sqrt(x\*e + d)\*F^(b\*c\*x + a\*c) + 3\*sqrt(pi)\*sqrt(-b\*c\*e^(-1)\*log(F))\*erf(sqrt(-b\*c\*e^(-1)\*log(F))\*sqrt(x\*e + d))\*e^2/F^((b\*c\*d - a\*c\*e)\*e^(-1)))/(b^3\*c^3\*log(F)^3)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}(d+ex)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(e\*x+d)\*\*(3/2),x)

[Out] Integral(F\*\*(c\*(a + b\*x))\*(d + e\*x)\*\*(3/2), x)

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 401 vs. 2(114) = 228.

time = 3.53, size = 401, normalized size = 2.91

$$\frac{1}{4} \left( \frac{4\sqrt{\pi} \operatorname{erf}\left(\sqrt{-bce^{(-1)} \log(F)} \sqrt{xe+d}\right) e^{bcx+ac}}{\sqrt{-bce^{(-1)} \log(F)}} - 2 \left( \frac{2(bcd-ace)e^{(-1)}}{\sqrt{-bce^{(-1)} \log(F)}} \right)^2 \right) \sqrt{xe+d} F^{bcx+ac} + \frac{3\sqrt{\pi} \sqrt{-bce^{(-1)} \log(F)} \operatorname{erf}\left(\sqrt{-bce^{(-1)} \log(F)} \sqrt{xe+d}\right) e^2}{F^{(bcd-ace)e^{(-1)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^(3/2),x, algorithm="giac")

[Out] -1/4\*(4\*sqrt(pi)\*d^2\*erf(-sqrt(-b\*c\*e\*log(F))\*sqrt(x\*e + d)\*e^(-1))\*e^(-(b\*c\*d\*log(F) - a\*c\*e\*log(F))\*e^(-1) + 1)/sqrt(-b\*c\*e\*log(F)) - 4\*d\*(sqrt(pi)\*(2\*b\*c\*d\*log(F) + e)\*erf(-sqrt(-b\*c\*e\*log(F))\*sqrt(x\*e + d)\*e^(-1))\*e^(-(b\*c\*d\*log(F) - a\*c\*e\*log(F))\*e^(-1) + 1)/(sqrt(-b\*c\*e\*log(F))\*b\*c\*log(F)) + 2\*sqrt(x\*e + d)\*e^(((x\*e + d)\*b\*c\*log(F) - b\*c\*d\*log(F) + a\*c\*e\*log(F))\*e^(-1) + 1)/(b\*c\*log(F))) + (sqrt(pi)\*(4\*b^2\*c^2\*d^2\*log(F)^2 + 4\*b\*c\*d\*e\*log(F)



```
) + 3*e^2)*erf(-sqrt(-b*c*e*log(F))*sqrt(x*e + d)*e^(-1))*e^(-(b*c*d*log(F)
- a*c*e*log(F) + 2*e)*e^(-1) + 1)/(sqrt(-b*c*e*log(F))*b^2*c^2*log(F)^2) -
2*(2*(x*e + d)^(3/2)*b*c*e*log(F) - 4*sqrt(x*e + d)*b*c*d*e*log(F) - 3*sqrt
(x*e + d)*e^2)*e^(((x*e + d)*b*c*log(F) - b*c*d*log(F) + a*c*e*log(F) - 2*
e)*e^(-1))/(b^2*c^2*log(F)^2))*e^2)*e^(-1)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int F^{c(a+bx)} (d+ex)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))\*(d + e\*x)^(3/2), x)

[Out] int(F^(c\*(a + b\*x))\*(d + e\*x)^(3/2), x)

### 3.42 $\int F^{c(a+bx)} \sqrt{d+ex} dx$

**Optimal.** Leaf size=105

$$-\frac{\sqrt{e} F^{c(a-\frac{bd}{e})} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c} \sqrt{d+ex} \sqrt{\log(F)}}{\sqrt{e}}\right)}{2b^{3/2} c^{3/2} \log^{3/2}(F)} + \frac{F^{c(a+bx)} \sqrt{d+ex}}{bc \log(F)}$$

[Out]  $-1/2 * F^{(c*(a-b*d/e))} * \operatorname{erfi}(b^{(1/2)} * c^{(1/2)} * (e*x+d)^{(1/2)} * \ln(F)^{(1/2)} / e^{(1/2)}) * e^{(1/2)} * \pi^{(1/2)} / b^{(3/2)} / c^{(3/2)} / \ln(F)^{(3/2)} + F^{(c*(b*x+a))} * (e*x+d)^{(1/2)} / b/c / \ln(F)$

**Rubi [A]**

time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2207, 2211, 2235}

$$\frac{\sqrt{d+ex} F^{c(a+bx)}}{bc \log(F)} - \frac{\sqrt{\pi} \sqrt{e} F^{c(a-\frac{bd}{e})} \operatorname{Erfi}\left(\frac{\sqrt{b} \sqrt{c} \sqrt{\log(F)} \sqrt{d+ex}}{\sqrt{e}}\right)}{2b^{3/2} c^{3/2} \log^{3/2}(F)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(c*(a + b*x))} * \operatorname{Sqrt}[d + e*x], x]$

[Out]  $-1/2 * (\operatorname{Sqrt}[e] * F^{(c*(a - (b*d)/e))} * \operatorname{Sqrt}[\pi] * \operatorname{Erfi}[(\operatorname{Sqrt}[b] * \operatorname{Sqrt}[c] * \operatorname{Sqrt}[d + e*x] * \operatorname{Sqrt}[\operatorname{Log}[F]]) / \operatorname{Sqrt}[e]]) / (b^{(3/2)} * c^{(3/2)} * \operatorname{Log}[F]^{(3/2)}) + (F^{(c*(a + b*x))} * \operatorname{Sqrt}[d + e*x]) / (b*c * \operatorname{Log}[F])$

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

F, a, b, c, d}, x] && PosQ[b]

Rubi steps

$$\begin{aligned} \int F^{c(a+bx)} \sqrt{d+ex} \, dx &= \frac{F^{c(a+bx)} \sqrt{d+ex}}{bc \log(F)} - \frac{e \int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} \, dx}{2bc \log(F)} \\ &= \frac{F^{c(a+bx)} \sqrt{d+ex}}{bc \log(F)} - \frac{\text{Subst}\left(\int F^{c\left(a-\frac{bd}{e}\right)+\frac{bcx^2}{e}} \, dx, x, \sqrt{d+ex}\right)}{bc \log(F)} \\ &= -\frac{\sqrt{e} F^{c\left(a-\frac{bd}{e}\right)} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c} \sqrt{d+ex} \sqrt{\log(F)}}{\sqrt{e}}\right)}{2b^{3/2} c^{3/2} \log^{\frac{3}{2}}(F)} + \frac{F^{c(a+bx)} \sqrt{d+ex}}{bc \log(F)} \end{aligned}$$

**Mathematica [A]**

time = 0.09, size = 63, normalized size = 0.60

$$-\frac{F^{c\left(a-\frac{bd}{e}\right)} (d+ex)^{3/2} \Gamma\left(\frac{3}{2}, -\frac{bc(d+ex) \log(F)}{e}\right)}{e \left(-\frac{bc(d+ex) \log(F)}{e}\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*Sqrt[d + e\*x], x]

[Out] -((F^(c\*(a - (b\*d)/e))\*(d + e\*x)^(3/2)\*Gamma[3/2, -((b\*c\*(d + e\*x)\*Log[F])/e)])/(e\*(-((b\*c\*(d + e\*x)\*Log[F])/e))^(3/2)))

**Maple [F]**

time = 0.01, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)} \sqrt{ex+d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*(e\*x+d)^(1/2), x)

[Out] int(F^(c\*(b\*x+a))\*(e\*x+d)^(1/2), x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x\*e + d)\*F^((b\*x + a)\*c), x)

**Fricas** [A]

time = 0.37, size = 91, normalized size = 0.87

$$\frac{2\sqrt{xe+d} F^{bcx+ac} bc \log(F) + \frac{\sqrt{\pi} \sqrt{-bce^{(-1)} \log(F)} \operatorname{erf}\left(\sqrt{-bce^{(-1)} \log(F)} \sqrt{xe+d}\right) e}{F^{(bcd-ace)e^{(-1)}}}}{2b^2c^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^(1/2),x, algorithm="fricas")

[Out] 1/2\*(2\*sqrt(x\*e + d)\*F^(b\*c\*x + a\*c)\*b\*c\*log(F) + sqrt(pi)\*sqrt(-b\*c\*e^(-1)\*log(F))\*erf(sqrt(-b\*c\*e^(-1)\*log(F))\*sqrt(x\*e + d))\*e/F^((b\*c\*d - a\*c\*e)\*^(-1)))/(b^2\*c^2\*log(F)^2)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)} \sqrt{d+ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(e\*x+d)\*\*(1/2),x)

[Out] Integral(F\*\*(c\*(a + b\*x))\*sqrt(d + e\*x), x)

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(83) = 166.

time = 4.24, size = 198, normalized size = 1.89

$$-\frac{1}{2} \left( \frac{2\sqrt{\pi} d \operatorname{erf}\left(-\sqrt{-bce \log(F)} \sqrt{xe+d} e^{(-1)}\right) e^{(-bcd \log(F) - ace \log(F))e^{(-1)+1}}}{\sqrt{-bce \log(F)}} - \frac{\sqrt{\pi} (2bcd \log(F) + e) \operatorname{erf}\left(-\sqrt{-bce \log(F)} \sqrt{xe+d} e^{(-1)}\right) e^{(-bcd \log(F) - ace \log(F))e^{(-1)+1}}}{\sqrt{-bce \log(F)} bc \log(F)} - \frac{2\sqrt{xe+d} e^{((x+e)d bc \log(F) - bcd \log(F) + ace \log(F))e^{(-1)+1}}}{bc \log(F)} \right) e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^(1/2),x, algorithm="giac")

[Out] -1/2\*(2\*sqrt(pi)\*d\*erf(-sqrt(-b\*c\*e\*log(F))\*sqrt(x\*e + d)\*e^(-1))\*e^(-(b\*c\*d\*log(F) - a\*c\*e\*log(F))\*e^(-1) + 1)/sqrt(-b\*c\*e\*log(F)) - sqrt(pi)\*(2\*b\*c\*d\*log(F) + e)\*erf(-sqrt(-b\*c\*e\*log(F))\*sqrt(x\*e + d)\*e^(-1))\*e^(-(b\*c\*d\*log(F) - a\*c\*e\*log(F))\*e^(-1) + 1)/(sqrt(-b\*c\*e\*log(F))\*b\*c\*log(F)) - 2\*sqrt(x\*e + d)\*e^(((x\*e + d)\*b\*c\*log(F) - b\*c\*d\*log(F) + a\*c\*e\*log(F))\*e^(-1) + 1)/(b\*c\*log(F))\*e^(-1)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int F^{c(a+bx)} \sqrt{d+ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))\*(d + e\*x)^(1/2), x)

[Out] int(F^(c\*(a + b\*x))\*(d + e\*x)^(1/2), x)

$$3.43 \quad \int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx$$

Optimal. Leaf size=72

$$\frac{F^{c(a-\frac{bd}{e})} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c} \sqrt{d+ex} \sqrt{\log(F)}}{\sqrt{e}}\right)}{\sqrt{b} \sqrt{c} \sqrt{e} \sqrt{\log(F)}}$$

[Out]  $F^{c*(a-b*d/e)} * \operatorname{erfi}(b^{(1/2)} * c^{(1/2)} * (e*x+d)^{(1/2)} * \ln(F)^{(1/2)} / e^{(1/2)}) * \pi^{(1/2)} / b^{(1/2)} / c^{(1/2)} / e^{(1/2)} / \ln(F)^{(1/2)}$

Rubi [A]

time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2211, 2235}

$$\frac{\sqrt{\pi} F^{c(a-\frac{bd}{e})} \operatorname{Erfi}\left(\frac{\sqrt{b} \sqrt{c} \sqrt{\log(F)} \sqrt{d+ex}}{\sqrt{e}}\right)}{\sqrt{b} \sqrt{c} \sqrt{e} \sqrt{\log(F)}}$$

Antiderivative was successfully verified.

[In] `Int[F^(c*(a + b*x))/Sqrt[d + e*x],x]`

[Out]  $(F^{c*(a - (b*d)/e)} * \operatorname{Sqrt}[\pi] * \operatorname{Erfi}[(\operatorname{Sqrt}[b] * \operatorname{Sqrt}[c] * \operatorname{Sqrt}[d + e*x] * \operatorname{Sqrt}[\operatorname{Log}[F]]) / \operatorname{Sqrt}[e]]) / (\operatorname{Sqrt}[b] * \operatorname{Sqrt}[c] * \operatorname{Sqrt}[e] * \operatorname{Sqrt}[\operatorname{Log}[F]])$

Rule 2211

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

Rule 2235

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

Rubi steps

$$\int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx = \frac{2 \text{Subst} \left( \int F^{c(a-\frac{bd}{e})+\frac{bcx^2}{e}} dx, x, \sqrt{d+ex} \right)}{e}$$

$$= \frac{F^{c(a-\frac{bd}{e})} \sqrt{\pi} \operatorname{erfi} \left( \frac{\sqrt{b} \sqrt{c} \sqrt{d+ex} \sqrt{\log(F)}}{\sqrt{e}} \right)}{\sqrt{b} \sqrt{c} \sqrt{e} \sqrt{\log(F)}}$$

**Mathematica [A]**

time = 0.09, size = 63, normalized size = 0.88

$$\frac{F^{c(a-\frac{bd}{e})} \sqrt{d+ex} \Gamma\left(\frac{1}{2}, -\frac{bc(d+ex)\log(F)}{e}\right)}{e \sqrt{-\frac{bc(d+ex)\log(F)}{e}}}$$

Antiderivative was successfully verified.

`[In] Integrate[F^(c*(a + b*x))/Sqrt[d + e*x], x]``[Out] -((F^(c*(a - (b*d)/e))*Sqrt[d + e*x]*Gamma[1/2, -((b*c*(d + e*x)*Log[F])/e])]/(e*Sqrt[-((b*c*(d + e*x)*Log[F])/e])))`**Maple [F]**

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{F^{c(bx+a)}}{\sqrt{ex+d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(F^(c*(b*x+a))/(e*x+d)^(1/2), x)``[Out] int(F^(c*(b*x+a))/(e*x+d)^(1/2), x)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(c*(b*x+a))/(e*x+d)^(1/2), x, algorithm="maxima")``[Out] integrate(F^((b*x + a)*c)/sqrt(x*e + d), x)`

**Fricas [A]**

time = 0.42, size = 63, normalized size = 0.88

$$\frac{\sqrt{\pi} \sqrt{-bce^{(-1)} \log(F)} \operatorname{erf}\left(\sqrt{-bce^{(-1)} \log(F)} \sqrt{xe+d}\right)}{F^{(bcd-ace)e^{(-1)}} bc \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(c*(b*x+a))/(e*x+d)^(1/2),x, algorithm="fricas")``[Out] -sqrt(pi)*sqrt(-b*c*e^(-1)*log(F))*erf(sqrt(-b*c*e^(-1)*log(F))*sqrt(x*e + d))/(F^((b*c*d - a*c*e)*e^(-1))*b*c*log(F))`**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F**(c*(b*x+a))/(e*x+d)**(1/2),x)``[Out] Integral(F**(c*(a + b*x))/sqrt(d + e*x), x)`**Giac [A]**

time = 3.05, size = 58, normalized size = 0.81

$$\frac{\sqrt{\pi} \operatorname{erf}\left(-\sqrt{-bce \log(F)} \sqrt{xe+d} e^{(-1)}\right) e^{(-(bcd \log(F) - ace \log(F))e^{(-1)})}}{\sqrt{-bce \log(F)}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(c*(b*x+a))/(e*x+d)^(1/2),x, algorithm="giac")``[Out] -sqrt(pi)*erf(-sqrt(-b*c*e*log(F))*sqrt(x*e + d)*e^(-1))*e^(-(b*c*d*log(F) - a*c*e*log(F))*e^(-1))/sqrt(-b*c*e*log(F))`**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(F^(c*(a + b*x))/(d + e*x)^(1/2),x)``[Out] int(F^(c*(a + b*x))/(d + e*x)^(1/2), x)`



$$3.44 \quad \int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx$$

**Optimal.** Leaf size=97

$$-\frac{2F^{c(a+bx)}}{e\sqrt{d+ex}} + \frac{2\sqrt{b}\sqrt{c}F^{c(a-\frac{bd}{e})}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{\log(F)}}{\sqrt{e}}\right)\sqrt{\log(F)}}{e^{3/2}}$$

[Out]  $-2F^{c(bx+a)}/e/(ex+d)^{(1/2)}+2F^{c(a-bd/e)}\operatorname{erfi}(b^{(1/2)}c^{(1/2)}(ex+d)^{(1/2)}\ln(F)^{(1/2)}/e^{(1/2)})b^{(1/2)}c^{(1/2)}\pi^{(1/2)}\ln(F)^{(1/2)}/e^{(3/2)}$

**Rubi [A]**

time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2208, 2211, 2235}

$$\frac{2\sqrt{\pi}\sqrt{b}\sqrt{c}\sqrt{\log(F)}F^{c(a-\frac{bd}{e})}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{e^{3/2}} - \frac{2F^{c(a+bx)}}{e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{c(a+bx)}/(d+ex)^{(3/2)}, x]$

[Out]  $(-2F^{c(a+bx)})/(e\sqrt{d+ex}) + (2\sqrt{b}\sqrt{c}F^{c(a-bd/e)}\sqrt{\pi}\operatorname{Erfi}[(\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{\log(F)})/\sqrt{e}]\sqrt{\log(F)})/e^{(3/2)}$

Rule 2208

$\operatorname{Int}[(b_1 F^{c_1(a_1+bx_1)})^{n_1}/(d_1+ex_1)^{m_1}, x\_Symbol] \rightarrow \operatorname{Simp}[(c_1+d_1x_1)^{m_1+1}((b_1 F^{c_1(a_1+bx_1)})^{n_1}/(d_1(m_1+1)))^{m_1+1}], x] - \operatorname{Dist}[f_1 g_1^n (\log[F]/(d_1(m_1+1))), \operatorname{Int}[(c_1+d_1x_1)^{m_1+1}((b_1 F^{c_1(a_1+bx_1)})^{n_1}/(d_1(m_1+1)))^{m_1+1}], x] /;$  FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !TrueQ[UseGamma]

Rule 2211

$\operatorname{Int}[F^{c(a+bx)}/\sqrt{d+ex}, x\_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{c(a+bx)}/\sqrt{c+dx}, x], x, \sqrt{c+dx}], x] /;$  FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[UseGamma]

Rule 2235

$\operatorname{Int}[F^{c(a+bx)}/(d+ex)^2, x\_Symbol] \rightarrow \operatorname{Simp}[F^a \sqrt{\pi} \operatorname{Erfi}[(c+dx) \operatorname{Rt}[b \log[F], 2]]/(2d \operatorname{Rt}[b \log[F], 2]), x] /;$  FreeQ[{

$F, a, b, c, d, x$  && PosQ[b]

Rubi steps

$$\begin{aligned} \int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx &= -\frac{2F^{c(a+bx)}}{e\sqrt{d+ex}} + \frac{(2bc \log(F)) \int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx}{e} \\ &= -\frac{2F^{c(a+bx)}}{e\sqrt{d+ex}} + \frac{(4bc \log(F)) \text{Subst}\left(\int F^{c\left(a-\frac{bd}{e}\right)+\frac{bcx^2}{e}} dx, x, \sqrt{d+ex}\right)}{e^2} \\ &= -\frac{2F^{c(a+bx)}}{e\sqrt{d+ex}} + \frac{2\sqrt{b} \sqrt{c} F^{c\left(a-\frac{bd}{e}\right)} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c} \sqrt{d+ex} \sqrt{\log(F)}}{\sqrt{e}}\right) \sqrt{\log(F)}}{e^{3/2}} \end{aligned}$$

Mathematica [A]

time = 0.17, size = 75, normalized size = 0.77

$$\frac{2 \left( F^{c(a+bx)} - F^{c\left(a-\frac{bd}{e}\right)} \Gamma\left(\frac{1}{2}, -\frac{bc(d+ex) \log(F)}{e}\right) \sqrt{-\frac{bc(d+ex) \log(F)}{e}} \right)}{e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))/(d + e\*x)^(3/2), x]

[Out] (-2\*(F^(c\*(a + b\*x)) - F^(c\*(a - (b\*d)/e))\*Gamma[1/2, -((b\*c\*(d + e\*x)\*Log[F])/e)]\*Sqrt[-((b\*c\*(d + e\*x)\*Log[F])/e]))/(e\*Sqrt[d + e\*x])

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{F^{c(bx+a)}}{(ex+d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))/(e\*x+d)^(3/2), x)

[Out] int(F^(c\*(b\*x+a))/(e\*x+d)^(3/2), x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^(3/2),x, algorithm="maxima")

[Out] integrate(F^((b\*x + a)\*c)/(x\*e + d)^(3/2), x)

**Fricas** [A]

time = 0.40, size = 91, normalized size = 0.94

$$2 \left( \frac{\sqrt{\pi} \sqrt{-bce^{(-1)} \log(F)} (xe+d) \operatorname{erf}\left(\sqrt{-bce^{(-1)} \log(F)} \sqrt{xe+d}\right)}{F^{(bcd-ace)e^{(-1)}}} + \sqrt{xe+d} F^{bcx+ac} \right) \frac{1}{xe^2 + de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^(3/2),x, algorithm="fricas")

[Out] -2\*(sqrt(pi)\*sqrt(-b\*c\*e^(-1)\*log(F))\*(x\*e + d)\*erf(sqrt(-b\*c\*e^(-1)\*log(F)))\*sqrt(x\*e + d))/F^((b\*c\*d - a\*c\*e)\*e^(-1)) + sqrt(x\*e + d)\*F^(b\*c\*x + a\*c)/(x\*e^2 + d\*e)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))/(e\*x+d)\*\*(3/2),x)

[Out] Integral(F\*\*(c\*(a + b\*x))/(d + e\*x)\*\*(3/2), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^(3/2),x, algorithm="giac")

[Out] integrate(F^((b\*x + a)\*c)/(x\*e + d)^(3/2), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/(d + e*x)^(3/2),x)
```

```
[Out] int(F^(c*(a + b*x))/(d + e*x)^(3/2), x)
```

### 3.45 $\int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx$

**Optimal.** Leaf size=130

$$\frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}} - \frac{4bcF^{c(a+bx)} \log(F)}{3e^2 \sqrt{d+ex}} + \frac{4b^{3/2} c^{3/2} F^{c(a-\frac{bd}{e})} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c} \sqrt{d+ex} \sqrt{\log(F)}}{\sqrt{e}}\right) \log^{\frac{3}{2}}(F)}{3e^{5/2}}$$

[Out]  $-2/3 * F^{(c*(b*x+a))} / e / (e*x+d)^{(3/2)} + 4/3 * b^{(3/2)} * c^{(3/2)} * F^{(c*(a-b*d/e))} * \operatorname{erfi}(b^{(1/2)} * c^{(1/2)} * (e*x+d)^{(1/2)} * \ln(F)^{(1/2)} / e^{(1/2)}) * \ln(F)^{(3/2)} * \pi^{(1/2)} / e^{(5/2)} - 4/3 * b * c * F^{(c*(b*x+a))} * \ln(F) / e^2 / (e*x+d)^{(1/2)}$

**Rubi [A]**

time = 0.09, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2208, 2211, 2235}

$$\frac{4\sqrt{\pi} b^{3/2} c^{3/2} \log^{\frac{3}{2}}(F) F^{c(a-\frac{bd}{e})} \operatorname{Erfi}\left(\frac{\sqrt{b} \sqrt{c} \sqrt{\log(F)} \sqrt{d+ex}}{\sqrt{e}}\right)}{3e^{5/2}} - \frac{4bc \log(F) F^{c(a+bx)}}{3e^2 \sqrt{d+ex}} - \frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(c*(a + b*x))} / (d + e*x)^{(5/2)}, x]$

[Out]  $(-2 * F^{(c*(a + b*x))} / (3 * e * (d + e*x)^{(3/2)}) - (4 * b * c * F^{(c*(a + b*x))} * \operatorname{Log}[F]) / (3 * e^2 * \operatorname{Sqrt}[d + e*x]) + (4 * b^{(3/2)} * c^{(3/2)} * F^{(c*(a - (b*d)/e)}) * \operatorname{Sqrt}[\pi] * \operatorname{Erfi}[(\operatorname{Sqrt}[b] * \operatorname{Sqrt}[c] * \operatorname{Sqrt}[d + e*x] * \operatorname{Sqrt}[\operatorname{Log}[F]]) / \operatorname{Sqrt}[e]] * \operatorname{Log}[F]^{(3/2)}) / (3 * e^{(5/2)}))$

**Rule 2208**

$\operatorname{Int}[(b_*) * (F_*)^{((g_*) * ((e_*) + (f_*) * (x_*)))^{(n_*) * ((c_*) + (d_*) * (x_*))^{(m_*)}}, x\_Symbol] :> \operatorname{Simp}[(c + d*x)^{(m+1)} * ((b * F^{(g*(e + f*x))})^n / (d * (m+1))) , x] - \operatorname{Dist}[f * g * n * (\operatorname{Log}[F] / (d * (m+1))), \operatorname{Int}[(c + d*x)^{(m+1)} * (b * F^{(g*(e + f*x))})^n, x], x] /; \operatorname{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{IntEgerQ}[2*m] \&\& !\operatorname{TrueQ}[\$UseGamma]$

**Rule 2211**

$\operatorname{Int}[(F_*)^{((g_*) * ((e_*) + (f_*) * (x_*))) / \operatorname{Sqrt}[(c_*) + (d_*) * (x_*)], x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\operatorname{TrueQ}[\$UseGamma]$

**Rule 2235**

$\operatorname{Int}[(F_*)^{((a_*) + (b_*) * ((c_*) + (d_*) * (x_*))^{(2)}), x\_Symbol] :> \operatorname{Simp}[F^a * \operatorname{Sqrt}[\pi] * (\operatorname{Erfi}[(c + d*x) * \operatorname{Rt}[b * \operatorname{Log}[F], 2]] / (2 * d * \operatorname{Rt}[b * \operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{$

F, a, b, c, d}, x] && PosQ[b]

Rubi steps

$$\begin{aligned}
 \int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx &= -\frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}} + \frac{(2bc \log(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx}{3e} \\
 &= -\frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}} - \frac{4bcF^{c(a+bx)} \log(F)}{3e^2 \sqrt{d+ex}} + \frac{(4b^2c^2 \log^2(F)) \int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx}{3e^2} \\
 &= -\frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}} - \frac{4bcF^{c(a+bx)} \log(F)}{3e^2 \sqrt{d+ex}} + \frac{(8b^2c^2 \log^2(F)) \text{Subst}\left(\int F^{c\left(a-\frac{bd}{e}\right)+\frac{bcx^2}{e}} dx, x, \sqrt{d+ex}\right)}{3e^3} \\
 &= -\frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}} - \frac{4bcF^{c(a+bx)} \log(F)}{3e^2 \sqrt{d+ex}} + \frac{4b^{3/2}c^{3/2}F^{c\left(a-\frac{bd}{e}\right)} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{1}}{\sqrt{e}}\right)}{3e^{5/2}}
 \end{aligned}$$

Mathematica [A]

time = 0.28, size = 92, normalized size = 0.71

$$\frac{2\left(2eF^{c\left(a-\frac{bd}{e}\right)}\Gamma\left(\frac{1}{2}, -\frac{bc(d+ex)\log(F)}{e}\right)\left(-\frac{bc(d+ex)\log(F)}{e}\right)^{3/2} + F^{c(a+bx)}(e + 2bc(d+ex)\log(F))\right)}{3e^2(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))/(d + e\*x)^(5/2), x]

[Out] (-2\*(2\*e\*F^(c\*(a - (b\*d)/e))\*Gamma[1/2, -((b\*c\*(d + e\*x)\*Log[F])/e)]\*(-((b\*c\*(d + e\*x)\*Log[F])/e))^(3/2) + F^(c\*(a + b\*x))\*(e + 2\*b\*c\*(d + e\*x)\*Log[F]))/(3\*e^2\*(d + e\*x)^(3/2))

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{F^{c(bx+a)}}{(ex+d)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))/(e\*x+d)^(5/2), x)

[Out] int(F^(c\*(b\*x+a))/(e\*x+d)^(5/2), x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(c*(b*x+a))/(e*x+d)^(5/2),x, algorithm="maxima")``[Out] integrate(F^((b*x + a)*c)/(x*e + d)^(5/2), x)`**Fricas [A]**

time = 0.39, size = 139, normalized size = 1.07

$$2 \left( \frac{2 \sqrt{\pi} (bcx^2e^2 + 2bcdxe + bcd^2) \sqrt{-bce^{(-1)} \log(F)} \operatorname{erf}\left(\sqrt{-bce^{(-1)} \log(F)} \sqrt{xe+d}\right) \log(F)}{F^{(bcd-ace)e^{(-1)}}} + \sqrt{xe+d} (2(bcxe + bcd) \log(F) + e) F^{bcx+ac} \right) \\ \hline 3(x^2e^4 + 2dxe^3 + d^2e^2)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(c*(b*x+a))/(e*x+d)^(5/2),x, algorithm="fricas")`

`[Out] -2/3*(2*sqrt(pi)*(b*c*x^2*e^2 + 2*b*c*d*x*e + b*c*d^2)*sqrt(-b*c*e^(-1)*log(F))*erf(sqrt(-b*c*e^(-1)*log(F))*sqrt(x*e + d))*log(F)/F^((b*c*d - a*c*e)*e^(-1)) + sqrt(x*e + d)*(2*(b*c*x*e + b*c*d)*log(F) + e)*F^(b*c*x + a*c))/(x^2*e^4 + 2*d*x*e^3 + d^2*e^2)`

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F**(c*(b*x+a))/(e*x+d)**(5/2),x)``[Out] Integral(F**(c*(a + b*x))/(d + e*x)**(5/2), x)`**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(F^(c*(b*x+a))/(e*x+d)^(5/2),x, algorithm="giac")``[Out] integrate(F^((b*x + a)*c)/(x*e + d)^(5/2), x)`

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))/(d + e\*x)^(5/2), x)

[Out] int(F^(c\*(a + b\*x))/(d + e\*x)^(5/2), x)



### 3.46 $\int \frac{F^{c(a+bx)}}{(d+ex)^{7/2}} dx$

**Optimal.** Leaf size=165

$$-\frac{2F^{c(a+bx)}}{5e(d+ex)^{5/2}} - \frac{4bcF^{c(a+bx)} \log(F)}{15e^2(d+ex)^{3/2}} - \frac{8b^2c^2F^{c(a+bx)} \log^2(F)}{15e^3\sqrt{d+ex}} + \frac{8b^{5/2}c^{5/2}F^{c(a-\frac{bd}{e})} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}}{\sqrt{e}}\right)}{15e^{7/2}}$$

[Out]  $-2/5 * F^{(c*(b*x+a))} / e / (e*x+d)^{(5/2)} - 4/15 * b * c * F^{(c*(b*x+a))} * \ln(F) / e^2 / (e*x+d)^{(3/2)} + 8/15 * b^{(5/2)} * c^{(5/2)} * F^{(c*(a-b*d/e))} * \operatorname{erfi}(b^{(1/2)} * c^{(1/2)} * (e*x+d)^{(1/2)} * \ln(F)^{(1/2)} / e^{(1/2)}) * \ln(F)^{(5/2)} * \pi^{(1/2)} / e^{(7/2)} - 8/15 * b^2 * c^2 * F^{(c*(b*x+a))} * \ln(F)^2 / e^3 / (e*x+d)^{(1/2)}$

**Rubi [A]**

time = 0.13, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2208, 2211, 2235}

$$\frac{8\sqrt{\pi} b^{5/2} c^{5/2} \log^2(F) F^{c(a-\frac{bd}{e})} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{15e^{7/2}} - \frac{8b^2c^2 \log^2(F) F^{c(a+bx)}}{15e^3\sqrt{d+ex}} - \frac{4bc \log(F) F^{c(a+bx)}}{15e^2(d+ex)^{3/2}} - \frac{2F^{c(a+bx)}}{5e(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(c*(a + b*x))} / (d + e*x)^{(7/2)}, x]$

[Out]  $(-2 * F^{(c*(a + b*x))} / (5 * e * (d + e*x)^{(5/2)}) - (4 * b * c * F^{(c*(a + b*x))} * \operatorname{Log}[F]) / (15 * e^2 * (d + e*x)^{(3/2)}) - (8 * b^2 * c^2 * F^{(c*(a + b*x))} * \operatorname{Log}[F]^2) / (15 * e^3 * \operatorname{Sqrt}[d + e*x]) + (8 * b^{(5/2)} * c^{(5/2)} * F^{(c*(a - (b*d)/e))} * \operatorname{Sqrt}[\pi] * \operatorname{Erfi}[(\operatorname{Sqrt}[b] * \operatorname{Sqrt}[c] * \operatorname{Sqrt}[d + e*x] * \operatorname{Sqrt}[\operatorname{Log}[F]]) / \operatorname{Sqrt}[e]] * \operatorname{Log}[F]^{(5/2)}) / (15 * e^{(7/2)})$

Rule 2208

$\operatorname{Int}[(b_*) * (F_*)^{((g_*) * ((e_*) + (f_*) * (x_*)))^{(n_*) * ((c_*) + (d_*) * (x_*))^{(m_*)}}, x\_Symbol] :> \operatorname{Simp}[(c + d*x)^{(m + 1)} * ((b * F^{(g*(e + f*x))})^n / (d * (m + 1))), x] - \operatorname{Dist}[f * g * n * (\operatorname{Log}[F] / (d * (m + 1))), \operatorname{Int}[(c + d*x)^{(m + 1)} * (b * F^{(g*(e + f*x))})^n, x], x] /; \operatorname{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{IntegerQ}[2 * m] \&\& !\operatorname{TrueQ}[\$UseGamma]$

Rule 2211

$\operatorname{Int}[(F_*)^{((g_*) * ((e_*) + (f_*) * (x_*))) / \operatorname{Sqrt}[(c_*) + (d_*) * (x_*)], x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - c*(f/d)) + f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\operatorname{TrueQ}[\$UseGamma]$

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{F^{c(a+bx)}}{(d+ex)^{7/2}} dx &= -\frac{2F^{c(a+bx)}}{5e(d+ex)^{5/2}} + \frac{(2bc \log(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx}{5e} \\ &= -\frac{2F^{c(a+bx)}}{5e(d+ex)^{5/2}} - \frac{4bcF^{c(a+bx)} \log(F)}{15e^2(d+ex)^{3/2}} + \frac{(4b^2c^2 \log^2(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx}{15e^2} \\ &= -\frac{2F^{c(a+bx)}}{5e(d+ex)^{5/2}} - \frac{4bcF^{c(a+bx)} \log(F)}{15e^2(d+ex)^{3/2}} - \frac{8b^2c^2 F^{c(a+bx)} \log^2(F)}{15e^3 \sqrt{d+ex}} + \frac{(8b^3c^3 \log^3(F)) \int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx}{15e^3} \\ &= -\frac{2F^{c(a+bx)}}{5e(d+ex)^{5/2}} - \frac{4bcF^{c(a+bx)} \log(F)}{15e^2(d+ex)^{3/2}} - \frac{8b^2c^2 F^{c(a+bx)} \log^2(F)}{15e^3 \sqrt{d+ex}} + \frac{(16b^3c^3 \log^3(F)) \text{Subst}(\int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx)}{15e^3} \\ &= -\frac{2F^{c(a+bx)}}{5e(d+ex)^{5/2}} - \frac{4bcF^{c(a+bx)} \log(F)}{15e^2(d+ex)^{3/2}} - \frac{8b^2c^2 F^{c(a+bx)} \log^2(F)}{15e^3 \sqrt{d+ex}} + \frac{8b^{5/2}c^{5/2} F^{c(a-\frac{bd}{e})} \sqrt{\pi} \text{erfi}(\frac{\sqrt{d+ex} \log(F)}{e})}{15e^3} \end{aligned}$$

**Mathematica [A]**

time = 0.27, size = 118, normalized size = 0.72

$$\frac{2 \left( -3e^2 F^{c(a+bx)} - 2bc(d+ex) \log(F) \left( 2e F^{c(a-\frac{bd}{e})} \Gamma\left(\frac{1}{2}, -\frac{bc(d+ex) \log(F)}{e}\right) \left( -\frac{bc(d+ex) \log(F)}{e} \right)^{3/2} + F^{c(a+bx)} (e + 2bc(d+ex) \log(F)) \right) \right)}{15e^3(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(d + e*x)^(7/2), x]
```

```
[Out] (2*(-3*e^2*F^(c*(a + b*x)) - 2*b*c*(d + e*x)*Log[F]*(2*e*F^(c*(a - (b*d)/e))
)*Gamma[1/2, -((b*c*(d + e*x)*Log[F])/e)]*(-((b*c*(d + e*x)*Log[F])/e))^(3/
2) + F^(c*(a + b*x))*(e + 2*b*c*(d + e*x)*Log[F]))/(15*e^3*(d + e*x)^(5/2
))
```

**Maple [F]**

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{F^{c(bx+a)}}{(ex+d)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(F^{(c*(b*x+a))}/(e*x+d)^{(7/2)}, x)$

[Out]  $\text{int}(F^{(c*(b*x+a))}/(e*x+d)^{(7/2)}, x)$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(c*(b*x+a))}/(e*x+d)^{(7/2)}, x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}(F^{((b*x + a)*c)}/(x*e + d)^{(7/2)}, x)$

**Fricas [A]**

time = 0.38, size = 224, normalized size = 1.36

$$2 \left( \frac{4 \sqrt{\pi} (b^2 c^2 x^3 e^3 + 3 b^2 c^2 d x^2 e^2 + 3 b^2 c^2 d^2 x e + b^2 c^2 d^3) \sqrt{-b c e^{(-1)} \log(F)} \operatorname{erf}\left(\sqrt{-b c e^{(-1)} \log(F)} \sqrt{x e + d}\right) \log(F)^2}{F^{(b c d - a c) e^{(-1)}}} + (4 (b^2 c^2 x^2 e^2 + 2 b^2 c^2 d x e + b^2 c^2 d^2) \log(F)^2 + 2 (b c x e^2 + b c d e) \log(F) + 3 e^2) \sqrt{x e + d} F^{b c x + a c} \right) \\ 15 (x^3 e^6 + 3 d x^2 e^5 + 3 d^2 x e^4 + d^3 e^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{(c*(b*x+a))}/(e*x+d)^{(7/2)}, x, \text{algorithm}="fricas")$

[Out]  $-2/15*(4*\text{sqrt}(\pi)*(b^2*c^2*x^3*e^3 + 3*b^2*c^2*d*x^2*e^2 + 3*b^2*c^2*d^2*x*e + b^2*c^2*d^3)*\text{sqrt}(-b*c*e^{(-1)}*\log(F))*\text{erf}(\text{sqrt}(-b*c*e^{(-1)}*\log(F))*\text{sqrt}(x*e + d))*\log(F)^2/F^{((b*c*d - a*c*e)*e^{(-1)})} + (4*(b^2*c^2*x^2*e^2 + 2*b^2*c^2*d*x*e + b^2*c^2*d^2)*\log(F)^2 + 2*(b*c*x*e^2 + b*c*d*e)*\log(F) + 3*e^2)*\text{sqrt}(x*e + d)*F^{(b*c*x + a*c)})/(x^3*e^6 + 3*d*x^2*e^5 + 3*d^2*x*e^4 + d^3*e^3)$

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(F^{**}(c*(b*x+a)))/(e*x+d)**(7/2), x)$

[Out] Timed out

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^(7/2),x, algorithm="giac")

[Out] integrate(F^((b\*x + a)\*c)/(x\*e + d)^(7/2), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))/(d + e\*x)^(7/2),x)

[Out] int(F^(c\*(a + b\*x))/(d + e\*x)^(7/2), x)

$$3.47 \quad \int \frac{F^{c(a+bx)}}{(d+ex)^{9/2}} dx$$

**Optimal.** Leaf size=200

$$-\frac{2F^{c(a+bx)}}{7e(d+ex)^{7/2}} - \frac{4bcF^{c(a+bx)} \log(F)}{35e^2(d+ex)^{5/2}} - \frac{8b^2c^2F^{c(a+bx)} \log^2(F)}{105e^3(d+ex)^{3/2}} - \frac{16b^3c^3F^{c(a+bx)} \log^3(F)}{105e^4\sqrt{d+ex}} + \frac{16b^{7/2}c^{7/2}F^{c(a-\frac{bd}{e})}}{\sqrt{e}}$$

[Out]  $-2/7 * F^{(c*(b*x+a))} / e / (e*x+d)^{(7/2)} - 4/35 * b * c * F^{(c*(b*x+a))} * \ln(F) / e^2 / (e*x+d)^{(5/2)} - 8/105 * b^2 * c^2 * F^{(c*(b*x+a))} * \ln(F)^2 / e^3 / (e*x+d)^{(3/2)} + 16/105 * b^{(7/2)} * c^{(7/2)} * F^{(c*(a-b*d/e))} * \operatorname{erfi}(b^{(1/2)} * c^{(1/2)} * (e*x+d)^{(1/2)} * \ln(F)^{(1/2)} / e^{(1/2)}) * \ln(F)^{(7/2)} * \operatorname{Pi}^{(1/2)} / e^{(9/2)} - 16/105 * b^3 * c^3 * F^{(c*(b*x+a))} * \ln(F)^3 / e^4 / (e*x+d)^{(1/2)}$

**Rubi [A]**

time = 0.16, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2208, 2211, 2235}

$$\frac{16\sqrt{\pi} b^{7/2} c^{7/2} \log^{3/2}(F) F^{c(a-\frac{bd}{e})} \operatorname{Erfi}\left(\frac{\sqrt{b} \sqrt{c} \sqrt{\log(F)} \sqrt{d+ex}}{\sqrt{e}}\right)}{105e^{9/2}} - \frac{16b^3c^3 \log^3(F) F^{c(a+bx)}}{105e^4\sqrt{d+ex}} - \frac{8b^2c^2 \log^2(F) F^{c(a+bx)}}{105e^3(d+ex)^{3/2}} - \frac{4bc \log(F) F^{c(a+bx)}}{35e^2(d+ex)^{5/2}} - \frac{2F^{c(a+bx)}}{7e(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[F^{(c*(a+b*x))} / (d+e*x)^{(9/2)}, x]$

[Out]  $(-2 * F^{(c*(a+b*x))} / (7 * e * (d+e*x)^{(7/2)}) - (4 * b * c * F^{(c*(a+b*x))} * \operatorname{Log}[F]) / (35 * e^2 * (d+e*x)^{(5/2)}) - (8 * b^2 * c^2 * F^{(c*(a+b*x))} * \operatorname{Log}[F]^2) / (105 * e^3 * (d+e*x)^{(3/2)}) - (16 * b^3 * c^3 * F^{(c*(a+b*x))} * \operatorname{Log}[F]^3) / (105 * e^4 * \operatorname{Sqrt}[d+e*x]) + (16 * b^{(7/2)} * c^{(7/2)} * F^{(c*(a-(b*d)/e)}) * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[(\operatorname{Sqrt}[b] * \operatorname{Sqrt}[c] * \operatorname{Sqrt}[d+e*x] * \operatorname{Sqrt}[\operatorname{Log}[F]]) / \operatorname{Sqrt}[e]] * \operatorname{Log}[F]^{(7/2)}) / (105 * e^{(9/2)}))$

**Rule 2208**

$\operatorname{Int}[(b * F^{(g * (e + f * x))})^{(n)} * (c + d * x)^{(m)}, x\_Symbol] :> \operatorname{Simp}[(c + d * x)^{(m+1)} * (b * F^{(g * (e + f * x))})^n / (d * (m+1)), x] - \operatorname{Dist}[f * g * n * (\operatorname{Log}[F] / (d * (m+1))), \operatorname{Int}[(c + d * x)^{(m+1)} * (b * F^{(g * (e + f * x))})^n, x], x] /;$  FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

**Rule 2211**

$\operatorname{Int}[F^{(g * (e + f * x))} / \operatorname{Sqrt}[c + d * x], x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g * (e - c * (f/d) + f * g * (x^2/d))}] , x], x, \operatorname{Sqrt}[c + d * x]], x] /;$  FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

## Rule 2235

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] := Simp[F^a\*sqrt[Pi]\*(Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]]/(2\*d\*Rt[b\*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

## Rubi steps

$$\begin{aligned}
 \int \frac{F^{c(a+bx)}}{(d+ex)^{9/2}} dx &= -\frac{2F^{c(a+bx)}}{7e(d+ex)^{7/2}} + \frac{(2bc \log(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^{7/2}} dx}{7e} \\
 &= -\frac{2F^{c(a+bx)}}{7e(d+ex)^{7/2}} - \frac{4bcF^{c(a+bx)} \log(F)}{35e^2(d+ex)^{5/2}} + \frac{(4b^2c^2 \log^2(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx}{35e^2} \\
 &= -\frac{2F^{c(a+bx)}}{7e(d+ex)^{7/2}} - \frac{4bcF^{c(a+bx)} \log(F)}{35e^2(d+ex)^{5/2}} - \frac{8b^2c^2 F^{c(a+bx)} \log^2(F)}{105e^3(d+ex)^{3/2}} + \frac{(8b^3c^3 \log^3(F)) \int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx}{105e^3} \\
 &= -\frac{2F^{c(a+bx)}}{7e(d+ex)^{7/2}} - \frac{4bcF^{c(a+bx)} \log(F)}{35e^2(d+ex)^{5/2}} - \frac{8b^2c^2 F^{c(a+bx)} \log^2(F)}{105e^3(d+ex)^{3/2}} - \frac{16b^3c^3 F^{c(a+bx)} \log^3(F)}{105e^4 \sqrt{d+ex}} \\
 &= -\frac{2F^{c(a+bx)}}{7e(d+ex)^{7/2}} - \frac{4bcF^{c(a+bx)} \log(F)}{35e^2(d+ex)^{5/2}} - \frac{8b^2c^2 F^{c(a+bx)} \log^2(F)}{105e^3(d+ex)^{3/2}} - \frac{16b^3c^3 F^{c(a+bx)} \log^3(F)}{105e^4 \sqrt{d+ex}} \\
 &= -\frac{2F^{c(a+bx)}}{7e(d+ex)^{7/2}} - \frac{4bcF^{c(a+bx)} \log(F)}{35e^2(d+ex)^{5/2}} - \frac{8b^2c^2 F^{c(a+bx)} \log^2(F)}{105e^3(d+ex)^{3/2}} - \frac{16b^3c^3 F^{c(a+bx)} \log^3(F)}{105e^4 \sqrt{d+ex}}
 \end{aligned}$$

**Mathematica [A]**

time = 0.36, size = 144, normalized size = 0.72

$$\frac{2 \left( -15e^{3F^{c(a+bx)}} + 2bc(d+ex) \log(F) \left( -3e^{2F^{c(a+bx)}} - 2bc(d+ex) \log(F) \left( 2e^{F^{c(a+bx)}} \Gamma\left(\frac{1}{2}, -\frac{bc(d+ex) \log(F)}{e}\right) \left( -\frac{bc(d+ex) \log(F)}{e} \right)^{3/2} + F^{c(a+bx)} (e + 2bc(d+ex) \log(F)) \right) \right) \right)}{105e^4(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))/(d + e\*x)^(9/2), x]

[Out] (2\*(-15\*e^3\*F^(c\*(a + b\*x)) + 2\*b\*c\*(d + e\*x)\*Log[F]\*(-3\*e^2\*F^(c\*(a + b\*x)) - 2\*b\*c\*(d + e\*x)\*Log[F]\*(2\*e\*F^(c\*(a + b\*x)/e))\*Gamma[1/2, -(b\*c\*(d + e\*x)\*Log[F])/e])\*(-((b\*c\*(d + e\*x)\*Log[F])/e))^(3/2) + F^(c\*(a + b\*x))\*(e + 2\*b\*c\*(d + e\*x)\*Log[F]))/(105\*e^4\*(d + e\*x)^(7/2))

**Maple [F]**

time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{F^{c(bx+a)}}{(ex+d)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a))/(e*x+d)^(9/2),x)`

[Out] `int(F^(c*(b*x+a))/(e*x+d)^(9/2),x)`

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))/(e*x+d)^(9/2),x, algorithm="maxima")`

[Out] `integrate(F^((b*x + a)*c)/(x*e + d)^(9/2), x)`

**Fricas** [A]

time = 0.37, size = 309, normalized size = 1.54

$$\frac{2 \left( \frac{\sqrt{\pi} (b^3 c^3 x^4 e^4 + 4 b^3 c^3 d x^3 e^3 + 6 b^3 c^3 d^2 x^2 e^2 + 4 b^3 c^3 d^3 x e + b^3 c^3 d^4) \sqrt{-b c e^{-1} \log(F)} \operatorname{erf}(\sqrt{-b c e^{-1} \log(F)} \sqrt{x e + d}) \operatorname{erf}(F^3) + (8 b^3 c^3 x^3 e^3 + 3 b^3 c^3 d x^2 e^2 + 3 b^3 c^3 d^2 x e + b^3 c^3 d^3) \log(F)^3 + 4 (b^3 c^3 x^3 e^3 + 2 b^3 c^3 d x^2 e^2 + b^3 c^3 d^2 x e) \log(F)^2 + 6 (b c x e^3 + b c d e^2) \log(F) + 15 e^3 \sqrt{x e + d} F^{6 c x + a c}}{105 (x^4 e^8 + 4 d x^3 e^7 + 6 d^2 x^2 e^6 + 4 d^3 x e^5 + d^4 e^4)} \right)}{105 (x^4 e^8 + 4 d x^3 e^7 + 6 d^2 x^2 e^6 + 4 d^3 x e^5 + d^4 e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))/(e*x+d)^(9/2),x, algorithm="fricas")`

[Out] `-2/105*(8*sqrt(pi)*(b^3*c^3*x^4*e^4 + 4*b^3*c^3*d*x^3*e^3 + 6*b^3*c^3*d^2*x^2*e^2 + 4*b^3*c^3*d^3*x*e + b^3*c^3*d^4)*sqrt(-b*c*e^(-1)*log(F))*erf(sqrt(-b*c*e^(-1)*log(F))*sqrt(x*e + d))*log(F)^3/F^((b*c*d - a*c*e)*e^(-1)) + (8*(b^3*c^3*x^3*e^3 + 3*b^3*c^3*d*x^2*e^2 + 3*b^3*c^3*d^2*x*e + b^3*c^3*d^3)*log(F)^3 + 4*(b^2*c^2*x^2*e^3 + 2*b^2*c^2*d*x*e^2 + b^2*c^2*d^2*e)*log(F)^2 + 6*(b*c*x*e^3 + b*c*d*e^2)*log(F) + 15*e^3)*sqrt(x*e + d)*F^(b*c*x + a*c))/(x^4*e^8 + 4*d*x^3*e^7 + 6*d^2*x^2*e^6 + 4*d^3*x*e^5 + d^4*e^4)`

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(c*(b*x+a))/(e*x+d)**(9/2),x)`

[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))/(e\*x+d)^(9/2),x, algorithm="giac")

[Out] integrate(F^((b\*x + a)\*c)/(x\*e + d)^(9/2), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))/(d + e\*x)^(9/2),x)

[Out] int(F^(c\*(a + b\*x))/(d + e\*x)^(9/2), x)



### 3.48 $\int e^{-bx} x^{13/2} dx$

Optimal. Leaf size=151

$$-\frac{135135e^{-bx}\sqrt{x}}{64b^7} - \frac{45045e^{-bx}x^{3/2}}{32b^6} - \frac{9009e^{-bx}x^{5/2}}{16b^5} - \frac{1287e^{-bx}x^{7/2}}{8b^4} - \frac{143e^{-bx}x^{9/2}}{4b^3} - \frac{13e^{-bx}x^{11/2}}{2b^2} - \frac{e^{-bx}x^{13/2}}{b} + \dots$$

[Out]  $-45045/32*x^{(3/2)}/b^6/\exp(b*x)-9009/16*x^{(5/2)}/b^5/\exp(b*x)-1287/8*x^{(7/2)}/b^4/\exp(b*x)-143/4*x^{(9/2)}/b^3/\exp(b*x)-13/2*x^{(11/2)}/b^2/\exp(b*x)-x^{(13/2)}/b/\exp(b*x)+135135/128*\operatorname{erf}(b^{(1/2)}*x^{(1/2)})*\operatorname{Pi}^{(1/2)}/b^{(15/2)}-135135/64*x^{(1/2)}/b^7/\exp(b*x)$

Rubi [A]

time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2207, 2211, 2236}

$$\frac{135135\sqrt{\pi}\operatorname{Erf}(\sqrt{b}\sqrt{x})}{128b^{15/2}} - \frac{135135\sqrt{x}e^{-bx}}{64b^7} - \frac{45045x^{3/2}e^{-bx}}{32b^6} - \frac{9009x^{5/2}e^{-bx}}{16b^5} - \frac{1287x^{7/2}e^{-bx}}{8b^4} - \frac{143x^{9/2}e^{-bx}}{4b^3} - \frac{13x^{11/2}e^{-bx}}{2b^2} - \frac{x^{13/2}e^{-bx}}{b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^{(13/2)}/E^{(b*x)}, x]$

[Out]  $(-135135*\operatorname{Sqrt}[x])/(64*b^7*E^{(b*x)}) - (45045*x^{(3/2)})/(32*b^6*E^{(b*x)}) - (9009*x^{(5/2)})/(16*b^5*E^{(b*x)}) - (1287*x^{(7/2)})/(8*b^4*E^{(b*x)}) - (143*x^{(9/2)})/(4*b^3*E^{(b*x)}) - (13*x^{(11/2)})/(2*b^2*E^{(b*x)}) - x^{(13/2)}/(b*E^{(b*x)}) + (135135*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]])/(128*b^{(15/2)})$

Rule 2207

$\operatorname{Int}[(b_.)*(F_.)^{(g_.)*((e_.)+(f_.)*(x_.))}^{(n_.)*((c_.)+(d_.)*(x_.))}^{(m_.)}, x\_Symbol] :> \operatorname{Simp}[(c+d*x)^m*((b*F^{(g*(e+f*x)))^n/(f*g*n*\operatorname{Log}[F])), x] - \operatorname{Dist}[d*(m/(f*g*n*\operatorname{Log}[F])), \operatorname{Int}[(c+d*x)^{(m-1)}*(b*F^{(g*(e+f*x)))^n}, x], x] /; \operatorname{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \operatorname{GtQ}[m, 0] \&\& \operatorname{IntegerQ}[2*m] \&\& \operatorname{!TrueQ}[\$UseGamma]$

Rule 2211

$\operatorname{Int}[(F_.)^{(g_.)*((e_.)+(f_.)*(x_.))}/\operatorname{Sqrt}[(c_.)+(d_.)*(x_.)], x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e-c*(f/d))+f*g*(x^2/d))}, x], x, \operatorname{Sqrt}[c+d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& \operatorname{!TrueQ}[\$UseGamma]$

Rule 2236

$\operatorname{Int}[(F_.)^{(a_.)+(b_.)*((c_.)+(d_.)*(x_.))^2}, x\_Symbol] :> \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erf}[(c+d*x)*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[(-b)*\operatorname{Log}[F], 2])), x] /; \operatorname{Fr}$

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rubi steps

$$\begin{aligned}
 \int e^{-bx} x^{13/2} dx &= -\frac{e^{-bx} x^{13/2}}{b} + \frac{13 \int e^{-bx} x^{11/2} dx}{2b} \\
 &= -\frac{13e^{-bx} x^{11/2}}{2b^2} - \frac{e^{-bx} x^{13/2}}{b} + \frac{143 \int e^{-bx} x^{9/2} dx}{4b^2} \\
 &= -\frac{143e^{-bx} x^{9/2}}{4b^3} - \frac{13e^{-bx} x^{11/2}}{2b^2} - \frac{e^{-bx} x^{13/2}}{b} + \frac{1287 \int e^{-bx} x^{7/2} dx}{8b^3} \\
 &= -\frac{1287e^{-bx} x^{7/2}}{8b^4} - \frac{143e^{-bx} x^{9/2}}{4b^3} - \frac{13e^{-bx} x^{11/2}}{2b^2} - \frac{e^{-bx} x^{13/2}}{b} + \frac{9009 \int e^{-bx} x^{5/2} dx}{16b^4} \\
 &= -\frac{9009e^{-bx} x^{5/2}}{16b^5} - \frac{1287e^{-bx} x^{7/2}}{8b^4} - \frac{143e^{-bx} x^{9/2}}{4b^3} - \frac{13e^{-bx} x^{11/2}}{2b^2} - \frac{e^{-bx} x^{13/2}}{b} + \frac{45045 \int e^{-bx} x^{3/2} dx}{32b^5} \\
 &= -\frac{45045e^{-bx} x^{3/2}}{32b^6} - \frac{9009e^{-bx} x^{5/2}}{16b^5} - \frac{1287e^{-bx} x^{7/2}}{8b^4} - \frac{143e^{-bx} x^{9/2}}{4b^3} - \frac{13e^{-bx} x^{11/2}}{2b^2} - \frac{e^{-bx} x^{13/2}}{b} \\
 &= -\frac{135135e^{-bx} \sqrt{x}}{64b^7} - \frac{45045e^{-bx} x^{3/2}}{32b^6} - \frac{9009e^{-bx} x^{5/2}}{16b^5} - \frac{1287e^{-bx} x^{7/2}}{8b^4} - \frac{143e^{-bx} x^{9/2}}{4b^3} - \frac{13e^{-bx} x^{11/2}}{2b^2} - \frac{e^{-bx} x^{13/2}}{b} \\
 &= -\frac{135135e^{-bx} \sqrt{x}}{64b^7} - \frac{45045e^{-bx} x^{3/2}}{32b^6} - \frac{9009e^{-bx} x^{5/2}}{16b^5} - \frac{1287e^{-bx} x^{7/2}}{8b^4} - \frac{143e^{-bx} x^{9/2}}{4b^3} - \frac{13e^{-bx} x^{11/2}}{2b^2} - \frac{e^{-bx} x^{13/2}}{b} \\
 &= -\frac{135135e^{-bx} \sqrt{x}}{64b^7} - \frac{45045e^{-bx} x^{3/2}}{32b^6} - \frac{9009e^{-bx} x^{5/2}}{16b^5} - \frac{1287e^{-bx} x^{7/2}}{8b^4} - \frac{143e^{-bx} x^{9/2}}{4b^3} - \frac{13e^{-bx} x^{11/2}}{2b^2} - \frac{e^{-bx} x^{13/2}}{b}
 \end{aligned}$$

**Mathematica** [A]

time = 0.60, size = 24, normalized size = 0.16

$$-\frac{\sqrt{bx} \Gamma\left(\frac{15}{2}, bx\right)}{b^8 \sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(13/2)/E^(b\*x), x]

[Out] -((Sqrt[b\*x]\*Gamma[15/2, b\*x])/(b^8\*Sqrt[x]))

**Maple** [A]

time = 0.10, size = 145, normalized size = 0.96

method	result
--------	--------

meijerg

$$-\frac{\sqrt{x} \sqrt{b} (960b^6x^6 + 6240b^5x^5 + 34320b^4x^4 + 154440b^3x^3 + 540540b^2x^2 + 1351350bx + 2027025)e^{-bx}}{960} + \frac{135135\sqrt{\pi} \operatorname{erf}(\sqrt{b}x)}{128} + \frac{15}{b^{\frac{15}{2}}}$$

derivativedivides

$$-x \frac{13}{2} \frac{e^{-bx}}{b} + \frac{-13x \frac{11}{2} e^{-bx}}{2b} +$$

$$13 \left[ -\frac{11x \frac{9}{2} e^{-bx}}{4b} + \right.$$

$$11 \left[ -\frac{9x \frac{7}{2} e^{-bx}}{4b} + \right.$$

$$9 \left[ -\frac{7x \frac{5}{2} e^{-bx}}{4b} + \right.$$

$$7 \left[ -\frac{5x \frac{3}{2} e^{-bx}}{4b} + \left( \frac{5}{2b} \left( -\frac{3\sqrt{x} e^{-bx}}{4b} + \frac{3\sqrt{\pi}}{2b} \right) \right) \right]$$

$$\frac{b}{b}$$

default

$$-\frac{x^{\frac{13}{2}} e^{-bx}}{b} + \frac{-\frac{13x^{\frac{11}{2}} e^{-bx}}{2b} + \dots}{b}$$

$$13 \left[ -\frac{11x^{\frac{9}{2}} e^{-bx}}{4b} + \dots \right]$$

$$11 \left[ -\frac{9x^{\frac{7}{2}} e^{-bx}}{4b} + \dots \right]$$

$$9 \left[ -\frac{7x^{\frac{5}{2}} e^{-bx}}{4b} + \dots \right]$$

$$7 \left[ -\frac{5x^{\frac{3}{2}} e^{-bx}}{4b} + \dots \right]$$

$$5 \left[ -\frac{3\sqrt{x} e^{-bx}}{4b} + \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(13/2)/exp(b*x),x,method=_RETURNVERBOSE)`

[Out]  $-1/b*x^{13/2}*exp(-b*x)+13/b*(-1/2/b*x^{11/2}*exp(-b*x)+11/2/b*(-1/2/b*x^{9/2}*exp(-b*x)+9/2/b*(-1/2/b*x^{7/2}*exp(-b*x)+7/2/b*(-1/2/b*x^{5/2}*exp(-b*x)+5/2/b*(-1/2/b*x^{3/2}*exp(-b*x)+3/2/b*(-1/2/b*x^{1/2}*exp(-b*x)+1/4/b^{3/2}*Pi^{1/2}*erf(b^{1/2}*x^{1/2}))))))$

**Maxima [A]**

time = 0.27, size = 79, normalized size = 0.52

$$\frac{\left(64b^6x^{\frac{13}{2}} + 416b^5x^{\frac{11}{2}} + 2288b^4x^{\frac{9}{2}} + 10296b^3x^{\frac{7}{2}} + 36036b^2x^{\frac{5}{2}} + 90090bx^{\frac{3}{2}} + 135135\sqrt{x}\right)e^{(-bx)}}{64b^7} + \frac{135135\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}\sqrt{x}\right)}{128b^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(13/2)/exp(b*x),x, algorithm="maxima")`

[Out]  $-1/64*(64*b^6*x^{13/2} + 416*b^5*x^{11/2} + 2288*b^4*x^{9/2} + 10296*b^3*x^{7/2} + 36036*b^2*x^{5/2} + 90090*b*x^{3/2} + 135135*\sqrt{x})*e^{(-b*x)}/b^7 + 135135/128*\sqrt{\pi}*erf(\sqrt{b}*\sqrt{x})/b^{15/2}$

**Fricas [A]**

time = 0.39, size = 82, normalized size = 0.54

$$\frac{2(64b^7x^6 + 416b^6x^5 + 2288b^5x^4 + 10296b^4x^3 + 36036b^3x^2 + 90090b^2x + 135135b)\sqrt{x}e^{(-bx)} - 135135\sqrt{\pi}\sqrt{b}\operatorname{erf}\left(\sqrt{b}\sqrt{x}\right)}{128b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(13/2)/exp(b*x),x, algorithm="fricas")`

[Out]  $-1/128*(2*(64*b^7*x^6 + 416*b^6*x^5 + 2288*b^5*x^4 + 10296*b^4*x^3 + 36036*b^3*x^2 + 90090*b^2*x + 135135*b)*\sqrt{x})*e^{(-b*x)} - 135135*\sqrt{\pi}*\sqrt{b}*\operatorname{erf}(\sqrt{b}*\sqrt{x})/b^8$

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(13/2)/exp(b*x),x)`

[Out] Exception raised: SystemError >> excessive stack use: stack is 4060 deep

**Giac [A]**

time = 2.74, size = 80, normalized size = 0.53

$$\frac{\left(64b^6x^{\frac{13}{2}} + 416b^5x^{\frac{11}{2}} + 2288b^4x^{\frac{9}{2}} + 10296b^3x^{\frac{7}{2}} + 36036b^2x^{\frac{5}{2}} + 90090bx^{\frac{3}{2}} + 135135\sqrt{x}\right)e^{(-bx)}}{64b^7} - \frac{135135\sqrt{\pi}\operatorname{erf}\left(-\sqrt{b}\sqrt{x}\right)}{128b^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(13/2)/exp(b\*x),x, algorithm="giac")

[Out]  $-1/64*(64*b^6*x^{(13/2)} + 416*b^5*x^{(11/2)} + 2288*b^4*x^{(9/2)} + 10296*b^3*x^{(7/2)} + 36036*b^2*x^{(5/2)} + 90090*b*x^{(3/2)} + 135135*\sqrt{x})*e^{(-b*x)}/b^7 - 135135/128*\sqrt{\pi}*erf(-\sqrt{b}*\sqrt{x})/b^{(15/2)}$

**Mupad [B]**

time = 3.43, size = 89, normalized size = 0.59

$$\frac{135135 x^{13/2} \sqrt{\pi} \operatorname{erfc}(\sqrt{b x})}{128 b (b x)^{13/2}} - \frac{x^{13/2} e^{-b x} \left( \frac{135135 \sqrt{b x}}{64} + \frac{45045 (b x)^{3/2}}{32} + \frac{9009 (b x)^{5/2}}{16} + \frac{1287 (b x)^{7/2}}{8} + \frac{143 (b x)^{9/2}}{4} + \frac{13 (b x)^{11/2}}{2} + (b x)^{13/2} \right)}{b (b x)^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(13/2)\*exp(-b\*x),x)

[Out]  $-(135135*x^{(13/2)}*\pi^{(1/2)}*\operatorname{erfc}((b*x)^{(1/2)}))/(128*b*(b*x)^{(13/2)}) - (x^{(13/2)}*\exp(-b*x)*((135135*(b*x)^{(1/2)})/64 + (45045*(b*x)^{(3/2)})/32 + (9009*(b*x)^{(5/2)})/16 + (1287*(b*x)^{(7/2)})/8 + (143*(b*x)^{(9/2)})/4 + (13*(b*x)^{(11/2)})/2 + (b*x)^{(13/2)}))/(b*(b*x)^{(13/2)})$

### 3.49 $\int F^{c(a+bx)}(d+ex)^{4/3} dx$

Optimal. Leaf size=71

$$\frac{eF^{c\left(a-\frac{bd}{e}\right)}\sqrt[3]{d+ex}\Gamma\left(\frac{7}{3},-\frac{bc(d+ex)\log(F)}{e}\right)}{b^2c^2\log^2(F)\sqrt[3]{-\frac{bc(d+ex)\log(F)}{e}}}$$

[Out]  $-eF^{c(a-b*d/e)}*(e*x+d)^{(1/3)}*GAMMA(7/3,-b*c*(e*x+d)*\ln(F)/e)/b^2/c^2/\ln(F)^2/(-b*c*(e*x+d)*\ln(F)/e)^{(1/3)}$

Rubi [A]

time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {2212}

$$\frac{e\sqrt[3]{d+ex}F^{c\left(a-\frac{bd}{e}\right)}\Gamma\left(\frac{7}{3},-\frac{bc\log(F)(d+ex)}{e}\right)}{b^2c^2\log^2(F)\sqrt[3]{-\frac{bc\log(F)(d+ex)}{e}}}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))\*(d + e\*x)^(4/3),x]

[Out]  $-((eF^{c(a-(b*d)/e)}*(d+e*x)^{(1/3)}*\Gamma[7/3,-((b*c*(d+e*x)*\text{Log}[F])/e)]))/(b^2*c^2*\text{Log}[F]^2*(-((b*c*(d+e*x)*\text{Log}[F])/e))^{(1/3)})$

Rule 2212

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol]  
 := Simp[(-F^(g\*(e - c\*(f/d))))\*((c + d\*x)^FracPart[m]/(d\*((-f)\*g\*(Log[F]/d)))^(IntPart[m] + 1)\*((-f)\*g\*Log[F]\*((c + d\*x)/d))^FracPart[m])\*Gamma[m + 1, ((-f)\*g\*(Log[F]/d))\*(c + d\*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

Rubi steps

$$\int F^{c(a+bx)}(d+ex)^{4/3} dx = -\frac{eF^{c\left(a-\frac{bd}{e}\right)}\sqrt[3]{d+ex}\Gamma\left(\frac{7}{3},-\frac{bc(d+ex)\log(F)}{e}\right)}{b^2c^2\log^2(F)\sqrt[3]{-\frac{bc(d+ex)\log(F)}{e}}}$$



**Mathematica [A]**

time = 0.17, size = 63, normalized size = 0.89

$$\frac{F^{c\left(a-\frac{bd}{e}\right)}(d+ex)^{7/3}\Gamma\left(\frac{7}{3},-\frac{bc(d+ex)\log(F)}{e}\right)}{e\left(-\frac{bc(d+ex)\log(F)}{e}\right)^{7/3}}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d + e\*x)^(4/3), x]

[Out] -((F^(c\*(a - (b\*d)/e))\*(d + e\*x)^(7/3)\*Gamma[7/3, -((b\*c\*(d + e\*x)\*Log[F])/e)])/(e\*(-((b\*c\*(d + e\*x)\*Log[F])/e))^(7/3)))

**Maple [F]**

time = 0.01, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}(ex+d)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*(e\*x+d)^(4/3), x)

[Out] int(F^(c\*(b\*x+a))\*(e\*x+d)^(4/3), x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^(4/3), x, algorithm="maxima")

[Out] integrate((x\*e + d)^(4/3)\*F^((b\*x + a)\*c), x)

**Fricas [A]**

time = 0.13, size = 118, normalized size = 1.66

$$\frac{3(4bce\log(F) - 3(b^2c^2xe + b^2c^2d)\log(F)^2)(xe+d)^{\frac{1}{3}}F^{bcx+ac} - \frac{4(-bce^{(-1)}\log(F))^{\frac{2}{3}}e^2\Gamma\left(\frac{1}{3},-(bcxe+bcd)e^{(-1)}\log(F)\right)}{F^{(bcd-ace)e^{(-1)}}}}{9b^3c^3\log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^(4/3), x, algorithm="fricas")

[Out] -1/9\*(3\*(4\*b\*c\*e\*log(F) - 3\*(b^2\*c^2\*x\*e + b^2\*c^2\*d)\*log(F)^2)\*(x\*e + d)^(1/3)\*F^(b\*c\*x + a\*c) - 4\*(-b\*c\*e^(-1)\*log(F))^(2/3)\*e^2\*gamma(1/3, -(b\*c\*x\*e + b\*c\*d)\*e^(-1)\*log(F))/F^((b\*c\*d - a\*c\*e)\*e^(-1)))/(b^3\*c^3\*log(F)^3)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}(d+ex)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(e\*x+d)\*\*(4/3), x)

[Out] Integral(F\*\*(c\*(a + b\*x))\*(d + e\*x)\*\*(4/3), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(e\*x+d)^(4/3), x, algorithm="giac")

[Out] integrate((x\*e + d)^(4/3)\*F^((b\*x + a)\*c), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int F^{c(a+bx)}(d+ex)^{4/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))\*(d + e\*x)^(4/3), x)

[Out] int(F^(c\*(a + b\*x))\*(d + e\*x)^(4/3), x)

### 3.50 $\int (F^{c(a+bx)})^n (d+ex)^{4/3} dx$

Optimal. Leaf size=98

$$\frac{e F^{c\left(a-\frac{bd}{e}\right)n-cn(a+bx)} (F^{c(a+bx)})^n \sqrt[3]{d+ex} \Gamma\left(\frac{7}{3}, -\frac{bcn(d+ex)\log(F)}{e}\right)}{b^2 c^2 n^2 \log^2(F) \sqrt[3]{-\frac{bcn(d+ex)\log(F)}{e}}}$$

[Out]  $-e F^{c(a-bd/e)n-cn(bx+a)} (F^{c(bx+a)})^n (e*x+d)^{1/3} \text{GAMMA}(7/3, -b*c*n*(e*x+d)*\ln(F)/e) / b^2/c^2/n^2/\ln(F)^2 / (-b*c*n*(e*x+d)*\ln(F)/e)^{1/3}$

Rubi [A]

time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2213, 2212}

$$\frac{e \sqrt[3]{d+ex} (F^{c(a+bx)})^n F^{cn\left(a-\frac{bd}{e}\right)-cn(a+bx)} \text{Gamma}\left(\frac{7}{3}, -\frac{bcn\log(F)(d+ex)}{e}\right)}{b^2 c^2 n^2 \log^2(F) \sqrt[3]{-\frac{bcn\log(F)(d+ex)}{e}}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(F^{c(a+bx)})^n (d+ex)^{4/3}, x]$

[Out]  $-((e F^{c(a-(b*d)/e)n-cn(a+bx)} (F^{c(a+bx)})^n (d+ex)^{1/3} \text{Gamma}[7/3, -((b*c*n*(d+ex)*\text{Log}[F])/e)]) / (b^2*c^2*n^2*\text{Log}[F]^2 * (-((b*c*n*(d+ex)*\text{Log}[F])/e))^{1/3}))$

Rule 2212

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d)))*((c + d*x)^FracPart[m]/(d*(-f)*g*(Log[F]/d))^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d)^FracPart[m]))*Gamma[m + 1, ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

Rule 2213

```
Int[((b_)*(F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> Dist[(b F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)), Int[(c + d*x)^m F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

Rubi steps

$$\int (F^{c(a+bx)})^n (d+ex)^{4/3} dx = \left( F^{-cn(a+bx)} (F^{c(a+bx)})^n \right) \int F^{cn(a+bx)} (d+ex)^{4/3} dx$$

$$= - \frac{e F^{c\left(a-\frac{bd}{e}\right)n-cn(a+bx)} (F^{c(a+bx)})^n \sqrt[3]{d+ex} \Gamma\left(\frac{7}{3}, -\frac{bcn(d+ex)\log(F)}{e}\right)}{b^2 c^2 n^2 \log^2(F) \sqrt[3]{-\frac{bcn(d+ex)\log(F)}{e}}}$$

**Mathematica [A]**

time = 0.20, size = 78, normalized size = 0.80

$$- \frac{F^{-\frac{bcn(d+ex)}{e}} (F^{c(a+bx)})^n (d+ex)^{7/3} \Gamma\left(\frac{7}{3}, -\frac{bcn(d+ex)\log(F)}{e}\right)}{e \left(-\frac{bcn(d+ex)\log(F)}{e}\right)^{7/3}}$$

Antiderivative was successfully verified.

`[In] Integrate[(F^(c*(a + b*x)))^n*(d + e*x)^(4/3), x]``[Out] -(((F^(c*(a + b*x)))^n*(d + e*x)^(7/3)*Gamma[7/3, -((b*c*n*(d + e*x)*Log[F])/e)])/(e))/((e*F^((b*c*n*(d + e*x))/e)*(-(b*c*n*(d + e*x)*Log[F])/e))^(7/3))`**Maple [F]**

time = 0.02, size = 0, normalized size = 0.00

$$\int (F^{c(bx+a)})^n (ex+d)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((F^(c*(b*x+a)))^n*(e*x+d)^(4/3), x)``[Out] int((F^(c*(b*x+a)))^n*(e*x+d)^(4/3), x)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((F^(c*(b*x+a)))^n*(e*x+d)^(4/3), x, algorithm="maxima")``[Out] integrate((x*e + d)^(4/3)*F^((b*x + a)*c*n), x)`

**Fricas [A]**

time = 0.10, size = 135, normalized size = 1.38

$$\frac{3(4bcne \log(F) - 3(b^2c^2n^2xe + b^2c^2dn^2) \log(F)^2)(xe + d)^{\frac{1}{3}} F^{bcnx+acn} - \frac{4(-bcne^{(-1)} \log(F))^{\frac{2}{3}} e^2 \Gamma(\frac{1}{3}, -(bcnxe+bcdn)e^{(-1)} \log(F))}{F^{(bcdn-acn)e^{(-1)}}}}{9b^3c^3n^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((F^(c\*(b\*x+a)))^n\*(e\*x+d)^(4/3), x, algorithm="fricas")

**[Out]**  $-1/9*(3*(4*b*c*n*e*\log(F) - 3*(b^2*c^2*n^2*x*e + b^2*c^2*d*n^2)*\log(F)^2)*(x*e + d)^{(1/3)}*F^{(b*c*n*x + a*c*n)} - 4*(-b*c*n*e^{(-1)}*\log(F))^{(2/3)}*e^2*\text{gamma}(1/3, -(b*c*n*x*e + b*c*d*n)*e^{(-1)}*\log(F))/F^{((b*c*d*n - a*c*n*e)*e^{(-1)})})/(b^3*c^3*n^3*\log(F)^3)$

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((F\*\*(c\*(b\*x+a)))\*\*n\*(e\*x+d)\*\*(4/3), x)**[Out]** Exception raised: SystemError >> excessive stack use: stack is 8008 deep**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((F^(c\*(b\*x+a)))^n\*(e\*x+d)^(4/3), x, algorithm="giac")**[Out]** integrate((x\*e + d)^(4/3)\*(F^((b\*x + a)\*c))^n, x)**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int (F^{c(a+bx)})^n (d + ex)^{4/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int((F^(c\*(a + b\*x)))^n\*(d + e\*x)^(4/3), x)**[Out]** int((F^(c\*(a + b\*x)))^n\*(d + e\*x)^(4/3), x)

### 3.51 $\int F^{c(a+bx)}(d+ex) dx$

**Optimal.** Leaf size=48

$$-\frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)}{bc \log(F)}$$

[Out]  $-eF^{c(bx+a)}/b^2/c^2/\ln(F)^2+F^{c(bx+a)}*(e*x+d)/b/c/\ln(F)$

**Rubi [A]**

time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2207, 2225}

$$\frac{(d+ex)F^{c(a+bx)}}{bc \log(F)} - \frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))\*(d + e\*x), x]

[Out]  $-((eF^{c(a + b*x)}))/(b^2*c^2*Log[F]^2) + (F^{c(a + b*x)}*(d + e*x))/(b*c*Log[F])$

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(c + d*x)^m*((bF^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(bF^(g*(e + f*x)))^n, x], x]
/; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol]
:> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int F^{c(a+bx)}(d+ex) dx &= \frac{F^{c(a+bx)}(d+ex)}{bc \log(F)} - \frac{e \int F^{c(a+bx)} dx}{bc \log(F)} \\ &= -\frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)}{bc \log(F)} \end{aligned}$$

**Mathematica [A]**

time = 0.01, size = 34, normalized size = 0.71

$$\frac{F^{c(a+bx)}(-e + bc(d + ex) \log(F))}{b^2 c^2 \log^2(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(d + e*x), x]
```

```
[Out] (F^(c*(a + b*x))*(-e + b*c*(d + e*x)*Log[F]))/(b^2*c^2*Log[F]^2)
```

**Maple [A]**

time = 0.00, size = 38, normalized size = 0.79

method	result	size
gospers	$\frac{(\ln(F)bcex + \ln(F)bcd - e)F^{c(bx+a)}}{c^2 b^2 \ln(F)^2}$	38
risch	$\frac{(\ln(F)bcex + \ln(F)bcd - e)F^{c(bx+a)}}{c^2 b^2 \ln(F)^2}$	38
norman	$\frac{(\ln(F)bcd - e)e^{c(bx+a) \ln(F)}}{c^2 b^2 \ln(F)^2} + \frac{ex e^{c(bx+a) \ln(F)}}{cb \ln(F)}$	56
meijerg	$\frac{F^{ca} e \left( 1 - \frac{(-2bcx \ln(F) + 2)e^{bcx \ln(F)}}{2} \right)}{c^2 b^2 \ln(F)^2} - \frac{F^{ca} d (1 - e^{bcx \ln(F)})}{cb \ln(F)}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(e*x+d), x, method=_RETURNVERBOSE)
```

```
[Out] (ln(F)*b*c*e*x + ln(F)*b*c*d - e)*F^(c*(b*x+a))/c^2/b^2/ln(F)^2
```

**Maxima [A]**

time = 0.27, size = 62, normalized size = 1.29

$$\frac{F^{bcx+ac}d}{bc \log(F)} + \frac{(F^{ac}bcx \log(F) - F^{ac})e^{(bcx \log(F)+1)}}{b^2 c^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x+d), x, algorithm="maxima")
```

```
[Out] F^(b*c*x + a*c)*d/(b*c*log(F)) + (F^(a*c)*b*c*x*log(F) - F^(a*c))*e^(b*c*x*log(F) + 1)/(b^2*c^2*log(F)^2)
```

**Fricas [A]**

time = 0.37, size = 40, normalized size = 0.83

$$\frac{((bcxe + bcd) \log(F) - e)F^{bcx+ac}}{b^2 c^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x+d),x, algorithm="fricas")
```

```
[Out] ((b*c*x*e + b*c*d)*log(F) - e)*F^(b*c*x + a*c)/(b^2*c^2*log(F)^2)
```

**Sympy [A]**

time = 0.05, size = 60, normalized size = 1.25

$$\begin{cases} \frac{F^{c(a+bx)}(bcd \log(F) + bce x \log(F) - e)}{b^2 c^2 \log(F)^2} & \text{for } b^2 c^2 \log(F)^2 \neq 0 \\ dx + \frac{ex^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(e*x+d),x)
```

```
[Out] Piecewise((F**(c*(a + b*x))*(b*c*d*log(F) + b*c*e*x*log(F) - e)/(b**2*c**2*log(F)**2), Ne(b**2*c**2*log(F)**2, 0)), (d*x + e*x**2/2, True))
```

**Giac [C]** Result contains complex when optimal does not.

time = 2.32, size = 1079, normalized size = 22.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x+d),x, algorithm="giac")
```

```
[Out] (2*((pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))*(pi*b*c*x*sgn(F) - pi*b*c*x)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2) + (pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)*(b*c*x*log(abs(F)) - 1)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2))*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c) + ((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)*(pi*b*c*x*sgn(F) - pi*b*c*x)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2) - 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))*(b*c*x*log(abs(F)) - 1)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2) + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2))*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)) + 1) - 1/2*I*((pi*b*c*x*sgn(F) - pi*b*c*x - 2*I*b*c*x*log(abs(F)) + 2*I)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(pi^2*b^2*c^2*sgn(F) + 2*I*pi*b^2*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 - 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^2*c^2*log(abs(F))^2) + (pi*b*c*x*sgn(F) - pi*b*c*x + 2*I*b*c*x*log(abs(F)) - 2*I)*e^(-1/2*I*pi*b
```



```

*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(pi^2*b^
2*c^2*sgn(F) - 2*I*pi*b^2*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 + 2*I*pi*b^
2*c^2*log(abs(F)) + 2*b^2*c^2*log(abs(F))^2))*e^(b*c*x*log(abs(F)) + a*c*lo
g(abs(F)) + 1) + 2*(2*b*c*d*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*p
i*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F)))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*s
gn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b*c)*d*sin(-1/2*pi*b*c*x*sgn(F) +
1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (
pi*b*c*sgn(F) - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*
d*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*p
i*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F))) - I*d*e^(-1/2*I*pi*
b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*
b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(
abs(F)))

```

**Mupad [B]**

time = 0.00, size = 38, normalized size = 0.79

$$\frac{F^{ac+bcx} (bcd \ln(F) - e + bcex \ln(F))}{b^2 c^2 \ln(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))\*(d + e\*x),x)

[Out] (F^(a\*c + b\*c\*x)\*(b\*c\*d\*log(F) - e + b\*c\*e\*x\*log(F)))/(b^2\*c^2\*log(F)^2)

### 3.52 $\int F^{c(a+bx)}(d + ex + fx^2) dx$

**Optimal.** Leaf size=135

$$\frac{2fF^{c(a+bx)}}{b^3c^3\log^3(F)} - \frac{eF^{c(a+bx)}}{b^2c^2\log^2(F)} - \frac{2fF^{c(a+bx)}x}{b^2c^2\log^2(F)} + \frac{dF^{c(a+bx)}}{bc\log(F)} + \frac{eF^{c(a+bx)}x}{bc\log(F)} + \frac{fF^{c(a+bx)}x^2}{bc\log(F)}$$

[Out]  $2*f*F^{(c*(b*x+a))}/b^3/c^3/\ln(F)^3 - e*F^{(c*(b*x+a))}/b^2/c^2/\ln(F)^2 - 2*f*F^{(c*(b*x+a))*x}/b^2/c^2/\ln(F)^2 + d*F^{(c*(b*x+a))}/b/c/\ln(F) + e*F^{(c*(b*x+a))*x}/b/c/\ln(F) + f*F^{(c*(b*x+a))*x^2}/b/c/\ln(F)$

**Rubi [A]**

time = 0.08, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {2227, 2225, 2207}

$$\frac{2fF^{c(a+bx)}}{b^3c^3\log^3(F)} - \frac{eF^{c(a+bx)}}{b^2c^2\log^2(F)} - \frac{2fxF^{c(a+bx)}}{b^2c^2\log^2(F)} + \frac{dF^{c(a+bx)}}{bc\log(F)} + \frac{exF^{c(a+bx)}}{bc\log(F)} + \frac{fx^2F^{c(a+bx)}}{bc\log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{(c*(a + b*x))}*(d + e*x + f*x^2), x]$

[Out]  $(2*f*F^{(c*(a + b*x))})/(b^3*c^3*\text{Log}[F]^3) - (e*F^{(c*(a + b*x))})/(b^2*c^2*\text{Log}[F]^2) - (2*f*F^{(c*(a + b*x))*x})/(b^2*c^2*\text{Log}[F]^2) + (d*F^{(c*(a + b*x))})/(b*c*\text{Log}[F]) + (e*F^{(c*(a + b*x))*x})/(b*c*\text{Log}[F]) + (f*F^{(c*(a + b*x))*x^2})/(b*c*\text{Log}[F])$

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2227

```
Int[(F_)^((c_.)*(v_))*(u_), x_Symbol] := Int[ExpandIntegrand[F^(c*ExpandToSum[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v, x] && !TrueQ[$UseGamma]
```

Rubi steps

$$\begin{aligned}
 \int F^{c(a+bx)}(d+ex+fx^2) dx &= \int (dF^{c(a+bx)} + eF^{c(a+bx)}x + fF^{c(a+bx)}x^2) dx \\
 &= d \int F^{c(a+bx)} dx + e \int F^{c(a+bx)}x dx + f \int F^{c(a+bx)}x^2 dx \\
 &= \frac{dF^{c(a+bx)}}{bc \log(F)} + \frac{eF^{c(a+bx)}x}{bc \log(F)} + \frac{fF^{c(a+bx)}x^2}{bc \log(F)} - \frac{e \int F^{c(a+bx)} dx}{bc \log(F)} - \frac{(2f) \int F^{c(a+bx)}x dx}{bc \log(F)} \\
 &= -\frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)} - \frac{2fF^{c(a+bx)}x}{b^2c^2 \log^2(F)} + \frac{dF^{c(a+bx)}}{bc \log(F)} + \frac{eF^{c(a+bx)}x}{bc \log(F)} + \frac{fF^{c(a+bx)}x^2}{bc \log(F)} + \frac{f}{b} \\
 &= \frac{2fF^{c(a+bx)}}{b^3c^3 \log^3(F)} - \frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)} - \frac{2fF^{c(a+bx)}x}{b^2c^2 \log^2(F)} + \frac{dF^{c(a+bx)}}{bc \log(F)} + \frac{eF^{c(a+bx)}x}{bc \log(F)} + \frac{f}{b}
 \end{aligned}$$

**Mathematica [A]**

time = 0.08, size = 56, normalized size = 0.41

$$\frac{F^{c(a+bx)}(2f - bc(e + 2fx) \log(F) + b^2c^2(d + x(e + fx)) \log^2(F))}{b^3c^3 \log^3(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d + e\*x + f\*x^2),x]

[Out] (F^(c\*(a + b\*x))\*(2\*f - b\*c\*(e + 2\*f\*x)\*Log[F] + b^2\*c^2\*(d + x\*(e + f\*x))\*Log[F]^2))/(b^3\*c^3\*Log[F]^3)

**Maple [A]**

time = 0.02, size = 80, normalized size = 0.59

method	result	size
gosper	$\frac{(fx^2c^2b^2 \ln(F)^2 + \ln(F)^2b^2c^2ex + c^2b^2 \ln(F)^2d - 2 \ln(F)bcfx - \ln(F)bce + 2f)F^{c(bx+a)}}{c^3b^3 \ln(F)^3}$	80
risch	$\frac{(fx^2c^2b^2 \ln(F)^2 + \ln(F)^2b^2c^2ex + c^2b^2 \ln(F)^2d - 2 \ln(F)bcfx - \ln(F)bce + 2f)F^{c(bx+a)}}{c^3b^3 \ln(F)^3}$	80
norman	$\frac{(c^2b^2 \ln(F)^2d - \ln(F)bce + 2f)e^{c(bx+a) \ln(F)}}{c^3b^3 \ln(F)^3} + \frac{fx^2e^{c(bx+a) \ln(F)}}{cb \ln(F)} + \frac{(\ln(F)bce - 2f)x e^{c(bx+a) \ln(F)}}{c^2b^2 \ln(F)^2}$	103
meijerg	$-\frac{F^{ca}f \left( 2 - \frac{(3b^2c^2x^2 \ln(F)^2 - 6bcx \ln(F) + 6)e^{bcx \ln(F)}}{3} \right)}{c^3b^3 \ln(F)^3} + \frac{F^{ca}e \left( 1 - \frac{(-2bcx \ln(F) + 2)e^{bcx \ln(F)}}{2} \right)}{c^2b^2 \ln(F)^2} - \frac{F^{ca}d(1 - e^{bcx \ln(F)})}{cb \ln(F)}$	121

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(b\*x+a))\*(f\*x^2+e\*x+d),x,method=\_RETURNVERBOSE)

[Out]  $(f*x^2*c^2*b^2*\ln(F)^2+\ln(F)^2*b^2*c^2*e*x+c^2*b^2*\ln(F)^2*d-2*\ln(F)*b*c*f*x-\ln(F)*b*c*e+2*f)*F^(c*(b*x+a))/c^3/b^3/\ln(F)^3$

**Maxima [A]**

time = 0.29, size = 119, normalized size = 0.88

$$\frac{F^{bcx+ac}d}{bc \log(F)} + \frac{(F^{ac}bcx \log(F) - F^{ac})e^{(bcx \log(F)+1)}}{b^2c^2 \log(F)^2} + \frac{(F^{ac}b^2c^2x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})F^{bcx}f}{b^3c^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(f\*x^2+e\*x+d),x, algorithm="maxima")

[Out]  $F^(b*c*x + a*c)*d/(b*c*\log(F)) + (F^(a*c)*b*c*x*\log(F) - F^(a*c))*e^(b*c*x*\log(F) + 1)/(b^2*c^2*\log(F)^2) + (F^(a*c)*b^2*c^2*x^2*\log(F)^2 - 2*F^(a*c)*b*c*x*\log(F) + 2*F^(a*c))*F^(b*c*x)*f/(b^3*c^3*\log(F)^3)$

**Fricas [A]**

time = 0.39, size = 76, normalized size = 0.56

$$\frac{((b^2c^2fx^2 + b^2c^2xe + b^2c^2d) \log(F)^2 - (2bcfx + bce) \log(F) + 2f) F^{bcx+ac}}{b^3c^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(f\*x^2+e\*x+d),x, algorithm="fricas")

[Out]  $((b^2*c^2*f*x^2 + b^2*c^2*x*e + b^2*c^2*d)*\log(F)^2 - (2*b*c*f*x + b*c*e)*\log(F) + 2*f)*F^(b*c*x + a*c)/(b^3*c^3*\log(F)^3)$

**Sympy [A]**

time = 0.06, size = 116, normalized size = 0.86

$$\begin{cases} \frac{F^{c(a+bx)}(b^2c^2d \log(F)^2 + b^2c^2ex \log(F)^2 + b^2c^2fx^2 \log(F)^2 - bce \log(F) - 2bcfx \log(F) + 2f)}{b^3c^3 \log(F)^3} & \text{for } b^3c^3 \log(F)^3 \neq 0 \\ dx + \frac{ex^2}{2} + \frac{fx^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*(f\*x\*\*2+e\*x+d),x)

[Out] Piecewise((F\*\*(c\*(a + b\*x))\*(b\*\*2\*c\*\*2\*d\*log(F)\*\*2 + b\*\*2\*c\*\*2\*e\*x\*log(F)\*\*2 + b\*\*2\*c\*\*2\*f\*x\*\*2\*log(F)\*\*2 - b\*c\*e\*log(F) - 2\*b\*c\*f\*x\*log(F) + 2\*f)/(b\*\*3\*c\*\*3\*log(F)\*\*3), Ne(b\*\*3\*c\*\*3\*log(F)\*\*3, 0)), (d\*x + e\*x\*\*2/2 + f\*x\*\*3/3, True))

**Giac [C]** Result contains complex when optimal does not.

time = 3.98, size = 2486, normalized size = 18.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*(f\*x^2+e\*x+d),x, algorithm="giac")

[Out] 
$$\begin{aligned} & (2*((\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))*(\pi*b*c*x*\text{sgn}(F) - \pi*b*c*x)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))^2) + (\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)*(b*c*x*\log(\text{abs}(F)) - 1)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))^2))*\cos(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c) + ((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)*(pi*b*c*x*\text{sgn}(F) - \pi*b*c*x)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))^2) - 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))*(b*c*x*\log(\text{abs}(F)) - 1)/((\pi^2*b^2*c^2*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*b^2*c^2*\log(\text{abs}(F))^2)^2 + 4*(\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^2*c^2*\log(\text{abs}(F)))^2))*\sin(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c))*e^{(b*c*x*\log(\text{abs}(F)) + a*c*\log(\text{abs}(F)) + 1) - 1/2*I*((\pi*b*c*x*\text{sgn}(F) - \pi*b*c*x - 2*I*b*c*x*\log(\text{abs}(F)) + 2*I)*e^{(1/2*I*\pi*b*c*x*\text{sgn}(F) - 1/2*I*\pi*b*c*x + 1/2*I*\pi*a*c*\text{sgn}(F) - 1/2*I*\pi*a*c)/(\pi^2*b^2*c^2*\text{sgn}(F) + 2*I*\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi^2*b^2*c^2 - 2*I*\pi*b^2*c^2*\log(\text{abs}(F)) + 2*b^2*c^2*\log(\text{abs}(F))^2) + (\pi*b*c*x*\text{sgn}(F) - \pi*b*c*x + 2*I*b*c*x*\log(\text{abs}(F)) - 2*I)*e^{(-1/2*I*\pi*b*c*x*\text{sgn}(F) + 1/2*I*\pi*b*c*x - 1/2*I*\pi*a*c*\text{sgn}(F) + 1/2*I*\pi*a*c)/(\pi^2*b^2*c^2*\text{sgn}(F) - 2*I*\pi*b^2*c^2*\log(\text{abs}(F))*\text{sgn}(F) - \pi^2*b^2*c^2 + 2*I*\pi*b^2*c^2*\log(\text{abs}(F)) + 2*b^2*c^2*\log(\text{abs}(F))^2))*e^{(b*c*x*\log(\text{abs}(F)) + a*c*\log(\text{abs}(F)) + 1) + (((\pi^2*b^2*c^2*f*x^2*\text{sgn}(F) - \pi^2*b^2*c^2*f*x^2 + 2*b^2*c^2*f*x^2*\log(\text{abs}(F))^2 + \pi^2*b^2*c^2*d*\text{sgn}(F) - \pi^2*b^2*c^2*d + 2*b^2*c^2*d*\log(\text{abs}(F))^2 - 4*b*c*f*x*\log(\text{abs}(F)) + 4*f)*(3*\pi^2*b^3*c^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*b^3*c^3*\log(\text{abs}(F)) + 2*b^3*c^3*\log(\text{abs}(F))^3)/((\pi^3*b^3*c^3*\text{sgn}(F) - 3*\pi*b^3*c^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*b^3*c^3 + 3*\pi*b^3*c^3*\log(\text{abs}(F))^2)^2 + (3*\pi^2*b^3*c^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*b^3*c^3*\log(\text{abs}(F)) + 2*b^3*c^3*\log(\text{abs}(F))^3)^2) - 2*(\pi^3*b^3*c^3*\text{sgn}(F) - 3*\pi*b^3*c^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*b^3*c^3 + 3*\pi*b^3*c^3*\log(\text{abs}(F))^2)*(pi*b^2*c^2*f*x^2*\log(\text{abs}(F))*\text{sgn}(F) - pi*b^2*c^2*f*x^2*\log(\text{abs}(F)) + pi*b^2*c^2*d*\log(\text{abs}(F))*\text{sgn}(F) - pi*b^2*c^2*d*\log(\text{abs}(F)) - pi*b*c*f*x*\text{sgn}(F) + pi*b*c*f*x)/((\pi^3*b^3*c^3*\text{sgn}(F) - 3*\pi*b^3*c^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*b^3*c^3 + 3*\pi*b^3*c^3*\log(\text{abs}(F))^2)^2 + (3*\pi^2*b^3*c^3*\log(\text{abs}(F))*\text{sgn}(F) - 3*\pi^2*b^3*c^3*\log(\text{abs}(F)) + 2*b^3*c^3*\log(\text{abs}(F))^3)^2))*\cos(-1/2*\pi*b*c*x*\text{sgn}(F) + 1/2*\pi*b*c*x - 1/2*\pi*a*c*\text{sgn}(F) + 1/2*\pi*a*c) + ((\pi^3*b^3*c^3*\text{sgn}(F) - 3*\pi*b^3*c^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*b^3*c^3 + 3*\pi*b^3*c^3*\log(\text{abs}(F))^2)*(pi^2*b^2*c^2*f*x^2*\text{sgn}(F) - \pi^2*b^2*c^2*f*x^2 + 2*b^2*c^2*f*x^2*\log(\text{abs}(F))^2 + \pi^2*b^2*c^2*d*\text{sgn}(F) - \pi^2*b^2*c^2*d + 2*b^2*c^2*d*\log(\text{abs}(F))^2 - 4*b*c*f*x*\log(\text{abs}(F)) + 4*f)/((\pi^3*b^3*c^3*\text{sgn}(F) - 3*\pi*b^3*c^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^3*b^3*c^3 + 3*\pi*b^3*c^3*\log(\text{abs}(F))^2)^2} \end{aligned}$$

+ (3\*pi^2\*b^3\*c^3\*log(abs(F))\*sgn(F) - 3\*pi^2\*b^3\*c^3\*log(abs(F)) + 2\*b^3\*c^3\*log(abs(F))^3)^2 + 2\*(3\*pi^2\*b^3\*c^3\*log(abs(F))\*sgn(F) - 3\*pi^2\*b^3\*c^3\*log(abs(F)) + 2\*b^3\*c^3\*log(abs(F))^3)\*(pi\*b^2\*c^2\*f\*x^2\*log(abs(F))\*sgn(F) - pi\*b^2\*c^2\*f\*x^2\*log(abs(F)) + pi\*b^2\*c^2\*d\*log(abs(F))\*sgn(F) - pi\*b^2\*c^2\*d\*log(abs(F)) - pi\*b\*c\*f\*x\*sgn(F) + pi\*b\*c\*f\*x)/((pi^3\*b^3\*c^3\*sgn(F) - 3\*pi\*b^3\*c^3\*log(abs(F))^2\*sgn(F) - pi^3\*b^3\*c^3 + 3\*pi\*b^3\*c^3\*log(abs(F))^2)^2 + (3\*pi^2\*b^3\*c^3\*log(abs(F))\*sgn(F) - 3\*pi^2\*b^3\*c^3\*log(abs(F)) + 2\*b^3\*c^3\*log(abs(F))^3)^2)\*sin(-1/2\*pi\*b\*c\*x\*sgn(F) + 1/2\*pi\*b\*c\*x - 1/2\*pi\*a\*c\*sgn(F) + 1/2\*pi\*a\*c))\*e^(b\*c\*x\*log(abs(F)) + a\*c\*log(abs(F))) - 2\*I\*((-I\*pi^2\*b^2\*c^2\*f\*x^2\*sgn(F) + 2\*pi\*b^2\*c^2\*f\*x^2\*log(abs(F))\*sgn(F) + I\*pi^2\*b^2\*c^2\*f\*x^2 - 2\*pi\*b^2\*c^2\*f\*x^2\*log(abs(F)) - 2\*I\*b^2\*c^2\*f\*x^2\*log(abs(F))^2 - I\*pi^2\*b^2\*c^2\*d\*sgn(F) + 2\*pi\*b^2\*c^2\*d\*log(abs(F))\*sgn(F) + I\*pi^2\*b^2\*c^2\*d - 2\*pi\*b^2\*c^2\*d\*log(abs(F)) - 2\*I\*b^2\*c^2\*d\*log(abs(F))^2 - 2\*pi\*b\*c\*f\*x\*sgn(F) + 2\*pi\*b\*c\*f\*x + 4\*I\*b\*c\*f\*x\*log(abs(F)) - 4\*I\*f)\*e^(1/2\*I\*pi\*b\*c\*x\*sgn(F) - 1/2\*I\*pi\*b\*c\*x + 1/2\*I\*pi\*a\*c\*sgn(F) - 1/2\*I\*pi\*a\*c)/(-4\*I\*pi^3\*b^3\*c^3\*sgn(F) + 12\*pi^2\*b^3\*c^3\*log(abs(F))\*sgn(F) + 12\*I\*pi\*b^3\*c^3\*log(abs(F))^2\*sgn(F) + 4\*I\*pi^3\*b^3\*c^3 - 12\*pi^2\*b^3\*c^3\*log(abs(F)) - 12\*I\*pi\*b^3\*c^3\*log(abs(F))^2 + 8\*b^3\*c^3\*log(abs(F))^3 - (-I\*pi^2\*b^2\*c^2\*f\*x^2\*sgn(F) - 2\*pi\*b^2\*c^2\*f\*x^2\*log(abs(F))\*sgn(F) + I\*pi^2\*b^2\*c^2\*f\*x^2 + 2\*pi\*b^2\*c^2\*f\*x^2\*log(abs(F)) - 2\*I\*b^2\*c^2\*f\*x^2\*log(abs(F))^2 - I\*pi^2\*b^2\*c^2\*d\*sgn(F) - 2\*pi\*b^2\*c^2\*d\*log(abs(F))\*sgn(F) + I\*pi^2\*b^2\*c^2\*d + 2\*pi\*b^2\*c^2\*d\*log(abs(F)) - 2\*I\*b^2\*c^2\*d\*log(abs(F))^2 + 2\*pi\*b\*c\*f\*x\*sgn(F) - 2\*pi\*b\*c\*f\*x + 4\*I\*b\*c\*f\*x\*log(abs(F)) - 4\*I\*f)\*e^(-1/2\*I\*pi\*b\*c\*x\*sgn(F) + 1/2\*I\*pi\*b\*c\*x - 1/2\*I\*pi\*a\*c\*sgn(F) + 1/2\*I\*pi\*a\*c)/(4\*I\*pi^3\*b^3\*c^3\*sgn(F) + 12\*pi^2\*b^3\*c^3\*log(abs(F))\*sgn(F) - 12\*I\*pi\*b^3\*c^3\*log(abs(F))^2\*sgn(F) - 4\*I\*pi^3\*b^3\*c^3 - 12\*p...

**Mupad [B]**

time = 3.40, size = 80, normalized size = 0.59

$$\frac{F^{a+bcx} (fb^2c^2x^2 \ln(F)^2 + eb^2c^2x \ln(F)^2 + db^2c^2 \ln(F)^2 - 2fbcx \ln(F) - ebc \ln(F) + 2f)}{b^3c^3 \ln(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c\*(a + b\*x))\*(d + e\*x + f\*x^2),x)

[Out] (F^(a\*c + b\*c\*x)\*(2\*f - b\*c\*e\*log(F) + b^2\*c^2\*d\*log(F)^2 + b^2\*c^2\*f\*x^2\*log(F)^2 - 2\*b\*c\*f\*x\*log(F) + b^2\*c^2\*e\*x\*log(F)^2))/(b^3\*c^3\*log(F)^3)

### 3.53 $\int F^{c(a+bx)}(d + ex + fx^2 + gx^3) dx$

**Optimal.** Leaf size=229

$$-\frac{6F^{c(a+bx)}g}{b^4c^4\log^4(F)} + \frac{2fF^{c(a+bx)}}{b^3c^3\log^3(F)} + \frac{6F^{c(a+bx)}gx}{b^3c^3\log^3(F)} - \frac{eF^{c(a+bx)}}{b^2c^2\log^2(F)} - \frac{2fF^{c(a+bx)}x}{b^2c^2\log^2(F)} - \frac{3F^{c(a+bx)}gx^2}{b^2c^2\log^2(F)} + \frac{dF^{c(a+bx)}}{bc\log(F)} + \frac{eF^{c(a+bx)}}{bc\log(F)}$$

[Out]  $-6F^{c(b*x+a)}*g/b^4/c^4/\ln(F)^4+2*f*F^{c(b*x+a)}/b^3/c^3/\ln(F)^3+6*F^{c(b*x+a)}*g*x/b^3/c^3/\ln(F)^3-e*F^{c(b*x+a)}/b^2/c^2/\ln(F)^2-2*f*F^{c(b*x+a)}*x/b^2/c^2/\ln(F)^2-3*F^{c(b*x+a)}*g*x^2/b^2/c^2/\ln(F)^2+d*F^{c(b*x+a)}/b/c/\ln(F)+e*F^{c(b*x+a)}*x/b/c/\ln(F)+f*F^{c(b*x+a)}*x^2/b/c/\ln(F)+F^{c(b*x+a)}*g*x^3/b/c/\ln(F)$

**Rubi** [A]

time = 0.14, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$ ,

Rules used = {2227, 2225, 2207}

$$-\frac{6gF^{c(a+bx)}}{b^4c^4\log^4(F)} + \frac{2fF^{c(a+bx)}}{b^3c^3\log^3(F)} + \frac{6gxF^{c(a+bx)}}{b^3c^3\log^3(F)} - \frac{eF^{c(a+bx)}}{b^2c^2\log^2(F)} - \frac{2fxF^{c(a+bx)}}{b^2c^2\log^2(F)} - \frac{3gx^2F^{c(a+bx)}}{b^2c^2\log^2(F)} + \frac{dF^{c(a+bx)}}{bc\log(F)} + \frac{exF^{c(a+bx)}}{bc\log(F)} + \frac{fx^2F^{c(a+bx)}}{bc\log(F)} + \frac{gx^3F^{c(a+bx)}}{bc\log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c(a + b*x)}*(d + e*x + f*x^2 + g*x^3), x]$

[Out]  $(-6*F^{c(a + b*x)}*g)/(b^4*c^4*\text{Log}[F]^4) + (2*f*F^{c(a + b*x)})/(b^3*c^3*\text{Log}[F]^3) + (6*F^{c(a + b*x)}*g*x)/(b^3*c^3*\text{Log}[F]^3) - (e*F^{c(a + b*x)})/(b^2*c^2*\text{Log}[F]^2) - (2*f*F^{c(a + b*x)}*x)/(b^2*c^2*\text{Log}[F]^2) - (3*F^{c(a + b*x)}*g*x^2)/(b^2*c^2*\text{Log}[F]^2) + (d*F^{c(a + b*x)})/(b*c*\text{Log}[F]) + (e*F^{c(a + b*x)}*x)/(b*c*\text{Log}[F]) + (f*F^{c(a + b*x)}*x^2)/(b*c*\text{Log}[F]) + (F^{c(a + b*x)}*g*x^3)/(b*c*\text{Log}[F])$

Rule 2207

$\text{Int}[(b_.)*(F_)^{((g_.)*((e_.) + (f_.)*(x_)))^{(n_.)*((c_.) + (d_.)*(x_))^{(m_.)}, x\_Symbol] :> \text{Simp}[(c + d*x)^m*((b*F^{g*(e + f*x)})^n/(f*g*n*\text{Log}[F])), x] - \text{Dist}[d*(m/(f*g*n*\text{Log}[F])), \text{Int}[(c + d*x)^{(m - 1)}*(b*F^{g*(e + f*x)})^n, x], x] /; \text{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0] \&\& \text{IntegerQ}[2*m] \&\& !\text{TrueQ}[\$UseGamma]$

Rule 2225

$\text{Int}[(F_)^{((c_.)*((a_.) + (b_.)*(x_)))^{(n_.)}, x\_Symbol] :> \text{Simp}[(F^{c(a + b*x)})^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

Rule 2227

$\text{Int}[(F_)^{((c_.)*(v_))*u_}, x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[F^{c*\text{ExpandToSum}[v, x]}, u, x], x] /; \text{FreeQ}\{F, c\}, x] \&\& \text{PolynomialQ}[u, x] \&\& \text{LinearQ}[v,$

x] && !TrueQ[\$UseGamma]

Rubi steps

$$\begin{aligned}
 \int F^{c(a+bx)}(d+ex+fx^2+gx^3) dx &= \int (dF^{c(a+bx)} + eF^{c(a+bx)}x + fF^{c(a+bx)}x^2 + F^{c(a+bx)}gx^3) dx \\
 &= d \int F^{c(a+bx)} dx + e \int F^{c(a+bx)}x dx + f \int F^{c(a+bx)}x^2 dx + g \int F^{c(a+bx)}x^3 dx \\
 &= \frac{dF^{c(a+bx)}}{bc \log(F)} + \frac{eF^{c(a+bx)}x}{bc \log(F)} + \frac{fF^{c(a+bx)}x^2}{bc \log(F)} + \frac{F^{c(a+bx)}gx^3}{bc \log(F)} - \frac{e \int F^{c(a+bx)} dx}{bc \log(F)} \\
 &= -\frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)} - \frac{2fF^{c(a+bx)}x}{b^2c^2 \log^2(F)} - \frac{3F^{c(a+bx)}gx^2}{b^2c^2 \log^2(F)} + \frac{dF^{c(a+bx)}}{bc \log(F)} + \frac{eF^{c(a+bx)}}{bc \log(F)} \\
 &= \frac{2fF^{c(a+bx)}}{b^3c^3 \log^3(F)} + \frac{6F^{c(a+bx)}gx}{b^3c^3 \log^3(F)} - \frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)} - \frac{2fF^{c(a+bx)}x}{b^2c^2 \log^2(F)} - \frac{3F^{c(a+bx)}}{b^2c^2 \log^2(F)} \\
 &= -\frac{6F^{c(a+bx)}g}{b^4c^4 \log^4(F)} + \frac{2fF^{c(a+bx)}}{b^3c^3 \log^3(F)} + \frac{6F^{c(a+bx)}gx}{b^3c^3 \log^3(F)} - \frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)} - \frac{2fF^{c(a+bx)}}{b^2c^2 \log^2(F)}
 \end{aligned}$$

Mathematica [A]

time = 0.13, size = 84, normalized size = 0.37

$$\frac{F^{c(a+bx)}(-6g + 2bc(f + 3gx) \log(F) - b^2c^2(e + x(2f + 3gx)) \log^2(F) + b^3c^3(d + x(e + x(f + gx))) \log^3(F))}{b^4c^4 \log^4(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*(d + e\*x + f\*x^2 + g\*x^3), x]

[Out] (F^(c\*(a + b\*x))\*(-6\*g + 2\*b\*c\*(f + 3\*g\*x)\*Log[F] - b^2\*c^2\*(e + x\*(2\*f + 3\*g\*x))\*Log[F]^2 + b^3\*c^3\*(d + x\*(e + x\*(f + g\*x)))\*Log[F]^3))/(b^4\*c^4\*Log[F]^4)

Maple [A]

time = 0.02, size = 138, normalized size = 0.60

method	result
gospers	$\frac{(gx^3c^3b^3 \ln(F)^3 + \ln(F)^3b^3c^3fx^2 + \ln(F)^3b^3c^3ex + c^3b^3 \ln(F)^3d - 3 \ln(F)^2b^2c^2gx^2 - 2 \ln(F)^2b^2c^2fx - c^2b^2 \ln(F)^2e + 6 \ln(F)bcgx + 2fcb^3c^3)}{c^4b^4 \ln(F)^4}$
risch	$\frac{(gx^3c^3b^3 \ln(F)^3 + \ln(F)^3b^3c^3fx^2 + \ln(F)^3b^3c^3ex + c^3b^3 \ln(F)^3d - 3 \ln(F)^2b^2c^2gx^2 - 2 \ln(F)^2b^2c^2fx - c^2b^2 \ln(F)^2e + 6 \ln(F)bcgx + 2fcb^3c^3)}{c^4b^4 \ln(F)^4}$
norman	$\frac{(c^3b^3 \ln(F)^3d - c^2b^2 \ln(F)^2e + 2fcb \ln(F) - 6g)e^{c(bx+a) \ln(F)}}{c^4b^4 \ln(F)^4} + \frac{gx^3e^{c(bx+a) \ln(F)}}{cb \ln(F)} + \frac{(fcb \ln(F) - 3g)x^2e^{c(bx+a) \ln(F)}}{c^2b^2 \ln(F)^2} + \frac{(c^2b^2 \ln(F)^2e - 6bcgx - 2fcb^3c^3)e^{c(bx+a) \ln(F)}}{c^4b^4 \ln(F)^4}$



meijerg	$\frac{F^{ca} g \left( 6 - \frac{(-4b^3 c^3 x^3 \ln(F)^3 + 12b^2 c^2 x^2 \ln(F)^2 - 24bcx \ln(F) + 24) e^{bcx \ln(F)}}{4} \right)}{c^4 b^4 \ln(F)^4} - \frac{F^{ca} f \left( 2 - \frac{(3b^2 c^2 x^2 \ln(F)^2 - 6bcx \ln(F) + 6) e^{bcx \ln(F)}}{3} \right)}{c^3 b^3 \ln(F)^3} +$
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a))*(g*x^3+f*x^2+e*x+d),x,method=_RETURNVERBOSE)`

[Out]  $(g*x^3*c^3*b^3*\ln(F)^3+\ln(F)^3*b^3*c^3*f*x^2+\ln(F)^3*b^3*c^3*e*x+c^3*b^3*\ln(F)^3*d-3*\ln(F)^2*b^2*c^2*g*x^2-2*\ln(F)^2*b^2*c^2*f*x-c^2*b^2*\ln(F)^2*e+6*\ln(F)*b*c*g*x+2*f*c*b*\ln(F)-6*g)*F^(c*(b*x+a))/c^4/b^4/\ln(F)^4$

**Maxima** [A]

time = 0.30, size = 196, normalized size = 0.86

$$\frac{F^{bcx+ac}d}{bc \log(F)} + \frac{(F^{ac}bcx \log(F) - F^{ac})e^{bcx \log(F)+1}}{b^2 c^2 \log(F)^2} + \frac{(F^{ac}b^2 c^2 x^2 \log(F)^2 - 2 F^{ac}bcx \log(F) + 2 F^{ac})F^{bcx} f}{b^3 c^3 \log(F)^3} + \frac{(F^{ac}b^3 c^3 x^3 \log(F)^3 - 3 F^{ac}b^2 c^2 x^2 \log(F)^2 + 6 F^{ac}bcx \log(F) - 6 F^{ac})F^{bcx} g}{b^4 c^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(g*x^3+f*x^2+e*x+d),x, algorithm="maxima")`

[Out]  $F^{(b*c*x + a*c)}*d/(b*c*\log(F)) + (F^{(a*c)}*b*c*x*\log(F) - F^{(a*c)})*e^{(b*c*x*\log(F) + 1)}/(b^2*c^2*\log(F)^2) + (F^{(a*c)}*b^2*c^2*x^2*\log(F)^2 - 2*F^{(a*c)}*b*c*x*\log(F) + 2*F^{(a*c)})*F^{(b*c*x)}*f/(b^3*c^3*\log(F)^3) + (F^{(a*c)}*b^3*c^3*x^3*\log(F)^3 - 3*F^{(a*c)}*b^2*c^2*x^2*\log(F)^2 + 6*F^{(a*c)}*b*c*x*\log(F) - 6*F^{(a*c)})*F^{(b*c*x)}*g/(b^4*c^4*\log(F)^4)$

**Fricas** [A]

time = 0.37, size = 124, normalized size = 0.54

$$\frac{((b^3 c^3 g x^3 + b^3 c^3 f x^2 + b^3 c^3 e x + b^3 c^3 d) \log(F)^3 - (3 b^2 c^2 g x^2 + 2 b^2 c^2 f x + b^2 c^2 e) \log(F)^2 + 2 (3 b c g x + b c f) \log(F) - 6 g) F^{b c x + a c}}{b^4 c^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(g*x^3+f*x^2+e*x+d),x, algorithm="fricas")`

[Out]  $((b^3*c^3*g*x^3 + b^3*c^3*f*x^2 + b^3*c^3*x*e + b^3*c^3*d)*\log(F)^3 - (3*b^2*c^2*g*x^2 + 2*b^2*c^2*f*x + b^2*c^2*e)*\log(F)^2 + 2*(3*b*c*g*x + b*c*f)*\log(F) - 6*g)*F^{(b*c*x + a*c)}/(b^4*c^4*\log(F)^4)$

**Sympy** [A]

time = 0.09, size = 190, normalized size = 0.83

$$\begin{cases} \frac{F^{c(a+bx)}(b^3 c^3 d \log(F)^3 + b^3 c^3 e x \log(F)^3 + b^3 c^3 f x^2 \log(F)^3 + b^3 c^3 g x^3 \log(F)^3 - b^2 c^2 e \log(F)^2 - 2 b^2 c^2 f x \log(F)^2 - 3 b^2 c^2 g x^2 \log(F)^2 + 2 b c f \log(F) + 6 b c g x \log(F) - 6 g)}{b^4 c^4 \log(F)^4} & \text{for } b^4 c^4 \log(F)^4 \neq 0 \\ dx + \frac{ex^2}{2} + \frac{fx^3}{3} + \frac{gx^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(c*(b*x+a))*(g*x**3+f*x**2+e*x+d),x)`

```
[Out] Piecewise((F**(c*(a + b*x))*(b**3*c**3*d*log(F)**3 + b**3*c**3*e*x*log(F)**3 + b**3*c**3*f*x**2*log(F)**3 + b**3*c**3*g*x**3*log(F)**3 - b**2*c**2*e*log(F)**2 - 2*b**2*c**2*f*x*log(F)**2 - 3*b**2*c**2*g*x**2*log(F)**2 + 2*b*c*f*log(F) + 6*b*c*g*x*log(F) - 6*g)/(b**4*c**4*log(F)**4), Ne(b**4*c**4*log(F)**4, 0)), (d*x + e*x**2/2 + f*x**3/3 + g*x**4/4, True))
```

**Giac** [C] Result contains complex when optimal does not.  
time = 3.20, size = 4275, normalized size = 18.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(g*x^3+f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] (2*((pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))*(pi*b*c*x*sgn(F) - pi*b*c*x)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2) + (pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)*(b*c*x*log(abs(F)) - 1)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2))*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c) + ((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)*(pi*b*c*x*sgn(F) - pi*b*c*x)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2) - 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))*(b*c*x*log(abs(F)) - 1)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2))*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)) + 1) - 1/2*I*((pi*b*c*x*sgn(F) - pi*b*c*x - 2*I*b*c*x*log(abs(F)) + 2*I)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(pi^2*b^2*c^2*sgn(F) + 2*I*pi*b^2*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 - 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^2*c^2*log(abs(F))^2) + (pi*b*c*x*sgn(F) - pi*b*c*x + 2*I*b*c*x*log(abs(F)) - 2*I)*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(pi^2*b^2*c^2*sgn(F) - 2*I*pi*b^2*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 + 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^2*c^2*log(abs(F))^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)) + 1) - (((3*pi^2*b^3*c^3*g*x^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*g*x^3*log(abs(F)) + 2*b^3*c^3*g*x^3*log(abs(F))^3 + 3*pi^2*b^3*c^3*f*x^2*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*f*x^2*log(abs(F)) + 2*b^3*c^3*f*x^2*log(abs(F))^3 + 3*pi^2*b^3*c^3*d*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*d*log(abs(F)) + 2*b^3*c^3*d*log(abs(F))^3 - 3*pi^2*b^2*c^2*g*x^2*sgn(F) + 3*pi^2*b^2*c^2*g*x^2 - 6*b^2*c^2*g*x^2*log(abs(F))^2 - 2*pi^2*b^2*c^2*f*x*sgn(F) + 2*pi^2*b^2*c^2*f*x - 4*b^2*c^2*f*x*log(abs(F))^2 + 12*b*c*g*x*log(abs(F)) + 4*b*c*f*log(abs(F)) - 12*g)*(pi^4*b^4*c^4*sgn(F) - 6*pi^2*b^4*c^4*log(abs(F))^2*sgn(F) - pi^4*b^4*c^4 + 6*pi^2*b^4*c^4*log(abs(F))^2 - 2*b^4*c^4*log(ab
```

$s(F))^4 / ((\pi^4 b^4 c^4 \operatorname{sgn}(F) - 6\pi^2 b^4 c^4 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^4 b^4 c^4 + 6\pi^2 b^4 c^4 \log(\operatorname{abs}(F))^2 - 2b^4 c^4 \log(\operatorname{abs}(F))^4)^2 + 16(\pi^3 b^4 c^4 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi b^4 c^4 \log(\operatorname{abs}(F))^3 \operatorname{sgn}(F) - \pi^3 b^4 c^4 \log(\operatorname{abs}(F)) + \pi b^4 c^4 \log(\operatorname{abs}(F))^3)^2) - 4(\pi^3 b^3 c^3 g x^3 \operatorname{sgn}(F) - 3\pi b^3 c^3 g x^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 b^3 c^3 g x^3 + 3\pi b^3 c^3 g x^3 \log(\operatorname{abs}(F))^2 + \pi^3 b^3 c^3 f x^2 \operatorname{sgn}(F) - 3\pi b^3 c^3 f x^2 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 b^3 c^3 f x^2 + 3\pi b^3 c^3 f x^2 \log(\operatorname{abs}(F)))^2 + \pi^3 b^3 c^3 d \operatorname{sgn}(F) - 3\pi b^3 c^3 d \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 b^3 c^3 d + 3\pi b^3 c^3 d \log(\operatorname{abs}(F))^2 + 6\pi b^2 c^2 g x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 6\pi b^2 c^2 g x^2 \log(\operatorname{abs}(F)) + 4\pi b^2 c^2 f x \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 4\pi b^2 c^2 f x \log(\operatorname{abs}(F)) - 6\pi b c g x \operatorname{sgn}(F) + 6\pi b c g x - 2\pi b c f \operatorname{sgn}(F) + 2\pi b c f)(\pi^3 b^4 c^4 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi b^4 c^4 \log(\operatorname{abs}(F))^3 \operatorname{sgn}(F) - \pi^3 b^4 c^4 \log(\operatorname{abs}(F)) + \pi b^4 c^4 \log(\operatorname{abs}(F))^3) / ((\pi^4 b^4 c^4 \operatorname{sgn}(F) - 6\pi^2 b^4 c^4 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^4 b^4 c^4 + 6\pi^2 b^4 c^4 \log(\operatorname{abs}(F))^2 - 2b^4 c^4 \log(\operatorname{abs}(F))^4)^2 + 16(\pi^3 b^4 c^4 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi b^4 c^4 \log(\operatorname{abs}(F))^3 \operatorname{sgn}(F) - \pi^3 b^4 c^4 \log(\operatorname{abs}(F)) + \pi b^4 c^4 \log(\operatorname{abs}(F))^3)^2) * \cos(-1/2 \pi b c x \operatorname{sgn}(F) + 1/2 \pi b c x - 1/2 \pi a c \operatorname{sgn}(F) + 1/2 \pi a c) - ((\pi^3 b^3 c^3 g x^3 \operatorname{sgn}(F) - 3\pi b^3 c^3 g x^3 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 b^3 c^3 g x^3 + 3\pi b^3 c^3 g x^3 \log(\operatorname{abs}(F))^2 + \pi^3 b^3 c^3 f x^2 \operatorname{sgn}(F) - 3\pi b^3 c^3 f x^2 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 b^3 c^3 f x^2 + 3\pi b^3 c^3 f x^2 \log(\operatorname{abs}(F))^2 + \pi^3 b^3 c^3 d \operatorname{sgn}(F) - 3\pi b^3 c^3 d \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^3 b^3 c^3 d + 3\pi b^3 c^3 d \log(\operatorname{abs}(F))^2 + 6\pi b^2 c^2 g x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 6\pi b^2 c^2 g x^2 \log(\operatorname{abs}(F)) + 4\pi b^2 c^2 f x \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 4\pi b^2 c^2 f x \log(\operatorname{abs}(F)) - 6\pi b c g x \operatorname{sgn}(F) + 6\pi b c g x - 2\pi b c f \operatorname{sgn}(F) + 2\pi b c f)(\pi^4 b^4 c^4 \operatorname{sgn}(F) - 6\pi^2 b^4 c^4 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^4 b^4 c^4 + 6\pi^2 b^4 c^4 \log(\operatorname{abs}(F))^2 - 2b^4 c^4 \log(\operatorname{abs}(F))^4) / ((\pi^4 b^4 c^4 \operatorname{sgn}(F) - 6\pi^2 b^4 c^4 \log(\operatorname{abs}(F))^2 \operatorname{sgn}(F) - \pi^4 b^4 c^4 + 6\pi^2 b^4 c^4 \log(\operatorname{abs}(F))^2 - 2b^4 c^4 \log(\operatorname{abs}(F))^4)^2 + 16(\pi^3 b^4 c^4 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - \pi b^4 c^4 \log(\operatorname{abs}(F))^3 \operatorname{sgn}(F) - \pi^3 b^4 c^4 \log(\operatorname{abs}(F)) + \pi b^4 c^4 \log(\operatorname{abs}(F))^3)^2) + 4(3\pi^2 b^3 c^3 g x^3 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3\pi^2 b^3 c^3 g x^3 \log(\operatorname{abs}(F)) + 2b^3 c^3 g x^3 \log(\operatorname{abs}(F))^3 + 3\pi^2 b^3 c^3 f x^2 \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3\pi^2 b^3 c^3 f x^2 \log(\operatorname{abs}(F)) + 2b^3 c^3 f x^2 \log(\operatorname{abs}(F))^3 + 3\pi^2 b^3 c^3 d \log(\operatorname{abs}(F)) \operatorname{sgn}(F) - 3\pi^2 b^3 c^3 d \log(\operatorname{abs}(F)) + 2b^3 c^3 d \log(\operatorname{abs}(F))^3 - 3\pi^2 b^2 c^2 g x^2 \operatorname{sgn}(F) + 3\pi^2 b^2 c^2 g x^2 - 6b^2 c^2 g \dots$

**Mupad [B]**

time = 3.54, size = 138, normalized size = 0.60

$$\frac{F^{a+bcx} (g b^3 c^3 x^3 \ln(F)^3 + f b^3 c^3 x^2 \ln(F)^3 + e b^3 c^3 x \ln(F)^3 + d b^3 c^3 \ln(F)^3 - 3 g b^2 c^2 x^2 \ln(F)^2 - 2 f b^2 c^2 x \ln(F)^2 - e b^2 c^2 \ln(F)^2 + 6 g b c x \ln(F) + 2 f b c \ln(F) - 6 g)}{b^4 c^4 \ln(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(a + b*x))*(d + e*x + f*x^2 + g*x^3),x)`

```
[Out] (F^(a*c + b*c*x)*(2*b*c*f*log(F) - 6*g + b^3*c^3*d*log(F)^3 - b^2*c^2*e*log
(F)^2 + b^3*c^3*f*x^2*log(F)^3 - 3*b^2*c^2*g*x^2*log(F)^2 + b^3*c^3*g*x^3*log(F)^3 + 6*b*c*g*x*log(F) + b^3*c^3*e*x*log(F)^3 - 2*b^2*c^2*f*x*log(F)^2)
)/(b^4*c^4*log(F)^4)
```

### 3.54 $\int F^{c(a+bx)}(d + ex + fx^2 + gx^3 + hx^4) dx$

**Optimal.** Leaf size=348

$$\frac{24F^{c(a+bx)}h}{b^5c^5\log^5(F)} - \frac{6F^{c(a+bx)}g}{b^4c^4\log^4(F)} - \frac{24F^{c(a+bx)}hx}{b^4c^4\log^4(F)} + \frac{2fF^{c(a+bx)}}{b^3c^3\log^3(F)} + \frac{6F^{c(a+bx)}gx}{b^3c^3\log^3(F)} + \frac{12F^{c(a+bx)}hx^2}{b^3c^3\log^3(F)} - \frac{eF^{c(a+bx)}}{b^2c^2\log^2(F)} - \frac{2fF^{c(a+bx)}}{b^2c^2\log^2(F)}$$

[Out]  $24F^{c(a+bx)}h/b^5c^5/\ln(F)^5 - 6F^{c(a+bx)}g/b^4c^4/\ln(F)^4 - 24F^{c(a+bx)}hx/b^4c^4/\ln(F)^4 + 2fF^{c(a+bx)}/b^3c^3/\ln(F)^3 + 6F^{c(a+bx)}gx/b^3c^3/\ln(F)^3 + 12F^{c(a+bx)}hx^2/b^3c^3/\ln(F)^3 - eF^{c(a+bx)}/b^2c^2/\ln(F)^2 - 2fF^{c(a+bx)}x/b^2c^2/\ln(F)^2 - 3F^{c(a+bx)}gx^2/b^2c^2/\ln(F)^2 - 4F^{c(a+bx)}hx^3/b^2c^2/\ln(F)^2 + dF^{c(a+bx)}/b^2c^2/\ln(F)^2 + eF^{c(a+bx)}x/b^2c^2/\ln(F)^2 + fF^{c(a+bx)}x^2/b^2c^2/\ln(F)^2 + F^{c(a+bx)}/b^2c^2/\ln(F)^2 + eF^{c(a+bx)}x/b^2c^2/\ln(F)^2 + fF^{c(a+bx)}x^2/b^2c^2/\ln(F)^2 + F^{c(a+bx)}/b^2c^2/\ln(F)^2 + eF^{c(a+bx)}x/b^2c^2/\ln(F)^2 + fF^{c(a+bx)}x^2/b^2c^2/\ln(F)^2 + F^{c(a+bx)}/b^2c^2/\ln(F)^2$

**Rubi** [A]

time = 0.24, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ ,

Rules used = {2227, 2225, 2207}

$$\frac{24hF^{c(a+bx)}}{b^5c^5\log^5(F)} - \frac{6gF^{c(a+bx)}}{b^4c^4\log^4(F)} - \frac{24hx^2F^{c(a+bx)}}{b^4c^4\log^4(F)} + \frac{2fF^{c(a+bx)}}{b^3c^3\log^3(F)} + \frac{6gx^2F^{c(a+bx)}}{b^3c^3\log^3(F)} + \frac{12hx^3F^{c(a+bx)}}{b^3c^3\log^3(F)} - \frac{eF^{c(a+bx)}}{b^2c^2\log^2(F)} - \frac{2fx^2F^{c(a+bx)}}{b^2c^2\log^2(F)} - \frac{3gx^3F^{c(a+bx)}}{b^2c^2\log^2(F)} - \frac{4hx^4F^{c(a+bx)}}{b^2c^2\log^2(F)} + \frac{dF^{c(a+bx)}}{bc\log(F)} + \frac{ex^2F^{c(a+bx)}}{bc\log(F)} + \frac{fx^3F^{c(a+bx)}}{bc\log(F)} + \frac{gx^4F^{c(a+bx)}}{bc\log(F)} + \frac{hx^4F^{c(a+bx)}}{bc\log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{c(a+bx)}(d + ex + fx^2 + gx^3 + hx^4), x]$

[Out]  $(24F^{c(a+bx)}h)/(b^5c^5\text{Log}[F]^5) - (6F^{c(a+bx)}g)/(b^4c^4\text{Log}[F]^4) - (24F^{c(a+bx)}hx)/(b^4c^4\text{Log}[F]^4) + (2fF^{c(a+bx)})/(b^3c^3\text{Log}[F]^3) + (6F^{c(a+bx)}gx)/(b^3c^3\text{Log}[F]^3) + (12F^{c(a+bx)}hx^2)/(b^3c^3\text{Log}[F]^3) - (eF^{c(a+bx)})/(b^2c^2\text{Log}[F]^2) - (2fF^{c(a+bx)}x)/(b^2c^2\text{Log}[F]^2) - (3F^{c(a+bx)}gx^2)/(b^2c^2\text{Log}[F]^2) - (4F^{c(a+bx)}hx^3)/(b^2c^2\text{Log}[F]^2) + (dF^{c(a+bx)})/(bc\text{Log}[F]) + (eF^{c(a+bx)}x)/(bc\text{Log}[F]) + (fF^{c(a+bx)}x^2)/(bc\text{Log}[F]) + (F^{c(a+bx)}gx^3)/(bc\text{Log}[F]) + (F^{c(a+bx)}hx^4)/(bc\text{Log}[F])$

**Rule 2207**

$\text{Int}[(b_.)(F_)^{((g_.)((e_.) + (f_.)(x_)))}^{(n_.)((c_.) + (d_.)(x_))}^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(c + dx)^m((bF^{g(e+fx)})^n/(f g^n \text{Log}[F])), x] - \text{Dist}[d(m/(f g^n \text{Log}[F])), \text{Int}[(c + dx)^{m-1}(bF^{g(e+fx)})^n, x], x] /; \text{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0] \&\& \text{IntegerQ}[2*m] \&\& !\text{TrueQ}[\$UseGamma]$

**Rule 2225**

$\text{Int}[(F_)^{((c_.)((a_.) + (b_.)(x_)))}^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(F^{c(a+bx)})^n/(b c^n \text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$



risch	$\frac{(hx^4c^4b^4\ln(F)^4 + \ln(F)^4b^4c^4gx^3 + \ln(F)^4b^4c^4fx^2 + \ln(F)^4b^4c^4ex + \ln(F)^4b^4c^4d - 4\ln(F)^3b^3c^3hx^3 - 3\ln(F)^3b^3c^3gx^2 - 2\ln(F)^3b^3c^3d - 4\ln(F)^2b^2c^2e + 2c^2b^2\ln(F)^2f - 6gcb\ln(F) + 24h)e^{c(bx+a)\ln(F)}}{c^5b^5\ln(F)^5}$
norman	$\frac{(\ln(F)^4b^4c^4d - \ln(F)^3b^3c^3e + 2c^2b^2\ln(F)^2f - 6gcb\ln(F) + 24h)e^{c(bx+a)\ln(F)}}{c^5b^5\ln(F)^5} + \frac{hx^4e^{c(bx+a)\ln(F)}}{cb\ln(F)} + \frac{(gcb\ln(F) - 4h)x^3e^{c(bx+a)\ln(F)}}{c^2b^2\ln(F)^2}$
meijerg	$-\frac{F^{ca}h\left(24 - \frac{(5b^4c^4x^4\ln(F)^4 - 20b^3c^3x^3\ln(F)^3 + 60b^2c^2x^2\ln(F)^2 - 120bcx\ln(F) + 120)e^{bcx\ln(F)}}{5}\right)}{c^5b^5\ln(F)^5} + \frac{F^{ca}g\left(6 - \frac{(-4b^3c^3x^3\ln(F)^3 + 12b^2c^2x^2\ln(F)^2 - 12bcx\ln(F) + 120)e^{bcx\ln(F)}}{5}\right)}{c^4b^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a))*(h*x^4+g*x^3+f*x^2+e*x+d),x,method=_RETURNVERBOSE)`

[Out]  $(hx^4c^4b^4\ln(F)^4 + \ln(F)^4b^4c^4gx^3 + \ln(F)^4b^4c^4fx^2 + \ln(F)^4b^4c^4d - 4\ln(F)^3b^3c^3hx^3 - 3\ln(F)^3b^3c^3gx^2 - 2\ln(F)^3b^3c^3d - 4\ln(F)^2b^2c^2e + 2c^2b^2\ln(F)^2f - 6gcb\ln(F) + 24h)e^{c(bx+a)\ln(F)}/c^5/b^5/\ln(F)^5$

**Maxima** [A]

time = 0.30, size = 293, normalized size = 0.84

$$\frac{F^{bcx+d}}{bc\log(F)} + \frac{(F^{bcx}\log(F) - F^{bcx})e^{(bcx+d)\log(F)}}{b^2c^2\log(F)^2} + \frac{(F^{bcx}\log(F)^2 - 2F^{bcx}\log(F) + 2F^{bcx})F^{bcx}f}{b^3c^3\log(F)^3} + \frac{(F^{bcx}\log(F)^3 - 3F^{bcx}\log(F)^2 + 6F^{bcx}\log(F) - 6F^{bcx})F^{bcx}g}{b^4c^4\log(F)^4} + \frac{(F^{bcx}\log(F)^4 - 4F^{bcx}\log(F)^3 + 12F^{bcx}\log(F)^2 - 24F^{bcx}\log(F) + 24F^{bcx})F^{bcx}h}{b^5c^5\log(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="maxima")`

[Out]  $F^{(b*c*x + a*c)*d}/(b*c*\log(F)) + (F^{(a*c)*b*c*x*\log(F) - F^{(a*c)}})*e^{(b*c*x*\log(F) + 1)}/(b^2*c^2*\log(F)^2) + (F^{(a*c)*b^2*c^2*x^2*\log(F)^2 - 2*F^{(a*c)*b*c*x*\log(F) + 2*F^{(a*c)}})*F^{(b*c*x)*f}/(b^3*c^3*\log(F)^3) + (F^{(a*c)*b^3*c^3*x^3*\log(F)^3 - 3*F^{(a*c)*b^2*c^2*x^2*\log(F)^2 + 6*F^{(a*c)*b*c*x*\log(F) - 6*F^{(a*c)}})*F^{(b*c*x)*g}/(b^4*c^4*\log(F)^4) + (F^{(a*c)*b^4*c^4*x^4*\log(F)^4 - 4*F^{(a*c)*b^3*c^3*x^3*\log(F)^3 + 12*F^{(a*c)*b^2*c^2*x^2*\log(F)^2 - 24*F^{(a*c)*b*c*x*\log(F) + 24*F^{(a*c)}})*F^{(b*c*x)*h}/(b^5*c^5*\log(F)^5)$

**Fricas** [A]

time = 0.40, size = 184, normalized size = 0.53

$$\frac{(b^4c^4hx^4 + b^4c^4gx^3 + b^4c^4fx^2 + b^4c^4ex + b^4c^4d)\log(F)^4 - (4b^3c^3hx^3 + 3b^3c^3gx^2 + 2b^3c^3fx + b^3c^3e)\log(F)^3 + 2(6b^2c^2hx^2 + 3b^2c^2gx + b^2c^2f)\log(F)^2 - 6(4bcx + bcg)\log(F) + 24h)F^{bcx+ac}}{b^5c^5\log(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*(h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="fricas")`

[Out]  $((b^4c^4h*x^4 + b^4c^4g*x^3 + b^4c^4f*x^2 + b^4c^4x*e + b^4c^4d)*\log(F)^4 - (4*b^3*c^3*h*x^3 + 3*b^3*c^3*g*x^2 + 2*b^3*c^3*f*x + b^3*c^3*e)*\log(F)^3 + 2*(6*b^2*c^2*h*x^2 + 3*b^2*c^2*g*x + b^2*c^2*f)*\log(F)^2 - 6*(4*b*c*h*x + b*c*g)*\log(F) + 24*h)*F^{(b*c*x + a*c)}/(b^5*c^5*\log(F)^5)$

**Sympy [A]**

time = 0.11, size = 284, normalized size = 0.82

$$\left\{ \begin{array}{l} \frac{F^{c(a+bx)}(b^4 d \log(F)^4 + b^4 c^2 x \log(F)^4 + b^4 c^2 x^2 \log(F)^4 + b^4 c^2 x^3 \log(F)^4 + b^4 c^2 x^4 \log(F)^4 - b^3 c^2 c \log(F)^3 - 2b^3 c^2 f x \log(F)^3 - 3b^3 c^2 g x^2 \log(F)^3 - 4b^3 c^2 h x^3 \log(F)^3 + 2b^2 c^2 f \log(F)^2 + 6b^2 c^2 g x \log(F)^2 + 12b^2 c^2 h x^2 \log(F)^2 - 6b c g \log(F) - 24b c h x \log(F) + 24b)}{b^5 \log(F)^5} \text{ for } b^5 c^5 \log(F)^5 \neq 0 \\ dx + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5} \text{ otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(F\*\*(c\*(b\*x+a))\*(h\*x\*\*4+g\*x\*\*3+f\*x\*\*2+e\*x+d), x)

**[Out]** Piecewise((F\*\*(c\*(a + b\*x))\*(b\*\*4\*c\*\*4\*d\*log(F)\*\*4 + b\*\*4\*c\*\*4\*e\*x\*log(F)\*\*4 + b\*\*4\*c\*\*4\*f\*x\*\*2\*log(F)\*\*4 + b\*\*4\*c\*\*4\*g\*x\*\*3\*log(F)\*\*4 + b\*\*4\*c\*\*4\*h\*x\*\*4\*log(F)\*\*4 - b\*\*3\*c\*\*3\*e\*log(F)\*\*3 - 2\*b\*\*3\*c\*\*3\*f\*x\*log(F)\*\*3 - 3\*b\*\*3\*c\*\*3\*g\*x\*\*2\*log(F)\*\*3 - 4\*b\*\*3\*c\*\*3\*h\*x\*\*3\*log(F)\*\*3 + 2\*b\*\*2\*c\*\*2\*f\*log(F)\*\*2 + 6\*b\*\*2\*c\*\*2\*g\*x\*log(F)\*\*2 + 12\*b\*\*2\*c\*\*2\*h\*x\*\*2\*log(F)\*\*2 - 6\*b\*c\*g\*log(F) - 24\*b\*c\*h\*x\*log(F) + 24\*h)/(b\*\*5\*c\*\*5\*log(F)\*\*5), Ne(b\*\*5\*c\*\*5\*log(F)\*\*5, 0)), (d\*x + e\*x\*\*2/2 + f\*x\*\*3/3 + g\*x\*\*4/4 + h\*x\*\*5/5, True))

**Giac [C]** Result contains complex when optimal does not.

time = 2.37, size = 7421, normalized size = 21.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(F^(c\*(b\*x+a))\*(h\*x^4+g\*x^3+f\*x^2+e\*x+d), x, algorithm="giac")

**[Out]** (2\*((pi\*b^2\*c^2\*log(abs(F))\*sgn(F) - pi\*b^2\*c^2\*log(abs(F)))\*(pi\*b\*c\*x\*sgn(F) - pi\*b\*c\*x)/((pi^2\*b^2\*c^2\*sgn(F) - pi^2\*b^2\*c^2 + 2\*b^2\*c^2\*log(abs(F))^2)^2 + 4\*(pi\*b^2\*c^2\*log(abs(F))\*sgn(F) - pi\*b^2\*c^2\*log(abs(F)))^2) + (pi^2\*b^2\*c^2\*sgn(F) - pi^2\*b^2\*c^2 + 2\*b^2\*c^2\*log(abs(F))^2)\*(b\*c\*x\*log(abs(F)) - 1)/((pi^2\*b^2\*c^2\*sgn(F) - pi^2\*b^2\*c^2 + 2\*b^2\*c^2\*log(abs(F))^2)^2 + 4\*(pi\*b^2\*c^2\*log(abs(F))\*sgn(F) - pi\*b^2\*c^2\*log(abs(F)))^2))\*cos(-1/2\*pi\*b\*c\*x\*sgn(F) + 1/2\*pi\*b\*c\*x - 1/2\*pi\*a\*c\*sgn(F) + 1/2\*pi\*a\*c) + ((pi^2\*b^2\*c^2\*sgn(F) - pi^2\*b^2\*c^2 + 2\*b^2\*c^2\*log(abs(F))^2)\*(pi\*b\*c\*x\*sgn(F) - pi\*b\*c\*x)/((pi^2\*b^2\*c^2\*sgn(F) - pi^2\*b^2\*c^2 + 2\*b^2\*c^2\*log(abs(F))^2)^2 + 4\*(pi\*b^2\*c^2\*log(abs(F))\*sgn(F) - pi\*b^2\*c^2\*log(abs(F)))^2) - 4\*(pi\*b^2\*c^2\*log(abs(F))\*sgn(F) - pi\*b^2\*c^2\*log(abs(F)))\*(b\*c\*x\*log(abs(F)) - 1)/((pi^2\*b^2\*c^2\*sgn(F) - pi^2\*b^2\*c^2 + 2\*b^2\*c^2\*log(abs(F))^2)^2 + 4\*(pi\*b^2\*c^2\*log(abs(F))\*sgn(F) - pi\*b^2\*c^2\*log(abs(F)))^2))\*sin(-1/2\*pi\*b\*c\*x\*sgn(F) + 1/2\*pi\*b\*c\*x - 1/2\*pi\*a\*c\*sgn(F) + 1/2\*pi\*a\*c))\*e^(b\*c\*x\*log(abs(F)) + a\*c\*log(abs(F)) + 1) - 1/2\*I\*((pi\*b\*c\*x\*sgn(F) - pi\*b\*c\*x - 2\*I\*b\*c\*x\*log(abs(F)) + 2\*I)\*e^(1/2\*I\*pi\*b\*c\*x\*sgn(F) - 1/2\*I\*pi\*b\*c\*x + 1/2\*I\*pi\*a\*c\*sgn(F) - 1/2\*I\*pi\*a\*c)/(pi^2\*b^2\*c^2\*sgn(F) + 2\*I\*pi\*b^2\*c^2\*log(abs(F))\*sgn(F) - pi^2\*b^2\*c^2 - 2\*I\*pi\*b^2\*c^2\*log(abs(F)) + 2\*b^2\*c^2\*log(abs(F))^2) + (pi\*b\*c\*x\*sgn(F) - pi\*b\*c\*x + 2\*I\*b\*c\*x\*log(abs(F)) - 2\*I)\*e^(-1/2\*I\*pi\*b\*c\*x\*sgn(F) + 1/2\*I\*pi\*b\*c\*x - 1/2\*I\*pi\*a\*c\*sgn(F) + 1/2\*I\*pi\*a\*c)/(pi^2\*b^2\*c^2\*sgn(F) - 2\*I\*pi\*b^2\*c^2\*log(abs(F))\*sgn(F) - pi^2\*b^2\*c^2 + 2\*I\*pi\*b^2\*c^2\*log(abs(F))



$$\begin{aligned}
& 2*c^2*\log(\text{abs}(F)) + 2*b^2*c^2*\log(\text{abs}(F))^2)) * e^{(b*c*x*\log(\text{abs}(F)) + a*c*\log(\text{abs}(F)) + 1) - (((4*\pi^3*b^4*c^4*h*x^4*\log(\text{abs}(F))*\text{sgn}(F) - 4*\pi*b^4*c^4*h*x^4*\log(\text{abs}(F))^3*\text{sgn}(F) - 4*\pi^3*b^4*c^4*h*x^4*\log(\text{abs}(F)) + 4*\pi*b^4*c^4*h*x^4*\log(\text{abs}(F))^3 + 4*\pi^3*b^4*c^4*g*x^3*\log(\text{abs}(F))*\text{sgn}(F) - 4*\pi*b^4*c^4*g*x^3*\log(\text{abs}(F))^3*\text{sgn}(F) - 4*\pi^3*b^4*c^4*g*x^3*\log(\text{abs}(F)) + 4*\pi*b^4*c^4*g*x^3*\log(\text{abs}(F))^3 + 4*\pi^3*b^4*c^4*f*x^2*\log(\text{abs}(F))*\text{sgn}(F) - 4*\pi*b^4*c^4*f*x^2*\log(\text{abs}(F))^3*\text{sgn}(F) - 4*\pi^3*b^4*c^4*f*x^2*\log(\text{abs}(F)) + 4*\pi*b^4*c^4*f*x^2*\log(\text{abs}(F))^3 - 4*\pi^3*b^3*c^3*h*x^3*\text{sgn}(F) + 4*\pi^3*b^4*c^4*d*\log(\text{abs}(F))*\text{sgn}(F) + 12*\pi*b^3*c^3*h*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) - 4*\pi*b^4*c^4*d*\log(\text{abs}(F))^3*\text{sgn}(F) + 4*\pi^3*b^3*c^3*h*x^3 - 4*\pi^3*b^4*c^4*d*\log(\text{abs}(F)) - 12*\pi*b^3*c^3*h*x^3*\log(\text{abs}(F))^2 + 4*\pi*b^4*c^4*d*\log(\text{abs}(F))^3 - 3*\pi^3*b^3*c^3*g*x^2*\text{sgn}(F) + 9*\pi*b^3*c^3*g*x^2*\log(\text{abs}(F))^2*\text{sgn}(F) + 3*\pi^3*b^3*c^3*g*x^2 - 9*\pi*b^3*c^3*g*x^2*\log(\text{abs}(F))^2 - 2*\pi^3*b^3*c^3*f*x*\text{sgn}(F) + 6*\pi*b^3*c^3*f*x*\log(\text{abs}(F))^2*\text{sgn}(F) + 2*\pi^3*b^3*c^3*f*x - 6*\pi*b^3*c^3*f*x*\log(\text{abs}(F))^2 - 24*\pi*b^2*c^2*h*x^2*\log(\text{abs}(F))*\text{sgn}(F) + 24*\pi*b^2*c^2*h*x^2*\log(\text{abs}(F)) - 12*\pi*b^2*c^2*g*x*\log(\text{abs}(F))*\text{sgn}(F) + 12*\pi*b^2*c^2*g*x*\log(\text{abs}(F)) - 4*\pi*b^2*c^2*f*\log(\text{abs}(F))*\text{sgn}(F) + 4*\pi*b^2*c^2*f*\log(\text{abs}(F)) + 24*\pi*b*c*h*x*\text{sgn}(F) - 24*\pi*b*c*h*x + 6*\pi*b*c*g*\text{sgn}(F) - 6*\pi*b*c*g*(\pi^5*b^5*c^5*\text{sgn}(F) - 10*\pi^3*b^5*c^5*\log(\text{abs}(F))^2*\text{sgn}(F) + 5*\pi*b^5*c^5*\log(\text{abs}(F))^4*\text{sgn}(F) - \pi^5*b^5*c^5 + 10*\pi^3*b^5*c^5*\log(\text{abs}(F))^2 - 5*\pi*b^5*c^5*\log(\text{abs}(F))^4)/((\pi^5*b^5*c^5*\text{sgn}(F) - 10*\pi^3*b^5*c^5*\log(\text{abs}(F))^2*\text{sgn}(F) + 5*\pi*b^5*c^5*\log(\text{abs}(F))^4*\text{sgn}(F) - \pi^5*b^5*c^5 + 10*\pi^3*b^5*c^5*\log(\text{abs}(F))^2 - 5*\pi*b^5*c^5*\log(\text{abs}(F))^4)^2 + (5*\pi^4*b^5*c^5*\log(\text{abs}(F))*\text{sgn}(F) - 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3*\text{sgn}(F) - 5*\pi^4*b^5*c^5*\log(\text{abs}(F)) + 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3 - 2*b^5*c^5*\log(\text{abs}(F))^5)^2) - (\pi^4*b^4*c^4*h*x^4*\text{sgn}(F) - 6*\pi^2*b^4*c^4*h*x^4*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^4*b^4*c^4*h*x^4 + 6*\pi^2*b^4*c^4*h*x^4*\log(\text{abs}(F))^2 - 2*b^4*c^4*h*x^4*\log(\text{abs}(F))^4 + \pi^4*b^4*c^4*g*x^3*\text{sgn}(F) - 6*\pi^2*b^4*c^4*g*x^3*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^4*b^4*c^4*g*x^3 + 6*\pi^2*b^4*c^4*g*x^3*\log(\text{abs}(F))^2 - 2*b^4*c^4*g*x^3*\log(\text{abs}(F))^4 + \pi^4*b^4*c^4*f*x^2*\text{sgn}(F) - 6*\pi^2*b^4*c^4*f*x^2*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^4*b^4*c^4*f*x^2 + 6*\pi^2*b^4*c^4*f*x^2*\log(\text{abs}(F))^2 - 2*b^4*c^4*f*x^2*\log(\text{abs}(F))^4 + \pi^4*b^4*c^4*d*\text{sgn}(F) + 12*\pi^2*b^3*c^3*h*x^3*\log(\text{abs}(F))*\text{sgn}(F) - 6*\pi^2*b^4*c^4*d*\log(\text{abs}(F))^2*\text{sgn}(F) - \pi^4*b^4*c^4*d - 12*\pi^2*b^3*c^3*h*x^3*\log(\text{abs}(F)) + 6*\pi^2*b^4*c^4*d*\log(\text{abs}(F))^2 + 8*b^3*c^3*h*x^3*\log(\text{abs}(F))^3 - 2*b^4*c^4*d*\log(\text{abs}(F))^4 + 9*\pi^2*b^3*c^3*g*x^2*\log(\text{abs}(F))*\text{sgn}(F) - 9*\pi^2*b^3*c^3*g*x^2*\log(\text{abs}(F)) + 6*b^3*c^3*g*x^2*\log(\text{abs}(F))^3 + 6*\pi^2*b^3*c^3*f*x*\log(\text{abs}(F))*\text{sgn}(F) - 6*\pi^2*b^3*c^3*f*x*\log(\text{abs}(F)) + 4*b^3*c^3*f*x*\log(\text{abs}(F))^3 - 12*\pi^2*b^2*c^2*h*x^2*\text{sgn}(F) + 12*\pi^2*b^2*c^2*h*x^2 - 24*b^2*c^2*h*x^2*\log(\text{abs}(F))^2 - 6*\pi^2*b^2*c^2*g*x*\text{sgn}(F) + 6*\pi^2*b^2*c^2*g*x - 12*b^2*c^2*g*x*\log(\text{abs}(F))^2 - 2*\pi^2*b^2*c^2*f*\text{sgn}(F) + 2*\pi^2*b^2*c^2*f - 4*b^2*c^2*f*\log(\text{abs}(F))^2 + 48*b*c*h*x*\log(\text{abs}(F)) + 12*b*c*g*\log(\text{abs}(F)) - 48*h)*(5*\pi^4*b^5*c^5*\log(\text{abs}(F))*\text{sgn}(F) - 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3*\text{sgn}(F) - 5*\pi^4*b^5*c^5*\log(\text{abs}(F)) + 10*\pi^2*b^5*c^5*\log(\text{abs}(F))^3 - 2*b^5*c^5*\log(\text{abs}(F))^5)/((\pi^5*b^5*c^5*\text{sgn}(F) - 10*\pi^3*b^5*c^5*\log(\text{abs}(F))^2*\text{sgn}(F) + 5*\pi*b^5*c^5*\log(\text{abs}(F))
\end{aligned}$$

$\text{sgn}(F) - \pi^5 b^5 c^5 + 10\pi^3 b^5 c^5 \log(\dots)$

**Mupad [B]**

time = 3.53, size = 212, normalized size = 0.61

$$\frac{F^{a+b*x} (h b^5 c^4 x^4 \ln(F)^4 + g b^5 c^4 x^3 \ln(F)^4 + f b^5 c^4 x^2 \ln(F)^4 + e b^5 c^4 x \ln(F)^4 + d b^5 c^4 \ln(F)^4 - 4 h b^5 c^4 x^3 \ln(F)^3 - 3 g b^5 c^4 x^2 \ln(F)^3 - 2 f b^5 c^4 x \ln(F)^3 - e b^5 c^4 \ln(F)^3 + 12 h b^5 c^4 x^2 \ln(F)^2 + 6 g b^5 c^4 x \ln(F)^2 + 2 f b^5 c^4 \ln(F)^2 - 24 h b c x \ln(F) - 6 g b c \ln(F) + 24 h)}{b^5 c^4 \ln(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(a + b*x))*(d + e*x + f*x^2 + g*x^3 + h*x^4),x)`

[Out]  $(F^{(a*c + b*c*x)} * (24*h - 6*b*c*g*\log(F) + b^4*c^4*d*\log(F)^4 - b^3*c^3*e*\log(F)^3 + 2*b^2*c^2*f*\log(F)^2 + b^4*c^4*f*x^2*\log(F)^4 - 3*b^3*c^3*g*x^2*\log(F)^3 + b^4*c^4*g*x^3*\log(F)^4 + 12*b^2*c^2*h*x^2*\log(F)^2 - 4*b^3*c^3*h*x^3*\log(F)^3 + b^4*c^4*h*x^4*\log(F)^4 - 24*b*c*h*x*\log(F) + b^4*c^4*e*x*\log(F)^4 - 2*b^3*c^3*f*x*\log(F)^3 + 6*b^2*c^2*g*x*\log(F)^2)) / (b^5*c^5*\log(F)^5)$

### 3.55 $\int e^{-a-bx} x^m (a + bx)^3 dx$

**Optimal.** Leaf size=116

$$\frac{a^3 e^{-a} x^m (bx)^{-m} \Gamma(1+m, bx)}{b} - \frac{3a^2 e^{-a} x^m (bx)^{-m} \Gamma(2+m, bx)}{b} - \frac{3a e^{-a} x^m (bx)^{-m} \Gamma(3+m, bx)}{b} - \frac{e^{-a} x^m (bx)^{-m} \Gamma(4+m, bx)}{b}$$

[Out]  $-a^3 x^m \text{GAMMA}(1+m, b*x)/b/\exp(a)/((b*x)^m) - 3*a^2 x^m \text{GAMMA}(2+m, b*x)/b/\exp(a)/((b*x)^m) - 3*a*x^m \text{GAMMA}(3+m, b*x)/b/\exp(a)/((b*x)^m) - x^m \text{GAMMA}(4+m, b*x)/b/\exp(a)/((b*x)^m)$

**Rubi [A]**

time = 0.13, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2230, 2212}

$$\frac{a^3 e^{-a} x^m (bx)^{-m} \text{Gamma}(m+1, bx)}{b} - \frac{3a^2 e^{-a} x^m (bx)^{-m} \text{Gamma}(m+2, bx)}{b} - \frac{3a e^{-a} x^m (bx)^{-m} \text{Gamma}(m+3, bx)}{b} - \frac{e^{-a} x^m (bx)^{-m} \text{Gamma}(m+4, bx)}{b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[E^{(-a - b*x)} * x^m * (a + b*x)^3, x]$

[Out]  $-((a^3 x^m \text{Gamma}[1 + m, b*x]) / (b * E^a * (b*x)^m)) - (3*a^2 x^m \text{Gamma}[2 + m, b*x]) / (b * E^a * (b*x)^m) - (3*a*x^m \text{Gamma}[3 + m, b*x]) / (b * E^a * (b*x)^m) - (x^m \text{Gamma}[4 + m, b*x]) / (b * E^a * (b*x)^m)$

**Rule 2212**

```
Int[(F_)^((g_)*(e_) + (f_)*(x_))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d))^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1, ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

**Rule 2230**

```
Int[(F_)^((c_)*(v_))*(u_)^(m_)*(w_), x_Symbol] :> Int[ExpandIntegrand[F^(c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && IntegerQ[m] && !TrueQ[$UseGamma]
```

Rubi steps

$$\begin{aligned} \int e^{-a-bx} x^m (a+bx)^3 dx &= \int (a^3 e^{-a-bx} x^m + 3a^2 b e^{-a-bx} x^{1+m} + 3ab^2 e^{-a-bx} x^{2+m} + b^3 e^{-a-bx} x^{3+m}) dx \\ &= a^3 \int e^{-a-bx} x^m dx + (3a^2 b) \int e^{-a-bx} x^{1+m} dx + (3ab^2) \int e^{-a-bx} x^{2+m} dx + b^3 \int e^{-a-bx} x^{3+m} dx \\ &= -\frac{a^3 e^{-a} x^m (bx)^{-m} \Gamma(1+m, bx)}{b} - \frac{3a^2 e^{-a} x^m (bx)^{-m} \Gamma(2+m, bx)}{b} - \frac{3ae^{-a} x^m (bx)^{-m} \Gamma(3+m, bx)}{b} - \frac{b^3 e^{-a} x^m (bx)^{-m} \Gamma(4+m, bx)}{b} \end{aligned}$$

**Mathematica [A]**

time = 0.10, size = 61, normalized size = 0.53

$$-\frac{e^{-a} x^m (bx)^{-m} (a^3 \Gamma(1+m, bx) + 3a^2 \Gamma(2+m, bx) + 3a \Gamma(3+m, bx) + \Gamma(4+m, bx))}{b}$$

Antiderivative was successfully verified.

`[In] Integrate[E^(-a - b*x)*x^m*(a + b*x)^3, x]``[Out] -(x^m*(a^3*Gamma[1 + m, b*x] + 3*a^2*Gamma[2 + m, b*x] + 3*a*Gamma[3 + m, b*x] + Gamma[4 + m, b*x]))/(b*E^a*(b*x)^m)`**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.09, size = 334, normalized size = 2.88

method	result
meijerg	$b^{-m-1} e^{-a} \left( x^m b^m (m^2 + 5m + 6) (bx)^{-\frac{m}{2}} e^{-\frac{bx}{2}} \text{WhittakerM}\left(\frac{m}{2}, \frac{m}{2} + \frac{1}{2}, bx\right) - x^m b^m (b^2 x^2 + bmx + 3) \text{WhittakerM}\left(\frac{m}{2}, \frac{m}{2} + \frac{1}{2}, bx\right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(exp(-b*x-a)*x^m*(b*x+a)^3, x, method=_RETURNVERBOSE)`

```
[Out] b^(-m-1)*exp(-a)*(x^m*b^m*(m^2+5*m+6)*(b*x)^(-1/2*m)*exp(-1/2*b*x)*WhittakerM(1/2*m, 1/2*m+1/2, b*x)-x^m*b^m*(b^2*x^2+b*m*x+3*b*x+m^2+5*m+6)*(b*x)^(-1/2*m)*exp(-1/2*b*x)*WhittakerM(1/2*m+1, 1/2*m+1/2, b*x))+3*b^(-m-1)*exp(-a)*a*(x^m*b^m*(2+m)*(b*x)^(-1/2*m)*exp(-1/2*b*x)*WhittakerM(1/2*m, 1/2*m+1/2, b*x)-x^m*b^m*(b*x+m+2)*(b*x)^(-1/2*m)*exp(-1/2*b*x)*WhittakerM(1/2*m+1, 1/2*m+1/2, b*x))+3*b^(-m-1)*exp(-a)*a^2*(x^m*b^m*(b*x)^(-1/2*m)*exp(-1/2*b*x)*WhittakerM(1/2*m, 1/2*m+1/2, b*x)+1/(2+m)*x^m*b^m*(-2-m)*(b*x)^(-1/2*m)*exp(-1/2*b*x)*WhittakerM(1/2*m+1, 1/2*m+1/2, b*x))+exp(-a-1/2*b*x)/b*a^3/(1+m)*x^m*(b*x)^(-1/2*m)*WhittakerM(1/2*m, 1/2*m+1/2, b*x)
```

**Maxima [A]**

time = 0.10, size = 123, normalized size = 1.06

$$-(bx)^{-m-4} b^3 x^{m+4} e^{(-a)} \Gamma(m+4, bx) - 3(bx)^{-m-3} ab^2 x^{m+3} e^{(-a)} \Gamma(m+3, bx) - 3(bx)^{-m-2} a^2 b x^{m+2} e^{(-a)} \Gamma(m+2, bx) - (bx)^{-m-1} a^3 x^{m+1} e^{(-a)} \Gamma(m+1, bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*x^m\*(b\*x+a)^3,x, algorithm="maxima")

[Out]  $-(b*x)^{-m-4}*b^3*x^{m+4}*e^{-a}*gamma(m+4, b*x) - 3*(b*x)^{-m-3}*a*b^2*x^{m+3}*e^{-a}*gamma(m+3, b*x) - 3*(b*x)^{-m-2}*a^2*b*x^{m+2}*e^{-a}*gamma(m+2, b*x) - (b*x)^{-m-1}*a^3*x^{m+1}*e^{-a}*gamma(m+1, b*x)$

**Fricas** [A]

time = 0.15, size = 126, normalized size = 1.09

$$\frac{(b^3x^3 + (3(a+1)b^2 + b^2m)x^2 + ((3a+5)bm + bm^2 + 3(a^2 + 2a+2)b)x)x^m e^{(-bx-a)} + (a^3 + 3(a+2)m^2 + m^3 + 3a^2 + (3a^2 + 9a + 11)m + 6a + 6)e^{(-m \log(b) - a)} \Gamma(m+1, bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*x^m\*(b\*x+a)^3,x, algorithm="fricas")

[Out]  $-\left((b^3x^3 + (3(a+1)b^2 + b^2m)x^2 + ((3a+5)b^2m + b^2m^2 + 3(a^2 + 2a+2)b)x)x^m e^{-bx-a} + (a^3 + 3(a+2)m^2 + m^3 + 3a^2 + (3a^2 + 9a + 11)m + 6a + 6)e^{-m \log(b) - a} \Gamma(m+1, bx)\right)/b$

**Sympy** [A]

time = 11.99, size = 99, normalized size = 0.85

$$\left(-\frac{a^3 x^m (bx)^{-m} \Gamma(m+1, bx)}{b} - 3a^2 x^{m+1} (bx)^{-m-1} \Gamma(m+2, bx) - 3abx^{m+2} (bx)^{-m-2} \Gamma(m+3, bx) - b^2 x^{m+3} (bx)^{-m-3} \Gamma(m+4, bx)\right) e^{-a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*x\*\*m\*(b\*x+a)\*\*3,x)

[Out]  $(-a**3*x**m*uppergamma(m+1, b*x)/(b*(b*x)**m) - 3*a**2*x**(m+1)*(b*x)**(-m-1)*uppergamma(m+2, b*x) - 3*a*b*x**(m+2)*(b*x)**(-m-2)*uppergamma(m+3, b*x) - b**2*x**(m+3)*(b*x)**(-m-3)*uppergamma(m+4, b*x))*exp(-a)$

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*x^m\*(b\*x+a)^3,x, algorithm="giac")

[Out] integrate((b\*x + a)^3\*x^m\*e^(-b\*x - a), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^m e^{-a-bx} (a+bx)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*exp(- a - b*x)*(a + b*x)^3,x)
```

```
[Out] int(x^m*exp(- a - b*x)*(a + b*x)^3, x)
```

### 3.56 $\int e^{-a-bx} x^3 (a + bx)^3 dx$

**Optimal.** Leaf size=397

$$\frac{720e^{-a-bx}}{b^4} - \frac{360ae^{-a-bx}}{b^4} - \frac{72a^2e^{-a-bx}}{b^4} - \frac{6a^3e^{-a-bx}}{b^4} - \frac{720e^{-a-bx}x}{b^3} - \frac{360ae^{-a-bx}x}{b^3} - \frac{72a^2e^{-a-bx}x}{b^3} - \frac{6a^3e^{-a-bx}x}{b^3}$$

[Out]  $-720*\exp(-b*x-a)/b^4-360*a*\exp(-b*x-a)/b^4-72*a^2*\exp(-b*x-a)/b^4-6*a^3*\exp(-b*x-a)/b^4-720*\exp(-b*x-a)*x/b^3-360*a*\exp(-b*x-a)*x/b^3-72*a^2*\exp(-b*x-a)*x/b^3-6*a^3*\exp(-b*x-a)*x/b^3-360*\exp(-b*x-a)*x^2/b^2-180*a*\exp(-b*x-a)*x^2/b^2-36*a^2*\exp(-b*x-a)*x^2/b^2-3*a^3*\exp(-b*x-a)*x^2/b^2-120*\exp(-b*x-a)*x^3/b-60*a*\exp(-b*x-a)*x^3/b-12*a^2*\exp(-b*x-a)*x^3/b-a^3*\exp(-b*x-a)*x^3/b-30*\exp(-b*x-a)*x^4-15*a*\exp(-b*x-a)*x^4-3*a^2*\exp(-b*x-a)*x^4-6*b*\exp(-b*x-a)*x^5-3*a*b*\exp(-b*x-a)*x^5-b^2*\exp(-b*x-a)*x^6$

**Rubi [A]**

time = 0.36, antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2227, 2207, 2225}

$$\frac{6a^3e^{-bx}}{b^4} - \frac{6a^2xe^{-bx}}{b^3} - \frac{3a^2x^2e^{-bx}}{b^2} - \frac{a^2x^3e^{-bx}}{b} - \frac{72a^2e^{-bx}}{b^4} - \frac{72a^2xe^{-bx}}{b^3} - \frac{36a^2x^2e^{-bx}}{b^2} - \frac{3a^2x^3e^{-bx}}{b} - \frac{12a^2x^4e^{-bx}}{b} - \frac{360ax^4e^{-bx}}{b^4} - \frac{720x^4e^{-bx}}{b^4} - \frac{360ax^4e^{-bx}}{b^3} - \frac{720x^4e^{-bx}}{b^3} - \frac{180ax^4e^{-bx}}{b^2} - \frac{360x^4e^{-bx}}{b^2} - \frac{360x^4e^{-bx}}{b^2} - \frac{60ax^4e^{-bx}}{b} - \frac{120x^4e^{-bx}}{b}$$

Antiderivative was successfully verified.

[In] Int[E<sup>-(a + b\*x)</sup>\*x<sup>3</sup>\*(a + b\*x)<sup>3</sup>,x]

[Out]  $(-720*E^{-(a + b*x)})/b^4 - (360*a*E^{-(a + b*x)})/b^4 - (72*a^2*E^{-(a + b*x)})/b^4 - (6*a^3*E^{-(a + b*x)})/b^4 - (720*E^{-(a + b*x)*x})/b^3 - (360*a*E^{-(a + b*x)*x})/b^3 - (72*a^2*E^{-(a + b*x)*x})/b^3 - (6*a^3*E^{-(a + b*x)*x})/b^3 - (360*E^{-(a + b*x)*x^2})/b^2 - (180*a*E^{-(a + b*x)*x^2})/b^2 - (36*a^2*E^{-(a + b*x)*x^2})/b^2 - (3*a^3*E^{-(a + b*x)*x^2})/b^2 - (120*E^{-(a + b*x)*x^3})/b - (60*a*E^{-(a + b*x)*x^3})/b - (12*a^2*E^{-(a + b*x)*x^3})/b - (a^3*E^{-(a + b*x)*x^3})/b - 30*E^{-(a + b*x)*x^4} - 15*a*E^{-(a + b*x)*x^4} - 3*a^2*E^{-(a + b*x)*x^4} - 6*b*E^{-(a + b*x)*x^5} - 3*a*b*E^{-(a + b*x)*x^5} - b^2*E^{-(a + b*x)*x^6}$

**Rule 2207**

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(c + d\*x)^m\*((b\*F^(g\*(e + f\*x)))^n/(f\*g\*n\*Log[F])), x] - Dist[d\*(m/(f\*g\*n\*Log[F])), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

**Rule 2225**

Int[(F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

## Rule 2227

```
Int[(F_)^((c_.)*(v_))*(u_), x_Symbol] := Int[ExpandIntegrand[F^(c*ExpandToSum[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v, x] && !TrueQ[$UseGamma]
```

## Rubi steps

$$\begin{aligned}
\int e^{-a-bx} x^3 (a+bx)^3 dx &= \int (a^3 e^{-a-bx} x^3 + 3a^2 b e^{-a-bx} x^4 + 3ab^2 e^{-a-bx} x^5 + b^3 e^{-a-bx} x^6) dx \\
&= a^3 \int e^{-a-bx} x^3 dx + (3a^2 b) \int e^{-a-bx} x^4 dx + (3ab^2) \int e^{-a-bx} x^5 dx + b^3 \int e^{-a-bx} x^6 dx \\
&= -\frac{a^3 e^{-a-bx} x^3}{b} - 3a^2 e^{-a-bx} x^4 - 3abe^{-a-bx} x^5 - b^2 e^{-a-bx} x^6 + (12a^2) \int e^{-a-bx} x^3 dx \\
&= -\frac{3a^3 e^{-a-bx} x^2}{b^2} - \frac{12a^2 e^{-a-bx} x^3}{b} - \frac{a^3 e^{-a-bx} x^3}{b} - 15ae^{-a-bx} x^4 - 3a^2 e^{-a-bx} x^4 - 6be^{-a-bx} x^5 \\
&= -\frac{6a^3 e^{-a-bx} x}{b^3} - \frac{36a^2 e^{-a-bx} x^2}{b^2} - \frac{3a^3 e^{-a-bx} x^2}{b^2} - \frac{60ae^{-a-bx} x^3}{b} - \frac{12a^2 e^{-a-bx} x^3}{b} - \frac{6be^{-a-bx} x^4}{b} \\
&= -\frac{6a^3 e^{-a-bx}}{b^4} - \frac{72a^2 e^{-a-bx} x}{b^3} - \frac{6a^3 e^{-a-bx} x}{b^3} - \frac{180ae^{-a-bx} x^2}{b^2} - \frac{36a^2 e^{-a-bx} x^2}{b^2} - \frac{3a^3 e^{-a-bx} x^2}{b^2} \\
&= -\frac{72a^2 e^{-a-bx}}{b^4} - \frac{6a^3 e^{-a-bx}}{b^4} - \frac{360ae^{-a-bx} x}{b^3} - \frac{72a^2 e^{-a-bx} x}{b^3} - \frac{6a^3 e^{-a-bx} x}{b^3} - \frac{360e^{-a-bx} x^2}{b^3} \\
&= -\frac{360ae^{-a-bx}}{b^4} - \frac{72a^2 e^{-a-bx}}{b^4} - \frac{6a^3 e^{-a-bx}}{b^4} - \frac{720e^{-a-bx} x}{b^3} - \frac{360ae^{-a-bx} x}{b^3} - \frac{72a^2 e^{-a-bx} x}{b^3} \\
&= -\frac{720e^{-a-bx}}{b^4} - \frac{360ae^{-a-bx}}{b^4} - \frac{72a^2 e^{-a-bx}}{b^4} - \frac{6a^3 e^{-a-bx}}{b^4} - \frac{720e^{-a-bx} x}{b^3} - \frac{360ae^{-a-bx} x}{b^3} - \frac{72a^2 e^{-a-bx} x}{b^3}
\end{aligned}$$

**Mathematica [A]**

time = 0.23, size = 121, normalized size = 0.30

$$e^{-a-bx} \left( -\frac{6(120+60a+12a^2+a^3)}{b^4} - \frac{6(120+60a+12a^2+a^3)x}{b^3} - \frac{3(120+60a+12a^2+a^3)x^2}{b^2} - \frac{(120+60a+12a^2+a^3)x^3}{b} - 3(10+5a+a^2)x^4 - 3(2+a)bx^5 - b^2x^6 \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-a - b*x)*x^3*(a + b*x)^3, x]
```

```
[Out] E^(-a - b*x)*((-6*(120 + 60*a + 12*a^2 + a^3))/b^4 - (6*(120 + 60*a + 12*a^2 + a^3)*x)/b^3 - (3*(120 + 60*a + 12*a^2 + a^3)*x^2)/b^2 - ((120 + 60*a + 12*a^2 + a^3)*x^3)/b - 3*(10 + 5*a + a^2)*x^4 - 3*(2 + a)*b*x^5 - b^2*x^6)
```

**Maple [A]**

time = 0.07, size = 432, normalized size = 1.09





[In] integrate(exp(-b\*x-a)\*x^3\*(b\*x+a)^3,x, algorithm="fricas")

[Out]  $-(b^6 x^6 + 3(a+2)b^5 x^5 + 3(a^2 + 5a + 10)b^4 x^4 + (a^3 + 12a^2 + 60a + 120)b^3 x^3 + 3(a^3 + 12a^2 + 60a + 120)b^2 x^2 + 6a^3 + 6(a^3 + 12a^2 + 60a + 120)b x + 72a^2 + 360a + 720)e^{-(b x - a)}/b^4$

**Sympy [A]**

time = 0.10, size = 236, normalized size = 0.59

$$\left\{ \begin{array}{l} \frac{(-a^3 b^3 x^3 - 3a^3 b^2 x^2 - 6a^3 b x - 6a^3 - 3a^2 b^3 x^3 - 12a^2 b^2 x^2 - 72a^2 b x - 72a^2 - 3ab^3 x^3 - 15ab^2 x^2 - 60ab^2 x^2 - 360abx - 360a - b^6 x^6 - 6b^5 x^5 - 30b^4 x^4 - 120b^3 x^3 - 360b^2 x^2 - 720bx - 720)e^{-a-bx}}{b^4} \text{ for } b^4 \neq 0 \\ \frac{a^3 x^4}{4} + \frac{3a^2 b x^5}{5} + \frac{ab^2 x^6}{2} + \frac{b^3 x^7}{7} \text{ otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*x\*\*3\*(b\*x+a)\*\*3,x)

[Out] Piecewise((( -a\*\*3\*b\*\*3\*x\*\*3 - 3\*a\*\*3\*b\*\*2\*x\*\*2 - 6\*a\*\*3\*b\*x - 6\*a\*\*3 - 3\*a\*\*2\*b\*\*4\*x\*\*4 - 12\*a\*\*2\*b\*\*3\*x\*\*3 - 36\*a\*\*2\*b\*\*2\*x\*\*2 - 72\*a\*\*2\*b\*x - 72\*a\*\*2 - 3\*a\*b\*\*5\*x\*\*5 - 15\*a\*b\*\*4\*x\*\*4 - 60\*a\*b\*\*3\*x\*\*3 - 180\*a\*b\*\*2\*x\*\*2 - 360\*a\*b\*x - 360\*a - b\*\*6\*x\*\*6 - 6\*b\*\*5\*x\*\*5 - 30\*b\*\*4\*x\*\*4 - 120\*b\*\*3\*x\*\*3 - 360\*b\*\*2\*x\*\*2 - 720\*b\*x - 720)\*exp(-a - b\*x)/b\*\*4, Ne(b\*\*4, 0)), (a\*\*3\*x\*\*4/4 + 3\*a\*\*2\*b\*x\*\*5/5 + a\*b\*\*2\*x\*\*6/2 + b\*\*3\*x\*\*7/7, True))

**Giac [A]**

time = 2.33, size = 202, normalized size = 0.51

$$\frac{(b^6 x^6 + 3ab^3 x^3 + 3a^2 b^2 x^2 + 6b^5 x^5 + a^3 b^3 x^3 + 15ab^2 x^2 + 12a^2 b^2 x^2 + 30b^4 x^4 + 3a^3 b^2 x^2 + 60ab^2 x^2 + 36a^2 b^2 x^2 + 120b^2 x^2 + 6a^3 b^4 x + 180ab^2 x + 72a^2 b^4 x + 360b^2 x + 360ab^2 + 720b^2)e^{(-bx-a)}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*x^3\*(b\*x+a)^3,x, algorithm="giac")

[Out]  $-(b^9 x^6 + 3a b^8 x^5 + 3a^2 b^7 x^4 + 6b^8 x^5 + a^3 b^6 x^3 + 15a b^7 x^4 + 12a^2 b^6 x^3 + 30b^7 x^4 + 3a^3 b^5 x^2 + 60a b^6 x^3 + 36a^2 b^5 x^2 + 120b^6 x^3 + 6a^3 b^4 x + 180a b^5 x^2 + 72a^2 b^4 x + 360b^5 x^2 + 6a^3 b^3 + 360a b^4 x + 72a^2 b^3 + 720b^4 x + 360a b^3 + 720b^3)e^{-(b x - a)}/b^7$

**Mupad [B]**

time = 3.54, size = 175, normalized size = 0.44

$$-x^4 e^{-a-bx} (3a^2 + 15a + 30) - b^2 x^6 e^{-a-bx} - \frac{6e^{-a-bx} (a^3 + 12a^2 + 60a + 120)}{b^4} - 3bx^5 e^{-a-bx} (a+2) - \frac{6xe^{-a-bx} (a^3 + 12a^2 + 60a + 120)}{b^3} - \frac{x^3 e^{-a-bx} (a^3 + 12a^2 + 60a + 120)}{b} - \frac{3x^2 e^{-a-bx} (a^3 + 12a^2 + 60a + 120)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*exp(- a - b\*x)\*(a + b\*x)^3,x)

[Out]  $-x^4 \exp(-a - b x) (15a + 3a^2 + 30) - b^2 x^6 \exp(-a - b x) - (6 \exp(-a - b x) (60a + 12a^2 + a^3 + 120))/b^4 - 3b x^5 \exp(-a - b x) (a + 2) - (6x \exp(-a - b x) (60a + 12a^2 + a^3 + 120))/b^3 - (x^3 \exp(-a - b x) (60a + 12a^2 + a^3 + 120))/b - (3x^2 \exp(-a - b x) (60a + 12a^2 + a^3 + 120))/b^2$

### 3.57 $\int e^{-a-bx} x^2 (a + bx)^3 dx$

**Optimal.** Leaf size=318

$$\frac{120e^{-a-bx}}{b^3} - \frac{72ae^{-a-bx}}{b^3} - \frac{18a^2e^{-a-bx}}{b^3} - \frac{2a^3e^{-a-bx}}{b^3} - \frac{120e^{-a-bx}x}{b^2} - \frac{72ae^{-a-bx}x}{b^2} - \frac{18a^2e^{-a-bx}x}{b^2} - \frac{2a^3e^{-a-bx}x}{b^2}$$

[Out]  $-120*\exp(-b*x-a)/b^3-72*a*\exp(-b*x-a)/b^3-18*a^2*\exp(-b*x-a)/b^3-2*a^3*\exp(-b*x-a)/b^3-120*\exp(-b*x-a)*x/b^2-72*a*\exp(-b*x-a)*x/b^2-18*a^2*\exp(-b*x-a)*x/b^2-2*a^3*\exp(-b*x-a)*x/b^2-60*\exp(-b*x-a)*x^2/b-36*a*\exp(-b*x-a)*x^2/b-9*a^2*\exp(-b*x-a)*x^2/b-a^3*\exp(-b*x-a)*x^2/b-20*\exp(-b*x-a)*x^3-12*a*\exp(-b*x-a)*x^3-3*a^2*\exp(-b*x-a)*x^3-5*b*\exp(-b*x-a)*x^4-3*a*b*\exp(-b*x-a)*x^4-b^2*\exp(-b*x-a)*x^5$

**Rubi** [A]

time = 0.29, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2227, 2207, 2225}

$$\frac{2a^2e^{-bx}}{b^3} - \frac{2a^2xe^{-bx}}{b^2} - \frac{a^2x^2e^{-bx}}{b} - \frac{18a^2e^{-bx}}{b^3} - \frac{18a^2xe^{-bx}}{b^2} - \frac{3a^2x^2e^{-bx}}{b} - \frac{9a^2x^2e^{-bx}}{b} - \frac{72ae^{-bx}}{b^3} - \frac{120e^{-bx}}{b^3} - \frac{72axe^{-bx}}{b^2} - \frac{120xe^{-bx}}{b^2} - \frac{3abx^2e^{-bx}}{b} - \frac{5bx^2e^{-bx}}{b} - \frac{12ax^2e^{-bx}}{b} - \frac{20x^3e^{-bx}}{b} - \frac{36ax^2e^{-bx}}{b} - \frac{60x^2e^{-bx}}{b}$$

Antiderivative was successfully verified.

[In] Int[E^(-a - b\*x)\*x^2\*(a + b\*x)^3,x]

[Out]  $(-120*E^(-a - b*x))/b^3 - (72*a*E^(-a - b*x))/b^3 - (18*a^2*E^(-a - b*x))/b^3 - (2*a^3*E^(-a - b*x))/b^3 - (120*E^(-a - b*x)*x)/b^2 - (72*a*E^(-a - b*x)*x)/b^2 - (18*a^2*E^(-a - b*x)*x)/b^2 - (2*a^3*E^(-a - b*x)*x)/b^2 - (60*E^(-a - b*x)*x^2)/b - (36*a*E^(-a - b*x)*x^2)/b - (9*a^2*E^(-a - b*x)*x^2)/b - (a^3*E^(-a - b*x)*x^2)/b - 20*E^(-a - b*x)*x^3 - 12*a*E^(-a - b*x)*x^3 - 3*a^2*E^(-a - b*x)*x^3 - 5*b*E^(-a - b*x)*x^4 - 3*a*b*E^(-a - b*x)*x^4 - b^2*E^(-a - b*x)*x^5$

Rule 2207

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(c + d\*x)^m\*((b\*F^(g\*(e + f\*x)))^n/(f\*g\*n\*Log[F])), x] - Dist[d\*(m/(f\*g\*n\*Log[F])), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

Rule 2225

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 2227

```
Int[(F_)^((c_.)*(v_))*(u_), x_Symbol] := Int[ExpandIntegrand[F^(c*ExpandToSum[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v, x] && !TrueQ[$UseGamma]
```

Rubi steps

$$\begin{aligned} \int e^{-a-bx}x^2(a+bx)^3 dx &= \int (a^3e^{-a-bx}x^2 + 3a^2be^{-a-bx}x^3 + 3ab^2e^{-a-bx}x^4 + b^3e^{-a-bx}x^5) dx \\ &= a^3 \int e^{-a-bx}x^2 dx + (3a^2b) \int e^{-a-bx}x^3 dx + (3ab^2) \int e^{-a-bx}x^4 dx + b^3 \int e^{-a-bx}x^5 dx \\ &= -\frac{a^3e^{-a-bx}x^2}{b} - 3a^2e^{-a-bx}x^3 - 3abe^{-a-bx}x^4 - b^2e^{-a-bx}x^5 + (9a^2) \int e^{-a-bx}x^2 dx \\ &= -\frac{2a^3e^{-a-bx}x}{b^2} - \frac{9a^2e^{-a-bx}x^2}{b} - \frac{a^3e^{-a-bx}x^2}{b} - 12ae^{-a-bx}x^3 - 3a^2e^{-a-bx}x^3 - 5be^{-a-bx}x^4 \\ &= -\frac{2a^3e^{-a-bx}}{b^3} - \frac{18a^2e^{-a-bx}x}{b^2} - \frac{2a^3e^{-a-bx}x}{b^2} - \frac{36ae^{-a-bx}x^2}{b} - \frac{9a^2e^{-a-bx}x^2}{b} - \frac{a^3e^{-a-bx}x^2}{b} \\ &= -\frac{18a^2e^{-a-bx}}{b^3} - \frac{2a^3e^{-a-bx}}{b^3} - \frac{72ae^{-a-bx}x}{b^2} - \frac{18a^2e^{-a-bx}x}{b^2} - \frac{2a^3e^{-a-bx}x}{b^2} - \frac{60e^{-a-bx}x^2}{b} \\ &= -\frac{72ae^{-a-bx}}{b^3} - \frac{18a^2e^{-a-bx}}{b^3} - \frac{2a^3e^{-a-bx}}{b^3} - \frac{120e^{-a-bx}x}{b^2} - \frac{72ae^{-a-bx}x}{b^2} - \frac{18a^2e^{-a-bx}x}{b^2} \\ &= -\frac{120e^{-a-bx}}{b^3} - \frac{72ae^{-a-bx}}{b^3} - \frac{18a^2e^{-a-bx}}{b^3} - \frac{2a^3e^{-a-bx}}{b^3} - \frac{120e^{-a-bx}x}{b^2} - \frac{72ae^{-a-bx}x}{b^2} \end{aligned}$$

**Mathematica [A]**

time = 0.19, size = 130, normalized size = 0.41

$$e^{-bx} \left( -\frac{2(60 + 36a + 9a^2 + a^3)e^{-a}}{b^3} - \frac{2(60 + 36a + 9a^2 + a^3)e^{-ax}}{b^2} - \frac{(60 + 36a + 9a^2 + a^3)e^{-ax^2}}{b} - (20 + 12a + 3a^2)e^{-ax^3} - (5 + 3a)be^{-ax^4} - b^2e^{-ax^5} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-a - b*x)*x^2*(a + b*x)^3,x]
```

```
[Out] ((-2*(60 + 36*a + 9*a^2 + a^3))/(b^3*E^a) - (2*(60 + 36*a + 9*a^2 + a^3)*x)/(b^2*E^a) - ((60 + 36*a + 9*a^2 + a^3)*x^2)/(b*E^a) - ((20 + 12*a + 3*a^2)*x^3)/E^a - ((5 + 3*a)*b*x^4)/E^a - (b^2*x^5)/E^a)/E^(b*x)
```

**Maple [A]**

time = 0.06, size = 291, normalized size = 0.92

method	result
gospers	$-\frac{(b^5x^5+3ab^4x^4+3a^2b^3x^3+5b^4x^4+a^3b^2x^2+12ab^3x^3+9a^2b^2x^2+20b^3x^3+2a^3bx+36ab^2x^2+18a^2bx+60b^2x^2+2a^3+72ab^2x^2)}{b^3}$

risch	$-\frac{(b^5x^5+3ab^4x^4+3a^2b^3x^3+5b^4x^4+a^3b^2x^2+12ab^3x^3+9a^2b^2x^2+20b^3x^3+2a^3bx+36ab^2x^2+18a^2bx+60b^2x^2+2a^3+72a^2)}{b^3}$
norman	$(-3ab-5b)x^4e^{-bx-a}+(-3a^2-12a-20)x^3e^{-bx-a}-b^2e^{-bx-a}x^5-\frac{2(a^3+9a^2+36a+60)e^{-bx-a}}{b^3}$
meijerg	$e^{-a}\left(\frac{120-\frac{(6b^5x^5+30b^4x^4+120b^3x^3+360b^2x^2+720bx+720)e^{-bx}}{6}}{b^3}\right)+\frac{3e^{-a}\left(24-\frac{(5b^4x^4+20b^3x^3+60b^2x^2+120bx+120)e^{-bx-a}}{5}\right)}{b^3}$
derivativedivides	$\frac{(-bx-a)^5e^{-bx-a}-5(-bx-a)^4e^{-bx-a}+20e^{-bx-a}(-bx-a)^3-60(-bx-a)^2e^{-bx-a}+120(-bx-a)e^{-bx-a}-120e^{-bx-a}}{b^3}$
default	$(-bx-a)^5e^{-bx-a}-5(-bx-a)^4e^{-bx-a}+20e^{-bx-a}(-bx-a)^3-60(-bx-a)^2e^{-bx-a}+120(-bx-a)e^{-bx-a}-120e^{-bx-a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(-b*x-a)*x^2*(b*x+a)^3,x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{b^3} * ((-bx-a)^5 * \exp(-bx-a) - 5 * (-bx-a)^4 * \exp(-bx-a) + 20 * \exp(-bx-a) * (-bx-a)^3 - 60 * (-bx-a)^2 * \exp(-bx-a) + 120 * (-bx-a) * \exp(-bx-a) - 120 * \exp(-bx-a) + a^2 * (\exp(-bx-a) * (-bx-a)^3 - 3 * (-bx-a)^2 * \exp(-bx-a) + 6 * (-bx-a) * \exp(-bx-a) - 6 * \exp(-bx-a)) + 2 * a * ((-bx-a)^4 * \exp(-bx-a) - 4 * \exp(-bx-a) * (-bx-a)^3 + 12 * (-bx-a)^2 * \exp(-bx-a) - 24 * (-bx-a) * \exp(-bx-a) + 24 * \exp(-bx-a)))$

Maxima [A]

time = 0.44, size = 164, normalized size = 0.52

$$\frac{(b^2x^2+2bx+2)a^3e^{-bx-a}}{b^3} - \frac{3(b^3x^3+3b^2x^2+6bx+6)a^2e^{-bx-a}}{b^3} - \frac{3(b^4x^4+4b^3x^3+12b^2x^2+24bx+24)ae^{-bx-a}}{b^3} - \frac{(b^5x^5+5b^4x^4+20b^3x^3+60b^2x^2+120bx+120)e^{-bx-a}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*x^2*(b*x+a)^3,x, algorithm="maxima")`

[Out]  $-(b^2x^2+2bx+2)a^3e^{-bx-a}/b^3 - 3*(b^3x^3+3b^2x^2+6bx+6)a^2e^{-bx-a}/b^3 - 3*(b^4x^4+4b^3x^3+12b^2x^2+24bx+24)ae^{-bx-a}/b^3 - (b^5x^5+5b^4x^4+20b^3x^3+60b^2x^2+120bx+120)e^{-bx-a}/b^3$

Fricas [A]

time = 0.38, size = 102, normalized size = 0.32

$$\frac{(b^5x^5+(3a+5)b^4x^4+(3a^2+12a+20)b^3x^3+(a^3+9a^2+36a+60)b^2x^2+2a^3+2(a^3+9a^2+36a+60)bx+18a^2+72a+120)e^{-bx-a}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*x^2*(b*x+a)^3,x, algorithm="fricas")`

[Out]  $-(b^5x^5+(3a+5)b^4x^4+(3a^2+12a+20)b^3x^3+(a^3+9a^2+36a+60)b^2x^2+2a^3+2(a^3+9a^2+36a+60)bx+18a^2+72a+120)e^{-bx-a}/b^3$

**Sympy [A]**

time = 0.10, size = 196, normalized size = 0.62

$$\begin{cases} \frac{(-a^3b^2x^2 - 2a^3bx - 2a^3 - 3a^2b^3x^3 - 9a^2b^2x^2 - 18a^2bx - 18a^2 - 3ab^4x^4 - 12ab^3x^3 - 36ab^2x^2 - 72abx - 72a - b^5x^5 - 5b^4x^4 - 20b^3x^3 - 60b^2x^2 - 120bx - 120)e^{-a-bx}}{b^3} & \text{for } b^3 \neq 0 \\ \frac{a^3x^3}{3} + \frac{3a^2bx^4}{4} + \frac{3ab^2x^5}{5} + \frac{b^3x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(exp(-b\*x-a)\*x\*\*2\*(b\*x+a)\*\*3,x)

**[Out]** Piecewise((((-a\*\*3\*b\*\*2\*x\*\*2 - 2\*a\*\*3\*b\*x - 2\*a\*\*3 - 3\*a\*\*2\*b\*\*3\*x\*\*3 - 9\*a\*\*2\*b\*\*2\*x\*\*2 - 18\*a\*\*2\*b\*x - 18\*a\*\*2 - 3\*a\*b\*\*4\*x\*\*4 - 12\*a\*b\*\*3\*x\*\*3 - 36\*a\*b\*\*2\*x\*\*2 - 72\*a\*b\*x - 72\*a - b\*\*5\*x\*\*5 - 5\*b\*\*4\*x\*\*4 - 20\*b\*\*3\*x\*\*3 - 60\*b\*\*2\*x\*\*2 - 120\*b\*x - 120)\*exp(-a - b\*x)/b\*\*3, Ne(b\*\*3, 0)), (a\*\*3\*x\*\*3/3 + 3\*a\*\*2\*b\*x\*\*4/4 + 3\*a\*b\*\*2\*x\*\*5/5 + b\*\*3\*x\*\*6/6, True))

**Giac [A]**

time = 2.28, size = 163, normalized size = 0.51

$$\frac{(b^5x^5 + 3ab^7x^4 + 3a^2b^6x^3 + 5b^7x^4 + a^3b^5x^2 + 12ab^6x^3 + 9a^2b^5x^2 + 20b^6x^3 + 2a^3b^4x + 36ab^5x^2 + 18a^2b^4x + 60b^5x^2 + 2a^3b^3 + 72ab^4x + 18a^2b^3 + 120b^4x + 72ab^3 + 120b^3)e^{(-bx-a)}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(exp(-b\*x-a)\*x^2\*(b\*x+a)^3,x, algorithm="giac")

**[Out]** -(b^8\*x^5 + 3\*a\*b^7\*x^4 + 3\*a^2\*b^6\*x^3 + 5\*b^7\*x^4 + a^3\*b^5\*x^2 + 12\*a\*b^6\*x^3 + 9\*a^2\*b^5\*x^2 + 20\*b^6\*x^3 + 2\*a^3\*b^4\*x + 36\*a\*b^5\*x^2 + 18\*a^2\*b^4\*x + 60\*b^5\*x^2 + 2\*a^3\*b^3 + 72\*a\*b^4\*x + 18\*a^2\*b^3 + 120\*b^4\*x + 72\*a\*b^3 + 120\*b^3)\*e^(-b\*x - a)/b^6

**Mupad [B]**

time = 3.52, size = 126, normalized size = 0.40

$$-x^3e^{-a-bx}(3a^2 + 3abx + 12a + b^2x^2 + 5bx + 20) - \frac{2e^{-a-bx}(a^3 + 9a^2 + 36a + 60)}{b^3} - \frac{2xe^{-a-bx}(a^3 + 9a^2 + 36a + 60)}{b^2} - \frac{x^2e^{-a-bx}(a^3 + 9a^2 + 36a + 60)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(x^2\*exp(- a - b\*x)\*(a + b\*x)^3,x)

**[Out]** - x^3\*exp(- a - b\*x)\*(12\*a + 5\*b\*x + 3\*a^2 + b^2\*x^2 + 3\*a\*b\*x + 20) - (2\*exp(- a - b\*x)\*(36\*a + 9\*a^2 + a^3 + 60))/b^3 - (2\*x\*exp(- a - b\*x)\*(36\*a + 9\*a^2 + a^3 + 60))/b^2 - (x^2\*exp(- a - b\*x)\*(36\*a + 9\*a^2 + a^3 + 60))/b

### 3.58 $\int e^{-a-bx} x(a+bx)^3 dx$

**Optimal.** Leaf size=184

$$-\frac{24e^{-a-bx}}{b^2} + \frac{6ae^{-a-bx}}{b^2} - \frac{24e^{-a-bx}(a+bx)}{b^2} + \frac{6ae^{-a-bx}(a+bx)}{b^2} - \frac{12e^{-a-bx}(a+bx)^2}{b^2} + \frac{3ae^{-a-bx}(a+bx)^2}{b^2} - \frac{4e^{-a-bx}(a+bx)^3}{b^2}$$

[Out]  $-24*\exp(-b*x-a)/b^2+6*a*\exp(-b*x-a)/b^2-24*\exp(-b*x-a)*(b*x+a)/b^2+6*a*\exp(-b*x-a)*(b*x+a)/b^2-12*\exp(-b*x-a)*(b*x+a)^2/b^2+3*a*\exp(-b*x-a)*(b*x+a)^2/b^2-4*\exp(-b*x-a)*(b*x+a)^3/b^2+a*\exp(-b*x-a)*(b*x+a)^3/b^2-\exp(-b*x-a)*(b*x+a)^4/b^2$

**Rubi [A]**

time = 0.17, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2227, 2207, 2225}

$$-\frac{e^{-a-bx}(a+bx)^4}{b^2} + \frac{ae^{-a-bx}(a+bx)^3}{b^2} - \frac{4e^{-a-bx}(a+bx)^3}{b^2} + \frac{3ae^{-a-bx}(a+bx)^2}{b^2} - \frac{12e^{-a-bx}(a+bx)^2}{b^2} + \frac{6ae^{-a-bx}(a+bx)}{b^2} - \frac{24e^{-a-bx}(a+bx)}{b^2} + \frac{6ae^{-a-bx}}{b^2} - \frac{24e^{-a-bx}}{b^2}$$

Antiderivative was successfully verified.

[In] `Int[E^(-a - b*x)*x*(a + b*x)^3,x]`

[Out]  $(-24*E^(-a - b*x))/b^2 + (6*a*E^(-a - b*x))/b^2 - (24*E^(-a - b*x)*(a + b*x))/b^2 + (6*a*E^(-a - b*x)*(a + b*x))/b^2 - (12*E^(-a - b*x)*(a + b*x)^2)/b^2 + (3*a*E^(-a - b*x)*(a + b*x)^2)/b^2 - (4*E^(-a - b*x)*(a + b*x)^3)/b^2 + (a*E^(-a - b*x)*(a + b*x)^3)/b^2 - (E^(-a - b*x)*(a + b*x)^4)/b^2$

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2227

```
Int[(F_)^((c_.)*(v_))*(u_), x_Symbol] := Int[ExpandIntegrand[F^(c*ExpandToSum[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v, x] && !TrueQ[$UseGamma]
```

Rubi steps

$$\begin{aligned}
\int e^{-a-bx} x(a+bx)^3 dx &= \int \left( -\frac{ae^{-a-bx}(a+bx)^3}{b} + \frac{e^{-a-bx}(a+bx)^4}{b} \right) dx \\
&= \frac{\int e^{-a-bx}(a+bx)^4 dx}{b} - \frac{a \int e^{-a-bx}(a+bx)^3 dx}{b} \\
&= \frac{ae^{-a-bx}(a+bx)^3}{b^2} - \frac{e^{-a-bx}(a+bx)^4}{b^2} + \frac{4 \int e^{-a-bx}(a+bx)^3 dx}{b} - \frac{(3a) \int e^{-a-bx}(a+bx)^2 dx}{b} \\
&= \frac{3ae^{-a-bx}(a+bx)^2}{b^2} - \frac{4e^{-a-bx}(a+bx)^3}{b^2} + \frac{ae^{-a-bx}(a+bx)^3}{b^2} - \frac{e^{-a-bx}(a+bx)^4}{b^2} + \frac{12 \int e^{-a-bx}(a+bx)^2 dx}{b} \\
&= \frac{6ae^{-a-bx}(a+bx)}{b^2} - \frac{12e^{-a-bx}(a+bx)^2}{b^2} + \frac{3ae^{-a-bx}(a+bx)^2}{b^2} - \frac{4e^{-a-bx}(a+bx)^3}{b^2} + \frac{12 \int e^{-a-bx}(a+bx) dx}{b} \\
&= \frac{6ae^{-a-bx}}{b^2} - \frac{24e^{-a-bx}(a+bx)}{b^2} + \frac{6ae^{-a-bx}(a+bx)}{b^2} - \frac{12e^{-a-bx}(a+bx)^2}{b^2} + \frac{3ae^{-a-bx}(a+bx)}{b^2} \\
&= -\frac{24e^{-a-bx}}{b^2} + \frac{6ae^{-a-bx}}{b^2} - \frac{24e^{-a-bx}(a+bx)}{b^2} + \frac{6ae^{-a-bx}(a+bx)}{b^2} - \frac{12e^{-a-bx}(a+bx)^2}{b^2} + \frac{3ae^{-a-bx}(a+bx)}{b^2}
\end{aligned}$$

**Mathematica [A]**

time = 0.15, size = 96, normalized size = 0.52

$$\frac{e^{-a-bx}(-24 - 24bx - 12b^2x^2 - 4b^3x^3 - b^4x^4 - a^3(1+bx) - 3a^2(2+2bx+b^2x^2) - 3a(6+6bx+3b^2x^2+b^3x^3))}{b^2}$$

Antiderivative was successfully verified.

`[In] Integrate[E^(-a - b*x)*x*(a + b*x)^3,x]`

```
[Out] (E^(-a - b*x)*(-24 - 24*b*x - 12*b^2*x^2 - 4*b^3*x^3 - b^4*x^4 - a^3*(1 + b
*x) - 3*a^2*(2 + 2*b*x + b^2*x^2) - 3*a*(6 + 6*b*x + 3*b^2*x^2 + b^3*x^3)))
/b^2
```

**Maple [A]**

time = 0.06, size = 173, normalized size = 0.94

method	result
gospers	$-\frac{(b^4x^4+3ab^3x^3+3a^2b^2x^2+4b^3x^3+a^3bx+9ab^2x^2+6a^2bx+12b^2x^2+a^3+18abx+6a^2+24bx+18a+24)e^{-bx-a}}{b^2}$
risch	$-\frac{(b^4x^4+3ab^3x^3+3a^2b^2x^2+4b^3x^3+a^3bx+9ab^2x^2+6a^2bx+12b^2x^2+a^3+18abx+6a^2+24bx+18a+24)e^{-bx-a}}{b^2}$
norman	$(-3ab - 4b)x^3e^{-bx-a} + (-3a^2 - 9a - 12)x^2e^{-bx-a} - b^2x^4e^{-bx-a} - \frac{(a^3+6a^2+18a+24)e^{-bx-a}}{b^2}$
meijerg	$\frac{e^{-a} \left( 24 - \frac{(5b^4x^4+20b^3x^3+60b^2x^2+120bx+120)e^{-bx}}{5} \right)}{b^2} + \frac{3e^{-a} \left( 6 - \frac{(4b^3x^3+12b^2x^2+24bx+24)e^{-bx}}{4} \right)}{b^2} + \frac{3e^{-a}a^2 \left( 2 - \frac{(3a^3+6a^2+18a+24)e^{-bx-a}}{b^2} \right)}{b^2}$
derivativedivides	$-\frac{(-bx-a)^4e^{-bx-a} - 4e^{-bx-a}(-bx-a)^3 + 12(-bx-a)^2e^{-bx-a} - 24(-bx-a)e^{-bx-a} + 24e^{-bx-a} + a(e^{-bx-a}(-bx-a)^3 - 3e^{-bx-a}(-bx-a)^2 + 6e^{-bx-a}(-bx-a) - 6e^{-bx-a})}{b^2}$



default	$-\frac{(-bx-a)^4 e^{-bx-a} - 4e^{-bx-a}(-bx-a)^3 + 12(-bx-a)^2 e^{-bx-a} - 24(-bx-a)e^{-bx-a} + 24e^{-bx-a} + a(e^{-bx-a}(-bx-a)^3)}{b^2}$
---------	---------------------------------------------------------------------------------------------------------------------------------------------

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(-b*x-a)*x*(b*x+a)^3,x,method=_RETURNVERBOSE)`

[Out] 
$$-1/b^2*((-b*x-a)^4*exp(-b*x-a)-4*exp(-b*x-a)*(-b*x-a)^3+12*(-b*x-a)^2*exp(-b*x-a)-24*(-b*x-a)*exp(-b*x-a)+24*exp(-b*x-a)+a*(exp(-b*x-a)*(-b*x-a)^3-3*(-b*x-a)^2*exp(-b*x-a)+6*(-b*x-a)*exp(-b*x-a)-6*exp(-b*x-a)))$$

**Maxima** [A]

time = 0.37, size = 132, normalized size = 0.72

$$-\frac{(bx+1)a^3e^{(-bx-a)}}{b^2} - \frac{3(b^2x^2+2bx+2)a^2e^{(-bx-a)}}{b^2} - \frac{3(b^3x^3+3b^2x^2+6bx+6)ae^{(-bx-a)}}{b^2} - \frac{(b^4x^4+4b^3x^3+12b^2x^2+24bx+24)e^{(-bx-a)}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*x*(b*x+a)^3,x, algorithm="maxima")`

[Out] 
$$-(b*x+1)*a^3*e^{(-b*x-a)}/b^2 - 3*(b^2*x^2+2*b*x+2)*a^2*e^{(-b*x-a)}/b^2 - 3*(b^3*x^3+3*b^2*x^2+6*b*x+6)*a*e^{(-b*x-a)}/b^2 - (b^4*x^4+4*b^3*x^3+12*b^2*x^2+24*b*x+24)*e^{(-b*x-a)}/b^2$$

**Fricas** [A]

time = 0.35, size = 78, normalized size = 0.42

$$\frac{(b^4x^4+(3a+4)b^3x^3+3(a^2+3a+4)b^2x^2+a^3+(a^3+6a^2+18a+24)bx+6a^2+18a+24)e^{(-bx-a)}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*x*(b*x+a)^3,x, algorithm="fricas")`

[Out] 
$$-(b^4*x^4+(3*a+4)*b^3*x^3+3*(a^2+3*a+4)*b^2*x^2+a^3+(a^3+6*a^2+18*a+24)*b*x+6*a^2+18*a+24)*e^{(-b*x-a)}/b^2$$

**Sympy** [A]

time = 0.08, size = 148, normalized size = 0.80

$$\begin{cases} \frac{(-a^3bx-a^3-3a^2b^2x^2-6a^2bx-6a^2-3ab^3x^3-9ab^2x^2-18abx-18a-b^4x^4-4b^3x^3-12b^2x^2-24bx-24)e^{-a-bx}}{b^2} & \text{for } b^2 \neq 0 \\ \frac{a^3x^2}{2} + a^2bx^3 + \frac{3ab^2x^4}{4} + \frac{b^3x^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*x*(b*x+a)**3,x)`

[Out] `Piecewise((( -a**3*b*x - a**3 - 3*a**2*b**2*x**2 - 6*a**2*b*x - 6*a**2 - 3*a**b**3*x**3 - 9*a*b**2*x**2 - 18*a*b*x - 18*a - b**4*x**4 - 4*b**3*x**3 - 12`

`*b**2*x**2 - 24*b*x - 24)*exp(-a - b*x)/b**2, Ne(b**2, 0)), (a**3*x**2/2 + a**2*b*x**3 + 3*a*b**2*x**4/4 + b**3*x**5/5, True))`

**Giac [A]**

time = 2.96, size = 123, normalized size = 0.67

$$\frac{(b^7x^4 + 3ab^6x^3 + 3a^2b^5x^2 + 4b^6x^3 + a^3b^4x + 9ab^5x^2 + 6a^2b^4x + 12b^5x^2 + a^3b^3 + 18ab^4x + 6a^2b^3 + 24b^4x + 18ab^3 + 24b^3)e^{(-bx-a)}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(-b*x-a)*x*(b*x+a)^3,x, algorithm="giac")`

`[Out] -(b^7*x^4 + 3*a*b^6*x^3 + 3*a^2*b^5*x^2 + 4*b^6*x^3 + a^3*b^4*x + 9*a*b^5*x^2 + 6*a^2*b^4*x + 12*b^5*x^2 + a^3*b^3 + 18*a*b^4*x + 6*a^2*b^3 + 24*b^4*x + 18*a*b^3 + 24*b^3)*e^(-b*x - a)/b^5`

**Mupad [B]**

time = 3.43, size = 117, normalized size = 0.64

$$-x^2 e^{-a-bx} (3a^2 + 9a + 12) - b^2 x^4 e^{-a-bx} - \frac{e^{-a-bx} (a^3 + 6a^2 + 18a + 24)}{b^2} - \frac{x e^{-a-bx} (a^3 + 6a^2 + 18a + 24)}{b} - b x^3 e^{-a-bx} (3a + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*exp(- a - b*x)*(a + b*x)^3,x)`

`[Out] - x^2*exp(- a - b*x)*(9*a + 3*a^2 + 12) - b^2*x^4*exp(- a - b*x) - (exp(- a - b*x)*(18*a + 6*a^2 + a^3 + 24))/b^2 - (x*exp(- a - b*x)*(18*a + 6*a^2 + a^3 + 24))/b - b*x^3*exp(- a - b*x)*(3*a + 4)`

### 3.59 $\int e^{-a-bx}(a+bx)^3 dx$

Optimal. Leaf size=80

$$-\frac{6e^{-a-bx}}{b} - \frac{6e^{-a-bx}(a+bx)}{b} - \frac{3e^{-a-bx}(a+bx)^2}{b} - \frac{e^{-a-bx}(a+bx)^3}{b}$$

[Out]  $-6*\exp(-b*x-a)/b-6*\exp(-b*x-a)*(b*x+a)/b-3*\exp(-b*x-a)*(b*x+a)^2/b-\exp(-b*x-a)*(b*x+a)^3/b$

Rubi [A]

time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ ,

Rules used = {2207, 2225}

$$-\frac{e^{-a-bx}(a+bx)^3}{b} - \frac{3e^{-a-bx}(a+bx)^2}{b} - \frac{6e^{-a-bx}(a+bx)}{b} - \frac{6e^{-a-bx}}{b}$$

Antiderivative was successfully verified.

[In] Int[E<sup>^</sup>(-a - b\*x)\*(a + b\*x)<sup>3</sup>, x]

[Out]  $(-6*E^{(-a - b*x)})/b - (6*E^{(-a - b*x)*(a + b*x)})/b - (3*E^{(-a - b*x)*(a + b*x)^2})/b - (E^{(-a - b*x)*(a + b*x)^3})/b$

Rule 2207

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(c + d\*x)^m\*((b\*F^(g\*(e + f\*x)))^n/(f\*g\*n\*Log[F])), x] - Dist[d\*(m/(f\*g\*n\*Log[F])), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

Rule 2225

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned} \int e^{-a-bx}(a+bx)^3 dx &= -\frac{e^{-a-bx}(a+bx)^3}{b} + 3 \int e^{-a-bx}(a+bx)^2 dx \\ &= -\frac{3e^{-a-bx}(a+bx)^2}{b} - \frac{e^{-a-bx}(a+bx)^3}{b} + 6 \int e^{-a-bx}(a+bx) dx \\ &= -\frac{6e^{-a-bx}(a+bx)}{b} - \frac{3e^{-a-bx}(a+bx)^2}{b} - \frac{e^{-a-bx}(a+bx)^3}{b} + 6 \int e^{-a-bx} dx \\ &= -\frac{6e^{-a-bx}}{b} - \frac{6e^{-a-bx}(a+bx)}{b} - \frac{3e^{-a-bx}(a+bx)^2}{b} - \frac{e^{-a-bx}(a+bx)^3}{b} \end{aligned}$$

**Mathematica [A]**

time = 0.15, size = 41, normalized size = 0.51

$$\frac{e^{-a-bx}(-6 - 6(a + bx) - 3(a + bx)^2 - (a + bx)^3)}{b}$$

Antiderivative was successfully verified.

**[In]** Integrate[E^(-a - b\*x)\*(a + b\*x)^3,x]**[Out]** (E^(-a - b\*x)\*(-6 - 6\*(a + b\*x) - 3\*(a + b\*x)^2 - (a + b\*x)^3))/b**Maple [A]**

time = 0.06, size = 77, normalized size = 0.96

method	result
gospers	$-\frac{(b^3x^3+3ab^2x^2+3a^2bx+3b^2x^2+a^3+6abx+3a^2+6bx+6a+6)e^{-bx-a}}{b}$
risch	$-\frac{(b^3x^3+3ab^2x^2+3a^2bx+3b^2x^2+a^3+6abx+3a^2+6bx+6a+6)e^{-bx-a}}{b}$
derivativedivides	$\frac{e^{-bx-a}(-bx-a)^3-3(-bx-a)^2e^{-bx-a}+6(-bx-a)e^{-bx-a}-6e^{-bx-a}}{b}$
default	$\frac{e^{-bx-a}(-bx-a)^3-3(-bx-a)^2e^{-bx-a}+6(-bx-a)e^{-bx-a}-6e^{-bx-a}}{b}$
norman	$(-3ab - 3b)x^2e^{-bx-a} + (-3a^2 - 6a - 6)xe^{-bx-a} - b^2x^3e^{-bx-a} - \frac{(a^3+3a^2+6a+6)e^{-bx-a}}{b}$
meijerg	$\frac{e^{-a}\left(6 - \frac{(4b^3x^3+12b^2x^2+24bx+24)e^{-bx}}{4}\right)}{b} + \frac{3e^{-a}a\left(2 - \frac{(3b^2x^2+6bx+6)e^{-bx}}{3}\right)}{b} + \frac{3e^{-a}a^2\left(1 - \frac{(2bx+2)e^{-bx}}{2}\right)}{b} + \frac{e^{-a}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(exp(-b\*x-a)\*(b\*x+a)^3,x,method=\_RETURNVERBOSE)**[Out]** 1/b\*(exp(-b\*x-a)\*(-b\*x-a)^3-3\*(-b\*x-a)^2\*exp(-b\*x-a)+6\*(-b\*x-a)\*exp(-b\*x-a)-6\*exp(-b\*x-a))**Maxima [A]**

time = 0.30, size = 103, normalized size = 1.29

$$\frac{3(bx+1)a^2e^{(-bx-a)}}{b} - \frac{a^3e^{(-bx-a)}}{b} - \frac{3(b^2x^2+2bx+2)ae^{(-bx-a)}}{b} - \frac{(b^3x^3+3b^2x^2+6bx+6)e^{(-bx-a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(exp(-b\*x-a)\*(b\*x+a)^3,x, algorithm="maxima")**[Out]** -3\*(b\*x + 1)\*a^2\*e^(-b\*x - a)/b - a^3\*e^(-b\*x - a)/b - 3\*(b^2\*x^2 + 2\*b\*x + 2)\*a\*e^(-b\*x - a)/b - (b^3\*x^3 + 3\*b^2\*x^2 + 6\*b\*x + 6)\*e^(-b\*x - a)/b

**Fricas** [A]

time = 0.41, size = 57, normalized size = 0.71

$$\frac{(b^3x^3 + 3(a+1)b^2x^2 + a^3 + 3(a^2 + 2a + 2)bx + 3a^2 + 6a + 6)e^{(-bx-a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^3,x, algorithm="fricas")

[Out]  $-(b^3x^3 + 3(a+1)b^2x^2 + a^3 + 3(a^2 + 2a + 2)bx + 3a^2 + 6a + 6)e^{(-bx-a)}/b$ **Sympy** [A]

time = 0.06, size = 104, normalized size = 1.30

$$\begin{cases} \frac{(-a^3 - 3a^2bx - 3a^2 - 3ab^2x^2 - 6abx - 6a - b^3x^3 - 3b^2x^2 - 6bx - 6)e^{-a-bx}}{b} & \text{for } b \neq 0 \\ a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)\*\*3,x)

[Out] Piecewise((( $-a^3 - 3a^2bx - 3a^2 - 3ab^2x^2 - 6abx - 6a - b^3x^3 - 3b^2x^2 - 6bx - 6$ )\*exp(-a - b\*x)/b, Ne(b, 0)), ( $a^3x + 3a^2bx^2/2 + ab^2x^3 + b^3x^4/4$ , True))**Giac** [A]

time = 3.56, size = 87, normalized size = 1.09

$$\frac{(b^6x^3 + 3ab^5x^2 + 3a^2b^4x + 3b^5x^2 + a^3b^3 + 6ab^4x + 3a^2b^3 + 6b^4x + 6ab^3 + 6b^3)e^{(-bx-a)}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^3,x, algorithm="giac")

[Out]  $-(b^6x^3 + 3a^2b^5x^2 + 3a^2b^4x + 3b^5x^2 + a^3b^3 + 6a^2b^4x + 3a^2b^3 + 6b^4x + 6a^2b^3 + 6b^3)e^{(-bx-a)}/b^4$ **Mupad** [B]

time = 0.11, size = 66, normalized size = 0.82

$$-xe^{-a-bx}(3a^2 + 3abx + 6a + b^2x^2 + 3bx + 6) - \frac{e^{-a-bx}(a^3 + 3a^2 + 6a + 6)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-a - b\*x)\*(a + b\*x)^3,x)

[Out]  $-x\exp(-a - b*x)*(6a + 3b*x + 3a^2 + b^2*x^2 + 3a*b*x + 6) - (\exp(-a - b*x)*(6a + 3a^2 + a^3 + 6))/b$

$$3.60 \quad \int \frac{e^{-a-bx}(a+bx)^3}{x} dx$$

**Optimal.** Leaf size=102

$$-2e^{-a-bx} - 3ae^{-a-bx} - 3a^2e^{-a-bx} - 2be^{-a-bx}x - 3abe^{-a-bx}x - b^2e^{-a-bx}x^2 + a^3e^{-a}\text{Ei}(-bx)$$

[Out] -2\*exp(-b\*x-a)-3\*a\*exp(-b\*x-a)-3\*a^2\*exp(-b\*x-a)-2\*b\*exp(-b\*x-a)\*x-3\*a\*b\*exp(-b\*x-a)\*x-b^2\*exp(-b\*x-a)\*x^2+a^3\*Ei(-b\*x)/exp(a)

**Rubi [A]**

time = 0.12, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$ , Rules used = {2230, 2225, 2209, 2207}

$$e^{-a}a^3\text{Ei}(-bx) - 3a^2e^{-a-bx} - b^2x^2e^{-a-bx} - 3ae^{-a-bx} - 3abxe^{-a-bx} - 2e^{-a-bx} - 2bx e^{-a-bx}$$

Antiderivative was successfully verified.

[In] Int[(E^(-a - b\*x)\*(a + b\*x)^3)/x,x]

[Out] -2\*E^(-a - b\*x) - 3\*a\*E^(-a - b\*x) - 3\*a^2\*E^(-a - b\*x) - 2\*b\*E^(-a - b\*x)\*x - 3\*a\*b\*E^(-a - b\*x)\*x - b^2\*E^(-a - b\*x)\*x^2 + (a^3\*ExpIntegralEi[-(b\*x)])/E^a

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol]
:> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2230

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol]
:> Int[ExpandIntegrand[F^(c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F,
```

c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && IntegerQ[m] && !TrueQ[\$UseGamma]

Rubi steps

$$\begin{aligned}
 \int \frac{e^{-a-bx}(a+bx)^3}{x} dx &= \int \left( 3a^2be^{-a-bx} + \frac{a^3e^{-a-bx}}{x} + 3ab^2e^{-a-bx}x + b^3e^{-a-bx}x^2 \right) dx \\
 &= a^3 \int \frac{e^{-a-bx}}{x} dx + (3a^2b) \int e^{-a-bx} dx + (3ab^2) \int e^{-a-bx}x dx + b^3 \int e^{-a-bx}x^2 dx \\
 &= -3a^2e^{-a-bx} - 3abe^{-a-bx}x - b^2e^{-a-bx}x^2 + a^3e^{-a}\text{Ei}(-bx) + (3ab) \int e^{-a-bx} dx + (2 \\
 &= -3ae^{-a-bx} - 3a^2e^{-a-bx} - 2be^{-a-bx}x - 3abe^{-a-bx}x - b^2e^{-a-bx}x^2 + a^3e^{-a}\text{Ei}(-bx) - \\
 &= -2e^{-a-bx} - 3ae^{-a-bx} - 3a^2e^{-a-bx} - 2be^{-a-bx}x - 3abe^{-a-bx}x - b^2e^{-a-bx}x^2 + a^3e^{-a}\text{Ei}(-bx)
 \end{aligned}$$

**Mathematica [A]**

time = 0.15, size = 52, normalized size = 0.51

$$e^{-a-bx}(-2 - 3a^2 - 2bx - b^2x^2 - 3a(1 + bx) + a^3e^{bx}\text{Ei}(-bx))$$

Antiderivative was successfully verified.

[In] Integrate[(E^(-a - b\*x)\*(a + b\*x)^3)/x,x]

[Out] E^(-a - b\*x)\*(-2 - 3\*a^2 - 2\*b\*x - b^2\*x^2 - 3\*a\*(1 + b\*x) + a^3\*E^(b\*x)\*ExpIntegralEi[-(b\*x)])

**Maple [A]**

time = 0.07, size = 113, normalized size = 1.11

method	result
meijerg	$e^{-a} \left( 2 - \frac{(3b^2x^2+6bx+6)e^{-bx}}{3} \right) + 3e^{-a}a \left( 1 - \frac{(2bx+2)e^{-bx}}{2} \right) + 3e^{-a}a^2(1 - e^{-bx}) + e^{-a}a^3(-\ln$
risch	$-2a^2e^{-bx-a} - abe^{-bx-a}x - ae^{-bx-a} - (-bx - a)^2e^{-bx-a} + 2(-bx - a)e^{-bx-a} - 2e^{-bx-a}$
derivativedivides	$-a^2e^{-bx-a} + a((-bx - a)e^{-bx-a} - e^{-bx-a}) - (-bx - a)^2e^{-bx-a} + 2(-bx - a)e^{-bx-a} -$
default	$-a^2e^{-bx-a} + a((-bx - a)e^{-bx-a} - e^{-bx-a}) - (-bx - a)^2e^{-bx-a} + 2(-bx - a)e^{-bx-a} -$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-b\*x-a)\*(b\*x+a)^3/x,x,method=\_RETURNVERBOSE)

[Out] -a^2\*exp(-b\*x-a)+a\*((-b\*x-a)\*exp(-b\*x-a)-exp(-b\*x-a))-(-b\*x-a)^2\*exp(-b\*x-a)+2\*(-b\*x-a)\*exp(-b\*x-a)-2\*exp(-b\*x-a)-a^3\*exp(-a)\*Ei(1,b\*x)

**Maxima [A]**

time = 0.34, size = 69, normalized size = 0.68

$$a^3 \text{Ei}(-bx) e^{(-a)} - 3(bx + 1) a e^{(-bx-a)} - 3a^2 e^{(-bx-a)} - (b^2 x^2 + 2bx + 2) e^{(-bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(-b*x-a)*(b*x+a)^3/x,x, algorithm="maxima")`

```
[Out] a^3*Ei(-b*x)*e^(-a) - 3*(b*x + 1)*a*e^(-b*x - a) - 3*a^2*e^(-b*x - a) - (b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)
```

**Fricas [A]**

time = 0.37, size = 50, normalized size = 0.49

$$a^3 \text{Ei}(-bx) e^{(-a)} - (b^2 x^2 + (3a + 2)bx + 3a^2 + 3a + 2) e^{(-bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(-b*x-a)*(b*x+a)^3/x,x, algorithm="fricas")`

```
[Out] a^3*Ei(-b*x)*e^(-a) - (b^2*x^2 + (3*a + 2)*b*x + 3*a^2 + 3*a + 2)*e^(-b*x - a)
```

**Sympy [A]**

time = 4.83, size = 70, normalized size = 0.69

$$(a^3 \text{Ei}(-bx) - 3a^2 e^{-bx} - 3a(bx e^{-bx} + e^{-bx}) - b^2 x^2 e^{-bx} - 2bx e^{-bx} - 2e^{-bx}) e^{-a}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(-b*x-a)*(b*x+a)**3/x,x)`

```
[Out] (a**3*Ei(-b*x) - 3*a**2*exp(-b*x) - 3*a*(b*x*exp(-b*x) + exp(-b*x)) - b**2*x**2*exp(-b*x) - 2*b*x*exp(-b*x) - 2*exp(-b*x))*exp(-a)
```

**Giac [A]**

time = 3.72, size = 95, normalized size = 0.93

$$-b^2 x^2 e^{(-bx-a)} + a^3 \text{Ei}(-bx) e^{(-a)} - 3abx e^{(-bx-a)} - 3a^2 e^{(-bx-a)} - 2bx e^{(-bx-a)} - 3a e^{(-bx-a)} - 2e^{(-bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(-b*x-a)*(b*x+a)^3/x,x, algorithm="giac")`

```
[Out] -b^2*x^2*e^(-b*x - a) + a^3*Ei(-b*x)*e^(-a) - 3*a*b*x*e^(-b*x - a) - 3*a^2*e^(-b*x - a) - 2*b*x*e^(-b*x - a) - 3*a*e^(-b*x - a) - 2*e^(-b*x - a)
```

**Mupad [B]**

time = 3.56, size = 69, normalized size = 0.68

$$-e^{-a-bx} (b^2 x^2 + 2bx + 2) - 3a^2 e^{-a-bx} - 3a e^{-a-bx} (bx + 1) - a^3 e^{-a} \text{expint}(bx)$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(- a - b*x)*(a + b*x)^3)/x,x)
```

```
[Out] - exp(- a - b*x)*(2*b*x + b^2*x^2 + 2) - 3*a^2*exp(- a - b*x) - 3*a*exp(- a  
- b*x)*(b*x + 1) - a^3*exp(-a)*expint(b*x)
```

### 3.61 $\int \frac{e^{-a-bx}(a+bx)^3}{x^2} dx$

**Optimal.** Leaf size=94

$$-be^{-a-bx} - 3abe^{-a-bx} - \frac{a^3e^{-a-bx}}{x} - b^2e^{-a-bx}x + 3a^2be^{-a}\text{Ei}(-bx) - a^3be^{-a}\text{Ei}(-bx)$$

[Out]  $-b*\exp(-b*x-a)-3*a*b*\exp(-b*x-a)-a^3*\exp(-b*x-a)/x-b^2*\exp(-b*x-a)*x+3*a^2*b*\text{Ei}(-b*x)/\exp(a)-a^3*b*\text{Ei}(-b*x)/\exp(a)$

**Rubi [A]**

time = 0.11, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {2230, 2225, 2208, 2209, 2207}

$$e^{-a}a^3(-b)\text{Ei}(-bx) - \frac{a^3e^{-a-bx}}{x} + 3e^{-a}a^2b\text{Ei}(-bx) - b^2xe^{-a-bx} - 3abe^{-a-bx} - be^{-a-bx}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(E^{-a - b*x})*(a + b*x)^3/x^2, x]$

[Out]  $-(b*E^{-a - b*x}) - 3*a*b*E^{-a - b*x} - (a^3*E^{-a - b*x})/x - b^2*E^{-a - b*x}*x + (3*a^2*b*\text{ExpIntegralEi}[-(b*x)]) / E^{-a} - (a^3*b*\text{ExpIntegralEi}[-(b*x)]) / E^{-a}$

Rule 2207

$\text{Int}[(b_*)*(F_*)^{((g_*)*((e_*) + (f_*)*(x_)))})^{(n_*)*((c_*) + (d_*)*(x_))}^{(m_*)}, x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*((b*F^{(g*(e + f*x)))})^n/(f*g*n*\text{Log}[F]), x] - \text{Dist}[d*(m/(f*g*n*\text{Log}[F])), \text{Int}[(c + d*x)^{(m - 1)}*(b*F^{(g*(e + f*x)))})^n, x], x] /; \text{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0] \&\& \text{IntegerQ}[2*m] \&\& !\text{TrueQ}[\$UseGamma]$

Rule 2208

$\text{Int}[(b_*)*(F_*)^{((g_*)*((e_*) + (f_*)*(x_)))})^{(n_*)*((c_*) + (d_*)*(x_))}^{(m_*)}, x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m + 1)}*((b*F^{(g*(e + f*x)))})^n/(d*(m + 1)), x] - \text{Dist}[f*g*n*(\text{Log}[F]/(d*(m + 1))), \text{Int}[(c + d*x)^{(m + 1)}*(b*F^{(g*(e + f*x)))})^n, x], x] /; \text{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \text{LtQ}[m, -1] \&\& \text{IntegerQ}[2*m] \&\& !\text{TrueQ}[\$UseGamma]$

Rule 2209

$\text{Int}[(F_*)^{((g_*)*((e_*) + (f_*)*(x_)))}/((c_*) + (d_*)*(x_)), x\_Symbol] \rightarrow \text{Simp}[(F^{(g*(e - c*(f/d))})/d)*\text{ExpIntegralEi}[f*g*(c + d*x)*(\text{Log}[F]/d)], x] /; \text{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\text{TrueQ}[\$UseGamma]$

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.), x_Symbol] := Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2230

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] := Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F,
c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] && !TrueQ[$UseGamma]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-a-bx}(a+bx)^3}{x^2} dx &= \int \left( 3ab^2e^{-a-bx} + \frac{a^3e^{-a-bx}}{x^2} + \frac{3a^2be^{-a-bx}}{x} + b^3e^{-a-bx}x \right) dx \\
&= a^3 \int \frac{e^{-a-bx}}{x^2} dx + (3a^2b) \int \frac{e^{-a-bx}}{x} dx + (3ab^2) \int e^{-a-bx} dx + b^3 \int e^{-a-bx} x dx \\
&= -3abe^{-a-bx} - \frac{a^3e^{-a-bx}}{x} - b^2e^{-a-bx}x + 3a^2be^{-a} \text{Ei}(-bx) - (a^3b) \int \frac{e^{-a-bx}}{x} dx + b^2 \\
&= -be^{-a-bx} - 3abe^{-a-bx} - \frac{a^3e^{-a-bx}}{x} - b^2e^{-a-bx}x + 3a^2be^{-a} \text{Ei}(-bx) - a^3be^{-a} \text{Ei}(-bx)
\end{aligned}$$

**Mathematica [A]**

time = 0.16, size = 54, normalized size = 0.57

$$\frac{e^{-a-bx}(-a^3 - 3abx - bx(1+bx) - (-3+a)a^2be^{bx}x \text{Ei}(-bx))}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(-a - b*x)*(a + b*x)^3)/x^2,x]
```

```
[Out] (E^(-a - b*x)*(-a^3 - 3*a*b*x - b*x*(1 + b*x) - (-3 + a)*a^2*b*E^(b*x)*x*ExpIntegralEi[-(b*x)]))/x
```

**Maple [A]**

time = 0.06, size = 92, normalized size = 0.98

method	result
risch	$-3abe^{-bx-a} - b^2e^{-bx-a}x - be^{-bx-a} - \frac{a^3e^{-bx-a}}{x} + ba^3e^{-a} \text{expIntegral}(1, bx) - 3ba^2e^{-a}$
derivativedivides	$b\left(-2ae^{-bx-a} + (-bx - a)e^{-bx-a} - e^{-bx-a} - a^3\left(\frac{e^{-bx-a}}{bx} - e^{-a} \text{expIntegral}(1, bx)\right) - 3a^2e^{-a}\right)$

default	$b\left(-2ae^{-bx-a} + (-bx-a)e^{-bx-a} - e^{-bx-a} - a^3\left(\frac{e^{-bx-a}}{bx} - e^{-a} \operatorname{expIntegral}(1, bx)\right) - 3a^2\right)$
meijerg	$b e^{-a} \left(1 - \frac{(2bx+2)e^{-bx}}{2}\right) + 3e^{-a}ba(1 - e^{-bx}) + 3be^{-a}a^2(-\ln(bx) - \operatorname{expIntegral}(1, bx) + 1)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(-b*x-a)*(b*x+a)^3/x^2,x,method=_RETURNVERBOSE)`

[Out] `b*(-2*a*exp(-b*x-a)+(-b*x-a)*exp(-b*x-a)-exp(-b*x-a)-a^3*(exp(-b*x-a)/b/x-exp(-a)*Ei(1,b*x))-3*a^2*exp(-a)*Ei(1,b*x))`

**Maxima** [A]

time = 0.41, size = 61, normalized size = 0.65

$$-a^3 b e^{(-a)} \Gamma(-1, bx) + 3 a^2 b \operatorname{Ei}(-bx) e^{(-a)} - (bx + 1) b e^{(-bx-a)} - 3 a b e^{(-bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^3/x^2,x, algorithm="maxima")`

[Out] `-a^3*b*e^(-a)*gamma(-1, b*x) + 3*a^2*b*Ei(-b*x)*e^(-a) - (b*x + 1)*b*e^(-b*x - a) - 3*a*b*e^(-b*x - a)`

**Fricas** [A]

time = 0.40, size = 56, normalized size = 0.60

$$\frac{(a^3 - 3a^2)bx \operatorname{Ei}(-bx) e^{(-a)} + (b^2x^2 + a^3 + (3a + 1)bx)e^{(-bx-a)}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^3/x^2,x, algorithm="fricas")`

[Out] `-((a^3 - 3*a^2)*b*x*Ei(-b*x)*e^(-a) + (b^2*x^2 + a^3 + (3*a + 1)*b*x)*e^(-b*x - a))/x`

**Sympy** [A]

time = 1.94, size = 99, normalized size = 1.05

$$-\frac{a^3 e^{-a} E_2(bx)}{x} + 3a^2 b e^{-a} \operatorname{Ei}(-bx) + 3ab^2 \left( \begin{cases} x & \text{for } b = 0 \\ -\frac{e^{-bx}}{b} & \text{otherwise} \end{cases} \right) e^{-a} + b^3 x \left( \begin{cases} x & \text{for } b = 0 \\ -\frac{e^{-bx}}{b} & \text{otherwise} \end{cases} \right) e^{-a} - b^3 \left( \begin{cases} \frac{x^2}{2} & \text{for } b = 0 \\ \begin{cases} -\frac{e^{-bx}}{b} & \text{for } b \neq 0 \\ x & \text{otherwise} \end{cases} & \text{otherwise} \end{cases} \right) e^{-a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)**3/x**2,x)`

[Out] `-a**3*exp(-a)*expint(2, b*x)/x + 3*a**2*b*exp(-a)*Ei(-b*x) + 3*a*b**2*Piecewise((x, Eq(b, 0)), (-exp(-b*x)/b, True))*exp(-a) + b**3*x*Piecewise((x, Eq`

(b, 0)), (-exp(-b\*x)/b, True))\*exp(-a) - b\*\*3\*Piecewise((x\*\*2/2, Eq(b, 0)),  
 (-Piecewise((-exp(-b\*x)/b, Ne(b, 0)), (x, True))/b, True))\*exp(-a)

**Giac [A]**

time = 3.66, size = 92, normalized size = 0.98

$$\frac{a^3 b x \operatorname{Ei}(-b x) e^{(-a)} - 3 a^2 b x \operatorname{Ei}(-b x) e^{(-a)} + b^2 x^2 e^{(-b x - a)} + a^3 e^{(-b x - a)} + 3 a b x e^{(-b x - a)} + b x e^{(-b x - a)}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^3/x^2,x, algorithm="giac")

[Out] -(a^3\*b\*x\*Ei(-b\*x)\*e^(-a) - 3\*a^2\*b\*x\*Ei(-b\*x)\*e^(-a) + b^2\*x^2\*e^(-b\*x - a)  
 ) + a^3\*e^(-b\*x - a) + 3\*a\*b\*x\*e^(-b\*x - a) + b\*x\*e^(-b\*x - a))/x

**Mupad [B]**

time = 3.64, size = 72, normalized size = 0.77

$$a^3 b e^{-a} \left( \operatorname{expint}(b x) - \frac{e^{-b x}}{b x} \right) - 3 a b e^{-a - b x} - b e^{-a - b x} (b x + 1) - 3 a^2 b e^{-a} \operatorname{expint}(b x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(- a - b\*x)\*(a + b\*x)^3)/x^2,x)

[Out] a^3\*b\*exp(-a)\*(expint(b\*x) - exp(-b\*x)/(b\*x)) - 3\*a\*b\*exp(- a - b\*x) - b\*exp(- a - b\*x)\*(b\*x + 1) - 3\*a^2\*b\*exp(-a)\*expint(b\*x)

$$3.62 \quad \int \frac{e^{-a-bx}(a+bx)^3}{x^3} dx$$

**Optimal.** Leaf size=130

$$-b^2e^{-a-bx} - \frac{a^3e^{-a-bx}}{2x^2} - \frac{3a^2be^{-a-bx}}{x} + \frac{a^3be^{-a-bx}}{2x} + 3ab^2e^{-a}\text{Ei}(-bx) - 3a^2b^2e^{-a}\text{Ei}(-bx) + \frac{1}{2}a^3b^2e^{-a}\text{Ei}(-bx)$$

[Out]  $-b^2*\exp(-b*x-a) - 1/2*a^3*\exp(-b*x-a)/x^2 - 3*a^2*b*\exp(-b*x-a)/x + 1/2*a^3*b*\exp(-b*x-a)/x + 3*a*b^2*\text{Ei}(-b*x)/\exp(a) - 3*a^2*b^2*\text{Ei}(-b*x)/\exp(a) + 1/2*a^3*b^2*\text{Ei}(-b*x)/\exp(a)$

**Rubi [A]**

time = 0.15, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$ , Rules used = {2230, 2225, 2208, 2209}

$$\frac{1}{2}e^{-a}a^3b^2\text{Ei}(-bx) - \frac{a^3e^{-a-bx}}{2x^2} + \frac{a^3be^{-a-bx}}{2x} - 3e^{-a}a^2b^2\text{Ei}(-bx) - \frac{3a^2be^{-a-bx}}{x} + 3e^{-a}ab^2\text{Ei}(-bx) - b^2e^{-a-bx}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(E^{-a - b*x})*(a + b*x)^3/x^3, x]$

[Out]  $-(b^2*E^{-a - b*x}) - (a^3*E^{-a - b*x})/(2*x^2) - (3*a^2*b*E^{-a - b*x})/x + (a^3*b*E^{-a - b*x})/(2*x) + (3*a*b^2*\text{ExpIntegralEi}[-(b*x)])/\text{E}^a - (3*a^2*b^2*\text{ExpIntegralEi}[-(b*x)])/\text{E}^a + (a^3*b^2*\text{ExpIntegralEi}[-(b*x)])/(2*\text{E}^a)$

Rule 2208

$\text{Int}[(b_*)*(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))^{(n_*)*((c_*) + (d_*)*(x_*))^{(m_*)}}, x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m + 1)}*((b*F^{(g*(e + f*x)))^n/(d*(m + 1)))], x] - \text{Dist}[f*g*n*(\text{Log}[F]/(d*(m + 1))), \text{Int}[(c + d*x)^{(m + 1)}*(b*F^{(g*(e + f*x)))^n}, x], x] /; \text{FreeQ}\{F, b, c, d, e, f, g, n\}, x\} \&\& \text{LtQ}[m, -1] \&\& \text{IntegerQ}[2*m] \&\& !\text{TrueQ}[\$UseGamma]$

Rule 2209

$\text{Int}[(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))^{(n_*)}}, x\_Symbol] \rightarrow \text{Simp}[(F^{(g*(e - c*(f/d))})/d)*\text{ExpIntegralEi}[f*g*(c + d*x)*(\text{Log}[F]/d)], x] /; \text{FreeQ}\{F, c, d, e, f, g\}, x\} \&\& !\text{TrueQ}[\$UseGamma]$

Rule 2225

$\text{Int}[(F_*)^{((c_*)*((a_*) + (b_*)*(x_*)))^{(n_*)}}, x\_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x))})^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x\}$

Rule 2230

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] := Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x] /; FreeQ[{F,
c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] && !TrueQ[$UseGamma]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-a-bx}(a+bx)^3}{x^3} dx &= \int \left( b^3 e^{-a-bx} + \frac{a^3 e^{-a-bx}}{x^3} + \frac{3a^2 b e^{-a-bx}}{x^2} + \frac{3ab^2 e^{-a-bx}}{x} \right) dx \\
&= a^3 \int \frac{e^{-a-bx}}{x^3} dx + (3a^2 b) \int \frac{e^{-a-bx}}{x^2} dx + (3ab^2) \int \frac{e^{-a-bx}}{x} dx + b^3 \int e^{-a-bx} dx \\
&= -b^2 e^{-a-bx} - \frac{a^3 e^{-a-bx}}{2x^2} - \frac{3a^2 b e^{-a-bx}}{x} + 3ab^2 e^{-a} \text{Ei}(-bx) - \frac{1}{2} (a^3 b) \int \frac{e^{-a-bx}}{x^2} dx - \dots \\
&= -b^2 e^{-a-bx} - \frac{a^3 e^{-a-bx}}{2x^2} - \frac{3a^2 b e^{-a-bx}}{x} + \frac{a^3 b e^{-a-bx}}{2x} + 3ab^2 e^{-a} \text{Ei}(-bx) - 3a^2 b^2 e^{-a} \text{Ei}(-bx) \\
&= -b^2 e^{-a-bx} - \frac{a^3 e^{-a-bx}}{2x^2} - \frac{3a^2 b e^{-a-bx}}{x} + \frac{a^3 b e^{-a-bx}}{2x} + 3ab^2 e^{-a} \text{Ei}(-bx) - 3a^2 b^2 e^{-a} \text{Ei}(-bx)
\end{aligned}$$

**Mathematica [A]**

time = 0.15, size = 68, normalized size = 0.52

$$\frac{e^{-a-bx}(-6a^2bx - 2b^2x^2 + a^3(-1 + bx) + a(6 - 6a + a^2)b^2e^{bx}x^2\text{Ei}(-bx))}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(E^(-a - b\*x)\*(a + b\*x)^3)/x^3,x]

[Out] (E^(-a - b\*x)\*(-6\*a^2\*b\*x - 2\*b^2\*x^2 + a^3\*(-1 + b\*x) + a\*(6 - 6\*a + a^2)\*b^2\*E^(b\*x)\*x^2\*ExpIntegralEi[-(b\*x)]))/(2\*x^2)

**Maple [A]**

time = 0.07, size = 112, normalized size = 0.86

method	result
derivativedivides	$-b^2 \left( e^{-bx-a} - a^3 \left( -\frac{e^{-bx-a}}{2b^2x^2} + \frac{e^{-bx-a}}{2bx} - \frac{e^{-a} \text{expIntegral}(1,bx)}{2} \right) \right) + 3a^2 \left( \frac{e^{-bx-a}}{bx} - e^{-a} \text{expIntegral}(1,bx) \right)$
default	$-b^2 \left( e^{-bx-a} - a^3 \left( -\frac{e^{-bx-a}}{2b^2x^2} + \frac{e^{-bx-a}}{2bx} - \frac{e^{-a} \text{expIntegral}(1,bx)}{2} \right) \right) + 3a^2 \left( \frac{e^{-bx-a}}{bx} - e^{-a} \text{expIntegral}(1,bx) \right)$
risch	$-b^2 e^{-bx-a} - \frac{a^3 e^{-bx-a}}{2x^2} + \frac{a^3 b e^{-bx-a}}{2x} - \frac{b^2 a^3 e^{-a} \text{expIntegral}(1,bx)}{2} - \frac{3a^2 b e^{-bx-a}}{x} + 3b^2 a^2 e^{-a} \text{expIntegral}(1,bx)$
meijerg	$e^{-a} b^2 (1 - e^{-bx}) + 3b^2 e^{-a} a (-\ln(bx) - \text{expIntegral}(1, bx) + \ln(x) + \ln(b)) + 3b^2 e^{-a} a^2 \text{expIntegral}(1, bx)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(-b*x-a)*(b*x+a)^3/x^3,x,method=_RETURNVERBOSE)`

[Out]  $-b^2*(\exp(-b*x-a)-a^3*(-1/2*\exp(-b*x-a)/b^2/x^2+1/2*\exp(-b*x-a)/b/x-1/2*\exp(-a)*\text{Ei}(1,b*x)))+3*a^2*(\exp(-b*x-a)/b/x-\exp(-a)*\text{Ei}(1,b*x))+3*a*\exp(-a)*\text{Ei}(1,b*x)$

**Maxima** [A]

time = 0.50, size = 64, normalized size = 0.49

$$-a^3 b^2 e^{(-a)} \Gamma(-2, bx) - 3 a^2 b^2 e^{(-a)} \Gamma(-1, bx) + 3 a b^2 \text{Ei}(-bx) e^{(-a)} - b^2 e^{(-bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^3/x^3,x, algorithm="maxima")`

[Out]  $-a^3*b^2*e^{(-a)}*\text{gamma}(-2, b*x) - 3*a^2*b^2*e^{(-a)}*\text{gamma}(-1, b*x) + 3*a*b^2*\text{Ei}(-b*x)*e^{(-a)} - b^2*e^{(-b*x - a)}$

**Fricas** [A]

time = 0.45, size = 70, normalized size = 0.54

$$\frac{(a^3 - 6 a^2 + 6 a) b^2 x^2 \text{Ei}(-bx) e^{(-a)} - (2 b^2 x^2 + a^3 - (a^3 - 6 a^2) b x) e^{(-bx-a)}}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^3/x^3,x, algorithm="fricas")`

[Out]  $1/2*((a^3 - 6*a^2 + 6*a)*b^2*x^2*\text{Ei}(-b*x)*e^{(-a)} - (2*b^2*x^2 + a^3 - (a^3 - 6*a^2)*b*x)*e^{(-b*x - a)})/x^2$

**Sympy** [A]

time = 1.75, size = 56, normalized size = 0.43

$$\left( -\frac{a^3 \text{E}_3(bx)}{x^2} - \frac{3a^2 b \text{E}_2(bx)}{x} + 3ab^2 \text{Ei}(-bx) + b^3 \left( \begin{cases} x & \text{for } b = 0 \\ -\frac{e^{-bx}}{b} & \text{otherwise} \end{cases} \right) \right) e^{-a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)**3/x**3,x)`

[Out]  $(-a**3*\text{expint}(3, b*x)/x**2 - 3*a**2*b*\text{expint}(2, b*x)/x + 3*a*b**2*\text{Ei}(-b*x) + b**3*\text{Piecewise}((x, \text{Eq}(b, 0)), (-\exp(-b*x)/b, \text{True}))) * \exp(-a)$

**Giac** [A]

time = 3.79, size = 125, normalized size = 0.96

$$\frac{a^3 b^2 x^2 \text{Ei}(-bx) e^{(-a)} - 6 a^2 b^2 x^2 \text{Ei}(-bx) e^{(-a)} + 6 a b^2 x^2 \text{Ei}(-bx) e^{(-a)} + a^3 b x e^{(-bx-a)} - 6 a^2 b x e^{(-bx-a)} - 2 b^2 x^2 e^{(-bx-a)} - a^3 e^{(-bx-a)}}{2 x^2}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^3/x^3,x, algorithm="giac")

[Out]  $\frac{1}{2}(a^3b^2x^2\text{Ei}(-bx)e^{-a} - 6a^2b^2x^2\text{Ei}(-bx)e^{-a} + 6ab^2x^2\text{Ei}(-bx)e^{-a} + a^3bx^2e^{-bx-a} - 6a^2bx^2e^{-bx-a} - 2b^2x^2e^{-bx-a} - a^3e^{-bx-a})/x^2$

**Mupad [B]**

time = 3.64, size = 100, normalized size = 0.77

$$3a^2b^2e^{-a}\left(\text{expint}(bx) - \frac{e^{-bx}}{bx}\right) - 3ab^2e^{-a}\text{expint}(bx) - b^2e^{-a-bx} + a^3b^2e^{-a}\left(e^{-bx}\left(\frac{1}{2bx} - \frac{1}{2b^2x^2}\right) - \frac{\text{expint}(bx)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(- a - b\*x)\*(a + b\*x)^3)/x^3,x)

[Out]  $3a^2b^2\exp(-a)(\text{expint}(bx) - \exp(-bx)/(bx)) - 3ab^2\exp(-a)\text{expint}(bx) - b^2\exp(-a - bx) + a^3b^2\exp(-a)(\exp(-bx)(1/(2bx) - 1/(2b^2x^2)) - \text{expint}(bx)/2)$

$$3.63 \quad \int \frac{e^{-a-bx}(a+bx)^3}{x^4} dx$$

**Optimal.** Leaf size=198

$$-\frac{a^3 e^{-a-bx}}{3x^3} - \frac{3a^2 b e^{-a-bx}}{2x^2} + \frac{a^3 b e^{-a-bx}}{6x^2} - \frac{3ab^2 e^{-a-bx}}{x} + \frac{3a^2 b^2 e^{-a-bx}}{2x} - \frac{a^3 b^2 e^{-a-bx}}{6x} + b^3 e^{-a} \text{Ei}(-bx) - 3ab^3 e^{-a} \text{Ei}(-bx)$$

[Out]  $-1/3*a^3*\exp(-b*x-a)/x^3-3/2*a^2*b*\exp(-b*x-a)/x^2+1/6*a^3*b*\exp(-b*x-a)/x^2-3*a*b^2*\exp(-b*x-a)/x+3/2*a^2*b^2*\exp(-b*x-a)/x-1/6*a^3*b^2*\exp(-b*x-a)/x+b^3*Ei(-b*x)/\exp(a)-3*a*b^3*Ei(-b*x)/\exp(a)+3/2*a^2*b^3*Ei(-b*x)/\exp(a)-1/6*a^3*b^3*Ei(-b*x)/\exp(a)$

**Rubi [A]**

time = 0.20, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2230, 2208, 2209}

$$-\frac{1}{6}e^{-a}a^3b^3\text{Ei}(-bx) - \frac{a^3b^2e^{-a-bx}}{6x} - \frac{a^3e^{-a-bx}}{3x^3} + \frac{a^3be^{-a-bx}}{6x^2} + \frac{3}{2}e^{-a}a^2b^3\text{Ei}(-bx) + \frac{3a^2b^2e^{-a-bx}}{2x} - \frac{3a^2be^{-a-bx}}{2x^2} - 3e^{-a}ab^3\text{Ei}(-bx) + e^{-a}b^3\text{Ei}(-bx) - \frac{3ab^2e^{-a-bx}}{x}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(E^{-a-b*x})*(a+b*x)^3/x^4, x]$

[Out]  $-1/3*(a^3*E^{-a-b*x})/x^3 - (3*a^2*b*E^{-a-b*x})/(2*x^2) + (a^3*b*E^{-a-b*x})/(6*x^2) - (3*a*b^2*E^{-a-b*x})/x + (3*a^2*b^2*E^{-a-b*x})/(2*x) - (a^3*b^2*E^{-a-b*x})/(6*x) + (b^3*ExpIntegralEi[-(b*x)])/E^a - (3*a*b^3*ExpIntegralEi[-(b*x)])/E^a + (3*a^2*b^3*ExpIntegralEi[-(b*x)])/E^a - (a^3*b^3*ExpIntegralEi[-(b*x)])/E^a$

**Rule 2208**

$\text{Int}[(b_*)*(F_)^{((g_*)*((e_*) + (f_*)*(x_)))}^{(n_*)}*((c_*) + (d_*)*(x_))^{(m_*)}, x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m+1)}*((b*F^{(g*(e+f*x)))^n/(d*(m+1))}, x] - \text{Dist}[f*g*n*(\text{Log}[F]/(d*(m+1))), \text{Int}[(c + d*x)^{(m+1)}*(b*F^{(g*(e+f*x)))^n}, x], x] /; \text{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \text{LtQ}[m, -1] \&\& \text{IntegerQ}[2*m] \&\& !\text{TrueQ}[\$UseGamma]$

**Rule 2209**

$\text{Int}[(F_)^{((g_*)*((e_*) + (f_*)*(x_)))}^{(n_*)}/((c_*) + (d_*)*(x_)), x\_Symbol] \rightarrow \text{Simp}[(F^{(g*(e-c*(f/d))})/d)*\text{ExpIntegralEi}[f*g*(c+d*x)*(\text{Log}[F]/d)], x] /; \text{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\text{TrueQ}[\$UseGamma]$

**Rule 2230**

$\text{Int}[(F_)^{((c_*)*(v_))*}^{(m_*)}*(u_)^{(m_*)}*(w_), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[F^{(c*\text{ExpandToSum}[v, x])}, w*\text{NormalizePowerOfLinear}[u, x]^m, x], x] /; \text{FreeQ}\{F,$

c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && IntegerQ[m] && !TrueQ[\$UseGamma]

Rubi steps

$$\begin{aligned}
 \int \frac{e^{-a-bx}(a+bx)^3}{x^4} dx &= \int \left( \frac{a^3 e^{-a-bx}}{x^4} + \frac{3a^2 b e^{-a-bx}}{x^3} + \frac{3ab^2 e^{-a-bx}}{x^2} + \frac{b^3 e^{-a-bx}}{x} \right) dx \\
 &= a^3 \int \frac{e^{-a-bx}}{x^4} dx + (3a^2 b) \int \frac{e^{-a-bx}}{x^3} dx + (3ab^2) \int \frac{e^{-a-bx}}{x^2} dx + b^3 \int \frac{e^{-a-bx}}{x} dx \\
 &= -\frac{a^3 e^{-a-bx}}{3x^3} - \frac{3a^2 b e^{-a-bx}}{2x^2} - \frac{3ab^2 e^{-a-bx}}{x} + b^3 e^{-a} \text{Ei}(-bx) - \frac{1}{3}(a^3 b) \int \frac{e^{-a-bx}}{x^3} dx - \\
 &= -\frac{a^3 e^{-a-bx}}{3x^3} - \frac{3a^2 b e^{-a-bx}}{2x^2} + \frac{a^3 b e^{-a-bx}}{6x^2} - \frac{3ab^2 e^{-a-bx}}{x} + \frac{3a^2 b^2 e^{-a-bx}}{2x} + b^3 e^{-a} \text{Ei}(-bx) \\
 &= -\frac{a^3 e^{-a-bx}}{3x^3} - \frac{3a^2 b e^{-a-bx}}{2x^2} + \frac{a^3 b e^{-a-bx}}{6x^2} - \frac{3ab^2 e^{-a-bx}}{x} + \frac{3a^2 b^2 e^{-a-bx}}{2x} - \frac{a^3 b^2 e^{-a-bx}}{6x} \\
 &= -\frac{a^3 e^{-a-bx}}{3x^3} - \frac{3a^2 b e^{-a-bx}}{2x^2} + \frac{a^3 b e^{-a-bx}}{6x^2} - \frac{3ab^2 e^{-a-bx}}{x} + \frac{3a^2 b^2 e^{-a-bx}}{2x} - \frac{a^3 b^2 e^{-a-bx}}{6x}
 \end{aligned}$$

**Mathematica [A]**

time = 0.18, size = 81, normalized size = 0.41

$$\frac{1}{6} e^{-a} \left( -\frac{ae^{-bx}(18b^2x^2 - 9abx(-1+bx) + a^2(2-bx+b^2x^2))}{x^3} - (-6 + 18a - 9a^2 + a^3) b^3 \text{Ei}(-bx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(E^(-a - b\*x)\*(a + b\*x)^3)/x^4,x]

[Out] (-(a\*(18\*b^2\*x^2 - 9\*a\*b\*x\*(-1 + b\*x) + a^2\*(2 - b\*x + b^2\*x^2)))/(E^(b\*x)\*x^3)) - (-6 + 18\*a - 9\*a^2 + a^3)\*b^3\*ExpIntegralEi[-(b\*x)]/(6\*E^a)

**Maple [A]**

time = 0.07, size = 167, normalized size = 0.84

method	result
derivativedivides	$b^3 \left( 3a^2 \left( -\frac{e^{-bx-a}}{2b^2x^2} + \frac{e^{-bx-a}}{2bx} - \frac{e^{-a} \text{expIntegral}(1,bx)}{2} \right) - a^3 \left( \frac{e^{-bx-a}}{3b^3x^3} - \frac{e^{-bx-a}}{6b^2x^2} + \frac{e^{-bx-a}}{6bx} - \frac{e^{-a} \text{expIntegral}(1,bx)}{6} \right) \right)$
default	$b^3 \left( 3a^2 \left( -\frac{e^{-bx-a}}{2b^2x^2} + \frac{e^{-bx-a}}{2bx} - \frac{e^{-a} \text{expIntegral}(1,bx)}{2} \right) - a^3 \left( \frac{e^{-bx-a}}{3b^3x^3} - \frac{e^{-bx-a}}{6b^2x^2} + \frac{e^{-bx-a}}{6bx} - \frac{e^{-a} \text{expIntegral}(1,bx)}{6} \right) \right)$
risch	$-\frac{3a^2 b e^{-bx-a}}{2x^2} + \frac{3a^2 b^2 e^{-bx-a}}{2x} - \frac{3b^3 a^2 e^{-a} \text{expIntegral}(1,bx)}{2} - \frac{a^3 e^{-bx-a}}{3x^3} + \frac{a^3 b e^{-bx-a}}{6x^2} - \frac{a^3 b^2 e^{-bx-a}}{6x} + b^3 e^{-a} \text{Ei}(-bx)$
meijerg	$b^3 e^{-a} \left( -\ln(bx) - \text{expIntegral}(1,bx) + \ln(x) + \ln(b) \right) + 3b^3 e^{-a} a \left( \frac{-2bx+2}{2bx} - \frac{e^{-bx}}{bx} + \ln(bx) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(-b*x-a)*(b*x+a)^3/x^4,x,method=_RETURNVERBOSE)`

[Out]  $b^3*(3*a^2*(-1/2*\exp(-b*x-a)/b^2/x^2+1/2*\exp(-b*x-a)/b/x-1/2*\exp(-a)*\text{Ei}(1,b*x))-a^3*(1/3*\exp(-b*x-a)/b^3/x^3-1/6*\exp(-b*x-a)/b^2/x^2+1/6*\exp(-b*x-a)/b/x-1/6*\exp(-a)*\text{Ei}(1,b*x))-3*a*(\exp(-b*x-a)/b/x-\exp(-a)*\text{Ei}(1,b*x))-\exp(-a)*\text{Ei}(1,b*x)$

**Maxima** [A]

time = 0.51, size = 63, normalized size = 0.32

$$-a^3 b^3 e^{(-a)} \Gamma(-3, bx) - 3 a^2 b^3 e^{(-a)} \Gamma(-2, bx) - 3 a b^3 e^{(-a)} \Gamma(-1, bx) + b^3 \text{Ei}(-bx) e^{(-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^3/x^4,x, algorithm="maxima")`

[Out]  $-a^3*b^3*e^{(-a)}*\text{gamma}(-3, b*x) - 3*a^2*b^3*e^{(-a)}*\text{gamma}(-2, b*x) - 3*a*b^3*e^{(-a)}*\text{gamma}(-1, b*x) + b^3*\text{Ei}(-b*x)*e^{(-a)}$

**Fricas** [A]

time = 0.36, size = 83, normalized size = 0.42

$$\frac{(a^3 - 9a^2 + 18a - 6)b^3 x^3 \text{Ei}(-bx) e^{(-a)} + ((a^3 - 9a^2 + 18a)b^2 x^2 + 2a^3 - (a^3 - 9a^2)bx) e^{(-bx-a)}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^3/x^4,x, algorithm="fricas")`

[Out]  $-1/6*((a^3 - 9a^2 + 18a - 6)*b^3*x^3*\text{Ei}(-b*x)*e^{(-a)} + ((a^3 - 9a^2 + 18a)*b^2*x^2 + 2a^3 - (a^3 - 9a^2)*b*x)*e^{(-b*x - a)})/x^3$

**Sympy** [A]

time = 1.79, size = 53, normalized size = 0.27

$$\left( -\frac{a^3 E_4(bx)}{x^3} - \frac{3a^2 b E_3(bx)}{x^2} - \frac{3ab^2 E_2(bx)}{x} + b^3 \text{Ei}(-bx) \right) e^{-a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)**3/x**4,x)`

[Out]  $(-a**3*\text{expint}(4, b*x)/x**3 - 3*a**2*b*\text{expint}(3, b*x)/x**2 - 3*a*b**2*\text{expint}(2, b*x)/x + b**3*\text{Ei}(-b*x))*\exp(-a)$

**Giac** [A]

time = 3.30, size = 183, normalized size = 0.92

$$\frac{a^3 b^3 x^3 \text{Ei}(-bx) e^{(-a)} - 9 a^2 b^3 x^3 \text{Ei}(-bx) e^{(-a)} + 18 a b^3 x^3 \text{Ei}(-bx) e^{(-a)} + a^3 b^2 x^2 e^{(-bx-a)} - 6 b^3 x^3 \text{Ei}(-bx) e^{(-a)} - 9 a^2 b^2 x^2 e^{(-bx-a)} - a^3 b x e^{(-bx-a)} + 18 a b^2 x^2 e^{(-bx-a)} + 9 a^2 b x e^{(-bx-a)} + 2 a^3 e^{(-bx-a)}}{6 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^3/x^4,x, algorithm="giac")

[Out]  $-1/6*(a^3*b^3*x^3*Ei(-b*x)*e^{-a} - 9*a^2*b^3*x^3*Ei(-b*x)*e^{-a} + 18*a*b^3*x^3*Ei(-b*x)*e^{-a} + a^3*b^2*x^2*e^{-b*x - a} - 6*b^3*x^3*Ei(-b*x)*e^{-a}) - 9*a^2*b^2*x^2*e^{-b*x - a} - a^3*b*x*e^{-b*x - a} + 18*a*b^2*x^2*e^{-b*x - a} + 9*a^2*b*x*e^{-b*x - a} + 2*a^3*e^{-b*x - a})/x^3$

**Mupad [B]**

time = 3.57, size = 142, normalized size = 0.72

$$3ab^3e^{-a}\left(\operatorname{expint}(bx) - \frac{e^{-bx}}{bx}\right) - b^3e^{-a}\operatorname{expint}(bx) + \frac{a^3b^3e^{-a}\operatorname{expint}(bx)}{6} + 3a^2b^3e^{-a}\left(e^{-bx}\left(\frac{1}{2bx} - \frac{1}{2b^2x^2}\right) - \frac{\operatorname{expint}(bx)}{2}\right) - a^3b^3e^{-a-bx}\left(\frac{1}{6bx} - \frac{1}{6b^2x^2} + \frac{1}{3b^3x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(- a - b\*x)\*(a + b\*x)^3)/x^4,x)

[Out]  $3*a*b^3*\exp(-a)*(\operatorname{expint}(b*x) - \exp(-b*x)/(b*x)) - b^3*\exp(-a)*\operatorname{expint}(b*x) + (a^3*b^3*\exp(-a)*\operatorname{expint}(b*x))/6 + 3*a^2*b^3*\exp(-a)*(\exp(-b*x)*(1/(2*b*x) - 1/(2*b^2*x^2)) - \operatorname{expint}(b*x)/2) - a^3*b^3*\exp(-a - b*x)*(1/(6*b*x) - 1/(6*b^2*x^2) + 1/(3*b^3*x^3))$

### 3.64 $\int F^{a+b(c+dx)} x^m (e + fx)^2 dx$

**Optimal.** Leaf size=139

$$\frac{f^2 F^{a+bc} x^m \Gamma(3+m, -bdx \log(F)) (-bdx \log(F))^{-m}}{b^3 d^3 \log^3(F)} - \frac{2ef F^{a+bc} x^m \Gamma(2+m, -bdx \log(F)) (-bdx \log(F))^{-m}}{b^2 d^2 \log^2(F)} +$$

```
[Out] f^2*F^(b*c+a)*x^m*GAMMA(3+m,-b*d*x*ln(F))/b^3/d^3/ln(F)^3/((-b*d*x*ln(F))^m)
)-2*e*f*F^(b*c+a)*x^m*GAMMA(2+m,-b*d*x*ln(F))/b^2/d^2/ln(F)^2/((-b*d*x*ln(F)
))^m)+e^2*F^(b*c+a)*x^m*GAMMA(1+m,-b*d*x*ln(F))/b/d/ln(F)/((-b*d*x*ln(F))^m
)
```

**Rubi [A]**

time = 0.21, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2230, 2212}

$$\frac{f^2 x^m F^{a+bc} (-bdx \log(F))^{-m} \Gamma(m+3, -bdx \log(F))}{b^3 d^3 \log^3(F)} - \frac{2ef x^m F^{a+bc} (-bdx \log(F))^{-m} \Gamma(m+2, -bdx \log(F))}{b^2 d^2 \log^2(F)} + \frac{e^2 x^m F^{a+bc} (-bdx \log(F))^{-m} \Gamma(m+1, -bdx \log(F))}{bd \log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(a + b*(c + d*x))*x^m*(e + f*x)^2,x]
```

```
[Out] (f^2*F^(a + b*c)*x^m*Gamma[3 + m, -(b*d*x*Log[F])])/(b^3*d^3*Log[F]^3*(-(b*
d*x*Log[F]))^m) - (2*e*f*F^(a + b*c)*x^m*Gamma[2 + m, -(b*d*x*Log[F])])/(b^
2*d^2*Log[F]^2*(-(b*d*x*Log[F]))^m) + (e^2*F^(a + b*c)*x^m*Gamma[1 + m, -(b
*d*x*Log[F])])/(b*d*Log[F]*(-(b*d*x*Log[F]))^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d)^FracPart[m]))*Gamma[m + 1,
((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2230

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] :> Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F,
c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] && !TrueQ[$UseGamma]
```

Rubi steps

$$\begin{aligned} \int F^{a+b(c+dx)} x^m (e+fx)^2 dx &= \int (e^2 F^{a+bc+bdx} x^m + 2ef F^{a+bc+bdx} x^{1+m} + f^2 F^{a+bc+bdx} x^{2+m}) dx \\ &= e^2 \int F^{a+bc+bdx} x^m dx + (2ef) \int F^{a+bc+bdx} x^{1+m} dx + f^2 \int F^{a+bc+bdx} x^{2+m} dx \\ &= \frac{f^2 F^{a+bc} x^m \Gamma(3+m, -bdx \log(F)) (-bdx \log(F))^{-m}}{b^3 d^3 \log^3(F)} - \frac{2ef F^{a+bc} x^m \Gamma(2+m, -bdx \log(F)) (-bdx \log(F))^{-m}}{b^3 d^3 \log^3(F)} \end{aligned}$$

**Mathematica [A]**

time = 0.24, size = 86, normalized size = 0.62

$$\frac{F^{a+bc} x^m (-bdx \log(F))^{-m} (f^2 \Gamma(3+m, -bdx \log(F)) + bde \log(F) (-2f \Gamma(2+m, -bdx \log(F)) + bde \Gamma(1+m, -bdx \log(F)) \log(F)))}{b^3 d^3 \log^3(F)}$$

Antiderivative was successfully verified.

**[In]** Integrate[F^(a + b\*(c + d\*x))\*x^m\*(e + f\*x)^2,x]

**[Out]** (F^(a + b\*c)\*x^m\*(f^2\*Gamma[3 + m, -(b\*d\*x\*Log[F])]) + b\*d\*e\*Log[F]\*(-2\*f\*Gamma[2 + m, -(b\*d\*x\*Log[F])]) + b\*d\*e\*Gamma[1 + m, -(b\*d\*x\*Log[F])]\*Log[F]))/(b^3\*d^3\*Log[F]^3\*(-(b\*d\*x\*Log[F]))^m)

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(139) = 278.

time = 0.10, size = 433, normalized size = 3.12

method	result
meijerg	$-\frac{\ln(F)^{-3-m} (-bd)^{-m} F^{cb+a} f^2 (x^m (-bd)^m \ln(F)^m m(m^2+3m+2) \Gamma(m) (-bdx \ln(F))^{-m} - x^m (-bd)^m \ln(F)^m (b^2 d^2 x^2 \ln(F)^2 - m b d x \ln(F) + m^2))}{b^3 d^3}$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(F^(a+b\*(d\*x+c))\*x^m\*(f\*x+e)^2,x,method=\_RETURNVERBOSE)

**[Out]** -1/b^3/d^3\*ln(F)^(-3-m)\*(-b\*d)^(-m)\*F^(b\*c+a)\*f^2\*(x^m\*(-b\*d)^m\*ln(F)^m\*(m^2+3\*m+2)\*GAMMA(m)\*(-b\*d\*x\*ln(F))^(-m)-x^m\*(-b\*d)^m\*ln(F)^m\*(b^2\*d^2\*x^2\*ln(F)^2-m\*b\*d\*x\*ln(F)+m^2-2\*b\*d\*x\*ln(F)+3\*m+2)\*exp(b\*d\*x\*ln(F))-x^m\*(-b\*d)^m\*ln(F)^m\*(m^2+3\*m+2)\*(-b\*d\*x\*ln(F))^(-m)\*GAMMA(m,-b\*d\*x\*ln(F)))+2/b^2/d^2\*ln(F)^(-2-m)\*(-b\*d)^(-m)\*F^(b\*c+a)\*f\*e\*(x^m\*(-b\*d)^m\*ln(F)^m\*(1+m)\*GAMMA(m)\*(-b\*d\*x\*ln(F))^(-m)+x^m\*(-b\*d)^m\*ln(F)^m\*(b\*d\*x\*ln(F)-1-m)\*exp(b\*d\*x\*ln(F))-x^m\*(-b\*d)^m\*ln(F)^m\*(1+m)\*(-b\*d\*x\*ln(F))^(-m)\*GAMMA(m,-b\*d\*x\*ln(F)))-F^(b\*c+a)\*(-b\*d)^(-m)\*ln(F)^(-m-1)\*e^2/b/d\*(x^m\*(-b\*d)^m\*ln(F)^m\*(m^2+3\*m+2)\*GAMMA(m)\*(-b\*d\*x\*ln(F))^(-m)-x^m\*(-b\*d)^m\*ln(F)^m\*exp(b\*d\*x\*ln(F))-x^m\*(-b\*d)^m\*ln(F)^m\*(m^2+3\*m+2)\*(-b\*d\*x\*ln(F))^(-m)\*GAMMA(m,-b\*d\*x\*ln(F)))

**Maxima [A]**

time = 0.18, size = 123, normalized size = 0.88

$$-(-bdx \log(F))^{-m-3} F^{bc+a} f^2 x^{m+3} \Gamma(m+3, -bdx \log(F)) - 2(-bdx \log(F))^{-m-2} F^{bc+a} f x^{m+2} e \Gamma(m+2, -bdx \log(F)) - (-bdx \log(F))^{-m-1} F^{bc+a} x^{m+1} e^2 \Gamma(m+1, -bdx \log(F))$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(F^(a+b\*(d\*x+c))\*x^m\*(f\*x+e)^2,x, algorithm="maxima")

**[Out]**  $-(-b*d*x*\log(F))^{(-m-3)}*F^{(b*c+a)}*f^2*x^{(m+3)}*\gamma(m+3, -b*d*x*\log(F)) - 2*(-b*d*x*\log(F))^{(-m-2)}*F^{(b*c+a)}*f*x^{(m+2)}*e*\gamma(m+2, -b*d*x*\log(F)) - (-b*d*x*\log(F))^{(-m-1)}*F^{(b*c+a)}*x^{(m+1)}*e^2*\gamma(m+1, -b*d*x*\log(F))$

**Fricas [A]**

time = 0.11, size = 161, normalized size = 1.16

$$\frac{((bdf^2m + 2bdf^2)x \log(F) - (b^2d^2f^2x^2 + 2b^2d^2fxe) \log(F)^2) F^{bdx+bc+ax^m} - (b^2d^2e^2 \log(F)^2 + f^2m^2 + 3f^2m - 2(bdfm + bdf)e \log(F) + 2f^2)e^{(-m \log(-bd \log(F)) + (bc+a) \log(F))} \Gamma(m+1, -bdx \log(F))}{b^3d^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(F^(a+b\*(d\*x+c))\*x^m\*(f\*x+e)^2,x, algorithm="fricas")

**[Out]**  $-(((b*d*f^2*m + 2*b*d*f^2)*x*\log(F) - (b^2*d^2*f^2*x^2 + 2*b^2*d^2*f*x*e)*\log(F)^2)*F^{(b*d*x + b*c + a)}*x^m - (b^2*d^2*e^2*\log(F)^2 + f^2*m^2 + 3*f^2*m - 2*(b*d*f*m + b*d*f)*e*\log(F) + 2*f^2)*e^{(-m*\log(-b*d*\log(F)) + (b*c + a)*\log(F))*\gamma(m + 1, -b*d*x*\log(F)))/(b^3*d^3*\log(F)^3)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int F^{a+b(c+dx)} x^m (e + fx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(F\*\*(a+b\*(d\*x+c))\*x\*\*m\*(f\*x+e)\*\*2,x)**[Out]** Integral(F\*\*(a + b\*(c + d\*x))\*x\*\*m\*(e + f\*x)\*\*2, x)**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(F^(a+b\*(d\*x+c))\*x^m\*(f\*x+e)^2,x, algorithm="giac")**[Out]** integrate((f\*x + e)^2\*F^((d\*x + c)\*b + a)\*x^m, x)



**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int F^{a+b(c+dx)} x^m (e+fx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a + b\*(c + d\*x))\*x^m\*(e + f\*x)^2,x)

[Out] int(F^(a + b\*(c + d\*x))\*x^m\*(e + f\*x)^2, x)

### 3.65 $\int F^{a+b(c+dx)} x^3 (e + fx)^2 dx$

Optimal. Leaf size=414

$$-\frac{120f^2 F^{a+bc+bdx}}{b^6 d^6 \log^6(F)} + \frac{48ef F^{a+bc+bdx}}{b^5 d^5 \log^5(F)} + \frac{120f^2 F^{a+bc+bdx} x}{b^5 d^5 \log^5(F)} - \frac{6e^2 F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} - \frac{48ef F^{a+bc+bdx} x}{b^4 d^4 \log^4(F)} - \frac{60f^2 F^{a+bc+bdx} x^2}{b^4 d^4 \log^4(F)} + \dots$$

[Out]  $-120*f^2*F^{(b*d*x+b*c+a)/b^6/d^6/\ln(F)^6} + 48*e*f*F^{(b*d*x+b*c+a)/b^5/d^5/\ln(F)^5} + 120*f^2*F^{(b*d*x+b*c+a)*x/b^5/d^5/\ln(F)^5} - 6*e^2*F^{(b*d*x+b*c+a)/b^4/d^4/\ln(F)^4} - 48*e*f*F^{(b*d*x+b*c+a)*x/b^4/d^4/\ln(F)^4} - 60*f^2*F^{(b*d*x+b*c+a)*x^2/b^4/d^4/\ln(F)^4} + 6*e^2*F^{(b*d*x+b*c+a)*x/b^3/d^3/\ln(F)^3} + 24*e*f*F^{(b*d*x+b*c+a)*x^2/b^3/d^3/\ln(F)^3} + 20*f^2*F^{(b*d*x+b*c+a)*x^3/b^3/d^3/\ln(F)^3} - 3*e^2*F^{(b*d*x+b*c+a)*x^2/b^2/d^2/\ln(F)^2} - 8*e*f*F^{(b*d*x+b*c+a)*x^3/b^2/d^2/\ln(F)^2} - 5*f^2*F^{(b*d*x+b*c+a)*x^4/b^2/d^2/\ln(F)^2} + e^2*F^{(b*d*x+b*c+a)*x^3/b/d/\ln(F)^2} + 2*e*f*F^{(b*d*x+b*c+a)*x^4/b/d/\ln(F)} + f^2*F^{(b*d*x+b*c+a)*x^5/b/d/\ln(F)}$

Rubi [A]

time = 0.48, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {2227, 2207, 2225}

$$-\frac{120f^2 F^{a+bc+bdx}}{b^6 d^6 \log^6(F)} + \frac{48ef F^{a+bc+bdx}}{b^5 d^5 \log^5(F)} + \frac{120f^2 F^{a+bc+bdx} x}{b^5 d^5 \log^5(F)} - \frac{6e^2 F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} - \frac{48ef F^{a+bc+bdx} x}{b^4 d^4 \log^4(F)} - \frac{60f^2 F^{a+bc+bdx} x^2}{b^4 d^4 \log^4(F)} + \frac{6e^2 F^{a+bc+bdx} x}{b^4 d^4 \log^4(F)} + \frac{24ef F^{a+bc+bdx} x^2}{b^4 d^4 \log^4(F)} + \frac{20f^2 F^{a+bc+bdx} x^3}{b^4 d^4 \log^4(F)} - \frac{3e^2 F^{a+bc+bdx} x^2}{b^3 d^3 \log^3(F)} - \frac{8ef F^{a+bc+bdx} x^3}{b^3 d^3 \log^3(F)} - \frac{5f^2 F^{a+bc+bdx} x^4}{b^3 d^3 \log^3(F)} + \frac{e^2 F^{a+bc+bdx} x^3}{bd \log(F)} + \frac{2ef F^{a+bc+bdx} x^4}{bd \log(F)} + \frac{f^2 F^{a+bc+bdx} x^5}{bd \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[F^{(a + b*(c + d*x))} x^3 (e + f*x)^2, x]$

[Out]  $(-120*f^2*F^{(a + b*c + b*d*x)})/(b^6*d^6*\text{Log}[F]^6) + (48*e*f*F^{(a + b*c + b*d*x)})/(b^5*d^5*\text{Log}[F]^5) + (120*f^2*F^{(a + b*c + b*d*x)*x})/(b^5*d^5*\text{Log}[F]^5) - (6*e^2*F^{(a + b*c + b*d*x)})/(b^4*d^4*\text{Log}[F]^4) - (48*e*f*F^{(a + b*c + b*d*x)*x})/(b^4*d^4*\text{Log}[F]^4) - (60*f^2*F^{(a + b*c + b*d*x)*x^2})/(b^4*d^4*\text{Log}[F]^4) + (6*e^2*F^{(a + b*c + b*d*x)*x})/(b^3*d^3*\text{Log}[F]^3) + (24*e*f*F^{(a + b*c + b*d*x)*x^2})/(b^3*d^3*\text{Log}[F]^3) + (20*f^2*F^{(a + b*c + b*d*x)*x^3})/(b^3*d^3*\text{Log}[F]^3) - (3*e^2*F^{(a + b*c + b*d*x)*x^2})/(b^2*d^2*\text{Log}[F]^2) - (8*e*f*F^{(a + b*c + b*d*x)*x^3})/(b^2*d^2*\text{Log}[F]^2) - (5*f^2*F^{(a + b*c + b*d*x)*x^4})/(b^2*d^2*\text{Log}[F]^2) + (e^2*F^{(a + b*c + b*d*x)*x^3})/(b*d*\text{Log}[F]) + (2*e*f*F^{(a + b*c + b*d*x)*x^4})/(b*d*\text{Log}[F]) + (f^2*F^{(a + b*c + b*d*x)*x^5})/(b*d*\text{Log}[F])$

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[UseGamma]
```

## Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.), x_Symbol] := Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

## Rule 2227

```
Int[(F_)^((c_.)*(v_))*(u_), x_Symbol] := Int[ExpandIntegrand[F^(c*ExpandToS
um[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v,
x] && !TrueQ[$UseGamma]
```

## Rubi steps

$$\begin{aligned}
\int F^{a+b(c+dx)} x^3 (e+fx)^2 dx &= \int (e^2 F^{a+bc+bdx} x^3 + 2ef F^{a+bc+bdx} x^4 + f^2 F^{a+bc+bdx} x^5) dx \\
&= e^2 \int F^{a+bc+bdx} x^3 dx + (2ef) \int F^{a+bc+bdx} x^4 dx + f^2 \int F^{a+bc+bdx} x^5 dx \\
&= \frac{e^2 F^{a+bc+bdx} x^3}{bd \log(F)} + \frac{2ef F^{a+bc+bdx} x^4}{bd \log(F)} + \frac{f^2 F^{a+bc+bdx} x^5}{bd \log(F)} - \frac{(3e^2) \int F^{a+bc+bdx} x^2 dx}{bd \log(F)} \\
&= -\frac{3e^2 F^{a+bc+bdx} x^2}{b^2 d^2 \log^2(F)} - \frac{8ef F^{a+bc+bdx} x^3}{b^2 d^2 \log^2(F)} - \frac{5f^2 F^{a+bc+bdx} x^4}{b^2 d^2 \log^2(F)} + \frac{e^2 F^{a+bc+bdx} x^3}{bd \log(F)} + \frac{2ef F^{a+bc+bdx} x^4}{bd \log(F)} \\
&= \frac{6e^2 F^{a+bc+bdx} x}{b^3 d^3 \log^3(F)} + \frac{24ef F^{a+bc+bdx} x^2}{b^3 d^3 \log^3(F)} + \frac{20f^2 F^{a+bc+bdx} x^3}{b^3 d^3 \log^3(F)} - \frac{3e^2 F^{a+bc+bdx} x^2}{b^2 d^2 \log^2(F)} - \frac{8ef F^{a+bc+bdx} x^3}{b^2 d^2 \log^2(F)} \\
&= -\frac{6e^2 F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} - \frac{48ef F^{a+bc+bdx} x}{b^4 d^4 \log^4(F)} - \frac{60f^2 F^{a+bc+bdx} x^2}{b^4 d^4 \log^4(F)} + \frac{6e^2 F^{a+bc+bdx} x}{b^3 d^3 \log^3(F)} + \frac{24ef F^{a+bc+bdx} x^2}{b^3 d^3 \log^3(F)} \\
&= \frac{48ef F^{a+bc+bdx}}{b^5 d^5 \log^5(F)} + \frac{120f^2 F^{a+bc+bdx} x}{b^5 d^5 \log^5(F)} - \frac{6e^2 F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} - \frac{48ef F^{a+bc+bdx} x}{b^4 d^4 \log^4(F)} - \frac{60f^2 F^{a+bc+bdx} x^2}{b^4 d^4 \log^4(F)} \\
&= -\frac{120f^2 F^{a+bc+bdx}}{b^6 d^6 \log^6(F)} + \frac{48ef F^{a+bc+bdx}}{b^5 d^5 \log^5(F)} + \frac{120f^2 F^{a+bc+bdx} x}{b^5 d^5 \log^5(F)} - \frac{6e^2 F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} - \frac{48ef F^{a+bc+bdx} x}{b^4 d^4 \log^4(F)} - \frac{60f^2 F^{a+bc+bdx} x^2}{b^4 d^4 \log^4(F)}
\end{aligned}$$

## Mathematica [A]

time = 0.27, size = 159, normalized size = 0.38

$$\frac{F^{a+b(c+dx)}(-120f^2 + 24bdf(2e + 5fx)\log(F) - 6b^2d^2(e^2 + 8efx + 10f^2x^2)\log^2(F) + 2b^3d^3x(3e^2 + 12efx + 10f^2x^2)\log^3(F) - b^4d^4x^2(3e^2 + 8efx + 5f^2x^2)\log^4(F) + b^5d^5x^3(e + fx)^2\log^5(F))}{b^6d^6\log^6(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(a + b*(c + d*x))*x^3*(e + f*x)^2,x]
```

```
[Out] (F^(a + b*(c + d*x))*(-120*f^2 + 24*b*d*f*(2*e + 5*f*x)*Log[F] - 6*b^2*d^2*
(e^2 + 8*e*f*x + 10*f^2*x^2)*Log[F]^2 + 2*b^3*d^3*x*(3*e^2 + 12*e*f*x + 10*
```



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c))\*x^3\*(f\*x+e)^2,x, algorithm="fricas")

[Out] ((b^5\*d^5\*f^2\*x^5 + 2\*b^5\*d^5\*f\*x^4\*e + b^5\*d^5\*x^3\*e^2)\*log(F)^5 - (5\*b^4\*d^4\*f^2\*x^4 + 8\*b^4\*d^4\*f\*x^3\*e + 3\*b^4\*d^4\*x^2\*e^2)\*log(F)^4 + 2\*(10\*b^3\*d^3\*f^2\*x^3 + 12\*b^3\*d^3\*f\*x^2\*e + 3\*b^3\*d^3\*x\*e^2)\*log(F)^3 - 6\*(10\*b^2\*d^2\*f^2\*x^2 + 8\*b^2\*d^2\*f\*x\*e + b^2\*d^2\*e^2)\*log(F)^2 - 120\*f^2 + 24\*(5\*b\*d\*f^2\*x + 2\*b\*d\*f\*e)\*log(F))\*F^(b\*d\*x + b\*c + a)/(b^6\*d^6\*log(F)^6)

Sympy [A]

time = 0.11, size = 323, normalized size = 0.78

$$\left\{ \frac{f^{5+4d} (b^5 d^5 e^2 \log(F)^5 + 2b^5 d^5 f e \log(F)^4 + b^5 d^5 f^2 \log(F)^3 - 3b^5 d^5 e^2 \log(F)^2 - 8b^5 d^5 f e \log(F) - 5b^5 d^5 f^2 \log(F) + 6b^5 d^5 e^2 \log(F) + 24b^5 d^5 f e \log(F) + 20b^5 d^5 f^2 \log(F) - 6b^5 d^5 e^2 \log(F) - 48b^5 d^5 f e \log(F) - 60b^5 d^5 f^2 \log(F) + 48b^5 d^5 e^2 \log(F) + 120b^5 d^5 f e \log(F) + 120b^5 d^5 f^2 \log(F) - 120f^2)}{b^6 d^6 \log(F)^6} \right\} \text{ for } b^6 d^6 \log(F)^6 \neq 0$$

$$\left\{ \frac{f^2}{4} + \frac{2fe}{5} + \frac{f^2 e}{6} \right\} \text{ otherwise}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c))\*x\*\*3\*(f\*x+e)\*\*2,x)

[Out] Piecewise((F\*\*(a + b\*(c + d\*x))\*(b\*\*5\*d\*\*5\*e\*\*2\*x\*\*3\*log(F)\*\*5 + 2\*b\*\*5\*d\*\*5\*e\*f\*x\*\*4\*log(F)\*\*5 + b\*\*5\*d\*\*5\*f\*\*2\*x\*\*5\*log(F)\*\*5 - 3\*b\*\*4\*d\*\*4\*e\*\*2\*x\*\*2\*log(F)\*\*4 - 8\*b\*\*4\*d\*\*4\*e\*f\*x\*\*3\*log(F)\*\*4 - 5\*b\*\*4\*d\*\*4\*f\*\*2\*x\*\*4\*log(F)\*\*4 + 6\*b\*\*3\*d\*\*3\*e\*\*2\*x\*log(F)\*\*3 + 24\*b\*\*3\*d\*\*3\*e\*f\*x\*\*2\*log(F)\*\*3 + 20\*b\*\*3\*d\*\*3\*f\*\*2\*x\*\*3\*log(F)\*\*3 - 6\*b\*\*2\*d\*\*2\*e\*\*2\*log(F)\*\*2 - 48\*b\*\*2\*d\*\*2\*e\*f\*x\*log(F)\*\*2 - 60\*b\*\*2\*d\*\*2\*f\*\*2\*x\*\*2\*log(F)\*\*2 + 48\*b\*d\*e\*f\*log(F) + 120\*b\*d\*f\*\*2\*x\*log(F) - 120\*f\*\*2)/(b\*\*6\*d\*\*6\*log(F)\*\*6), Ne(b\*\*6\*d\*\*6\*log(F)\*\*6, 0)), (e\*\*2\*x\*\*4/4 + 2\*e\*f\*x\*\*5/5 + f\*\*2\*x\*\*6/6, True))

Giac [C] Result contains complex when optimal does not.

time = 3.93, size = 9945, normalized size = 24.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c))\*x^3\*(f\*x+e)^2,x, algorithm="giac")

[Out] ((4\*(pi^3\*b^3\*d^3\*x^3\*sgn(F) - 3\*pi\*b^3\*d^3\*x^3\*log(abs(F)))^2\*sgn(F) - pi^3\*b^3\*d^3\*x^3 + 3\*pi\*b^3\*d^3\*x^3\*log(abs(F)))^2 + 6\*pi\*b^2\*d^2\*x^2\*log(abs(F))\*sgn(F) - 6\*pi\*b^2\*d^2\*x^2\*log(abs(F)) - 6\*pi\*b\*d\*x\*sgn(F) + 6\*pi\*b\*d\*x)\*(pi^3\*b^4\*d^4\*log(abs(F))\*sgn(F) - pi\*b^4\*d^4\*log(abs(F))^3\*sgn(F) - pi^3\*b^4\*d^4\*log(abs(F)) + pi\*b^4\*d^4\*log(abs(F))^3)/((pi^4\*b^4\*d^4\*sgn(F) - 6\*pi^2\*b^4\*d^4\*log(abs(F))^2\*sgn(F) - pi^4\*b^4\*d^4 + 6\*pi^2\*b^4\*d^4\*log(abs(F))^2 - 2\*b^4\*d^4\*log(abs(F))^4)^2 + 16\*(pi^3\*b^4\*d^4\*log(abs(F))\*sgn(F) - pi\*b^4\*d^4\*log(abs(F))^3\*sgn(F) - pi^3\*b^4\*d^4\*log(abs(F)) + pi\*b^4\*d^4\*log(abs(F))^3)^2) - (pi^4\*b^4\*d^4\*sgn(F) - 6\*pi^2\*b^4\*d^4\*log(abs(F))^2\*sgn(F) - pi^4\*b^4\*d^4 + 6\*pi^2\*b^4\*d^4\*log(abs(F))^2 - 2\*b^4\*d^4\*log(abs(F))^4)\*(3\*pi^2\*b^3\*d^3\*x^3\*log(abs(F))\*sgn(F) - 3\*pi^2\*b^3\*d^3\*x^3\*log(abs(F)) + 2\*b^3\*d^3\*x^3\*log(abs(F))



```

sgn(F) + 4*I*pi*b^4*d^4*log(abs(F))^3*sgn(F) - pi^4*b^4*d^4 + 4*I*pi^3*b^4*
d^4*log(abs(F)) + 6*pi^2*b^4*d^4*log(abs(F))^2 - 4*I*pi*b^4*d^4*log(abs(F))
^3 - 2*b^4*d^4*log(abs(F))^4)*e^(b*d*x*log(abs(F)) + b*c*log(abs(F)) + a*log
(abs(F)) + 2) - 2*((4*(pi^3*b^4*d^4*f*x^4*log(abs(F))*sgn(F) - pi*b^4*d^4
*f*x^4*log(abs(F))^3*sgn(F) - pi^3*b^4*d^4*f*x^4*log(abs(F)) + pi*b^4*d^4*f
*x^4*log(abs(F))^3 - pi^3*b^3*d^3*f*x^3*sgn(F) + 3*pi*b^3*d^3*f*x^3*log(abs
(F))^2*sgn(F) + pi^3*b^3*d^3*f*x^3 - 3*pi*b^3*d^3*f*x^3*log(abs(F))^2 - 6*pi
i*b^2*d^2*f*x^2*log(abs(F))*sgn(F) + 6*pi*b^2*d^2*f*x^2*log(abs(F)) + 6*pi*
b*d*f*x*sgn(F) - 6*pi*b*d*f*x)*(pi^5*b^5*d^5*sgn(F) - 10*pi^3*b^5*d^5*log(a
bs(F))^2*sgn(F) + 5*pi*b^5*d^5*log(abs(F))^4*sgn(F) - pi^5*b^5*d^5 + 10*pi^
3*b^5*d^5*log(abs(F))^2 - 5*pi*b^5*d^5*log(abs(...

```

**Mupad [B]**

time = 3.65, size = 249, normalized size = 0.60

$\frac{f^{6+4bd} (b^6 d^6 e^2 x^6 \ln(F)^5 + 24 b^5 d^5 e f x^5 \ln(F)^4 + 24 b^4 d^4 e^2 x^4 \ln(F)^3 + 24 b^3 d^3 e f x^3 \ln(F)^2 + 20 b^2 d^2 e^2 x^2 \ln(F)^2 + 20 b d e f x \ln(F)^2 - 6 b^2 d^2 e^2 \ln(F)^2 - 48 b d e f x \ln(F)^2 - 60 b d^2 f^2 x \ln(F)^2 + 48 b d e f \ln(F) + 120 b d f x \ln(F) - 120 f^2) e^{b d x \ln(F)}}{b^6 d^6 \ln(F)^6}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a + b\*(c + d\*x))\*x^3\*(e + f\*x)^2,x)

[Out] (F^(a + b\*c + b\*d\*x)\*(120\*b\*d\*f^2\*x\*log(F) - 6\*b^2\*d^2\*e^2\*log(F)^2 - 120\*f^2 + 6\*b^3\*d^3\*e^2\*x\*log(F)^3 - 3\*b^4\*d^4\*e^2\*x^2\*log(F)^4 + b^5\*d^5\*e^2\*x^3\*log(F)^5 - 60\*b^2\*d^2\*f^2\*x^2\*log(F)^2 + 20\*b^3\*d^3\*f^2\*x^3\*log(F)^3 - 5\*b^4\*d^4\*f^2\*x^4\*log(F)^4 + b^5\*d^5\*f^2\*x^5\*log(F)^5 + 48\*b\*d\*e\*f\*log(F) - 48\*b^2\*d^2\*e\*f\*x\*log(F)^2 + 24\*b^3\*d^3\*e\*f\*x^2\*log(F)^3 - 8\*b^4\*d^4\*e\*f\*x^3\*log(F)^4 + 2\*b^5\*d^5\*e\*f\*x^4\*log(F)^5))/(b^6\*d^6\*log(F)^6)

### 3.66 $\int F^{a+b(c+dx)} x^2 (e + fx)^2 dx$

**Optimal.** Leaf size=328

$$\frac{24f^2 F^{a+bc+bdx}}{b^5 d^5 \log^5(F)} - \frac{12ef F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} - \frac{24f^2 F^{a+bc+bdx} x}{b^4 d^4 \log^4(F)} + \frac{2e^2 F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} + \frac{12ef F^{a+bc+bdx} x}{b^3 d^3 \log^3(F)} + \frac{12f^2 F^{a+bc+bdx} x^2}{b^3 d^3 \log^3(F)} - \frac{2e^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)}$$

[Out]  $24f^2 F^{a+bc+bdx} x^2 / (b^5 d^5 \ln(F)^5) - 12ef F^{a+bc+bdx} x / (b^4 d^4 \ln(F)^4) + 12f^2 F^{a+bc+bdx} x^2 / (b^4 d^4 \ln(F)^4) + 2e^2 F^{a+bc+bdx} / (b^3 d^3 \ln(F)^3) + 12ef F^{a+bc+bdx} x / (b^3 d^3 \ln(F)^3) + 12f^2 F^{a+bc+bdx} x^2 / (b^3 d^3 \ln(F)^3) - 2e^2 F^{a+bc+bdx} x / (b^2 d^2 \ln(F)^2) - 6ef F^{a+bc+bdx} x^2 / (b^2 d^2 \ln(F)^2) - 4f^2 F^{a+bc+bdx} x^3 / (b^2 d^2 \ln(F)^2) + e^2 F^{a+bc+bdx} x^2 / (b d \ln(F)) + 2ef F^{a+bc+bdx} x^3 / (b d \ln(F)) + f^2 F^{a+bc+bdx} x^4 / (b d \ln(F))$

**Rubi [A]**

time = 0.37, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {2227, 2207, 2225}

$$\frac{24f^2 F^{a+bc+bdx}}{b^5 d^5 \log^5(F)} - \frac{12ef F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} - \frac{24f^2 F^{a+bc+bdx} x}{b^4 d^4 \log^4(F)} + \frac{2e^2 F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} + \frac{12ef F^{a+bc+bdx} x}{b^3 d^3 \log^3(F)} + \frac{12f^2 F^{a+bc+bdx} x^2}{b^3 d^3 \log^3(F)} - \frac{2e^2 F^{a+bc+bdx} x}{b^2 d^2 \log^2(F)} - \frac{6ef F^{a+bc+bdx} x^2}{b^2 d^2 \log^2(F)} - \frac{4f^2 F^{a+bc+bdx} x^3}{b^2 d^2 \log^2(F)} + \frac{e^2 F^{a+bc+bdx} x^2}{b d \log(F)} + \frac{2ef F^{a+bc+bdx} x^3}{b d \log(F)} + \frac{f^2 F^{a+bc+bdx} x^4}{b d \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x))\*x^2\*(e + f\*x)^2, x]

[Out]  $(24f^2 F^{a+bc+bdx} x^2) / (b^5 d^5 \text{Log}[F]^5) - (12ef F^{a+bc+bdx} x) / (b^4 d^4 \text{Log}[F]^4) - (24f^2 F^{a+bc+bdx} x^2) / (b^4 d^4 \text{Log}[F]^4) + (2e^2 F^{a+bc+bdx}) / (b^3 d^3 \text{Log}[F]^3) + (12ef F^{a+bc+bdx} x) / (b^3 d^3 \text{Log}[F]^3) + (12f^2 F^{a+bc+bdx} x^2) / (b^3 d^3 \text{Log}[F]^3) - (2e^2 F^{a+bc+bdx} x) / (b^2 d^2 \text{Log}[F]^2) - (6ef F^{a+bc+bdx} x^2) / (b^2 d^2 \text{Log}[F]^2) - (4f^2 F^{a+bc+bdx} x^3) / (b^2 d^2 \text{Log}[F]^2) + (e^2 F^{a+bc+bdx} x^2) / (b d \text{Log}[F]) + (2ef F^{a+bc+bdx} x^3) / (b d \text{Log}[F]) + (f^2 F^{a+bc+bdx} x^4) / (b d \text{Log}[F])$

Rule 2207

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(c + d\*x)^m\*((b F^(g\*(e + f\*x)))^n/(f\*g\*n\*Log[F])), x] - Dist[d\*(m/(f\*g\*n\*Log[F])), Int[(c + d\*x)^(m-1)\*(b F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

Rule 2225

Int[(F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]



## Rule 2227

`Int[(F_)^((c_.)*(v_))*(u_), x_Symbol] := Int[ExpandIntegrand[F^(c*ExpandToSum[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

## Rubi steps

$$\begin{aligned}
\int F^{a+b(c+dx)} x^2 (e+fx)^2 dx &= \int (e^2 F^{a+bc+bdx} x^2 + 2ef F^{a+bc+bdx} x^3 + f^2 F^{a+bc+bdx} x^4) dx \\
&= e^2 \int F^{a+bc+bdx} x^2 dx + (2ef) \int F^{a+bc+bdx} x^3 dx + f^2 \int F^{a+bc+bdx} x^4 dx \\
&= \frac{e^2 F^{a+bc+bdx} x^2}{bd \log(F)} + \frac{2ef F^{a+bc+bdx} x^3}{bd \log(F)} + \frac{f^2 F^{a+bc+bdx} x^4}{bd \log(F)} - \frac{(2e^2) \int F^{a+bc+bdx} x dx}{bd \log(F)} \\
&= -\frac{2e^2 F^{a+bc+bdx} x}{b^2 d^2 \log^2(F)} - \frac{6ef F^{a+bc+bdx} x^2}{b^2 d^2 \log^2(F)} - \frac{4f^2 F^{a+bc+bdx} x^3}{b^2 d^2 \log^2(F)} + \frac{e^2 F^{a+bc+bdx} x^2}{bd \log(F)} + \frac{2ef F^{a+bc+bdx} x^3}{b^2 d^2 \log^2(F)} \\
&= \frac{2e^2 F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} + \frac{12ef F^{a+bc+bdx} x}{b^3 d^3 \log^3(F)} + \frac{12f^2 F^{a+bc+bdx} x^2}{b^3 d^3 \log^3(F)} - \frac{2e^2 F^{a+bc+bdx} x}{b^2 d^2 \log^2(F)} - \frac{6ef F^{a+bc+bdx} x^2}{b^2 d^2 \log^2(F)} \\
&= -\frac{12ef F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} - \frac{24f^2 F^{a+bc+bdx} x}{b^4 d^4 \log^4(F)} + \frac{2e^2 F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} + \frac{12ef F^{a+bc+bdx} x}{b^3 d^3 \log^3(F)} + \frac{12f^2 F^{a+bc+bdx} x^2}{b^3 d^3 \log^3(F)} \\
&= \frac{24f^2 F^{a+bc+bdx}}{b^5 d^5 \log^5(F)} - \frac{12ef F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} - \frac{24f^2 F^{a+bc+bdx} x}{b^4 d^4 \log^4(F)} + \frac{2e^2 F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} + \frac{12ef F^{a+bc+bdx} x}{b^3 d^3 \log^3(F)} + \frac{12f^2 F^{a+bc+bdx} x^2}{b^3 d^3 \log^3(F)}
\end{aligned}$$

**Mathematica** [A]

time = 0.24, size = 121, normalized size = 0.37

$$\frac{F^{a+b(c+dx)} (24f^2 - 12bdf(e+2fx) \log(F) + 2b^2d^2(e^2 + 6efx + 6f^2x^2) \log^2(F) - 2b^3d^3x(e^2 + 3efx + 2f^2x^2) \log^3(F) + b^4d^4x^2(e+fx)^2 \log^4(F))}{b^5d^5 \log^5(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x))\*x^2\*(e + f\*x)^2,x]

[Out] (F^(a + b\*(c + d\*x))\*(24\*f^2 - 12\*b\*d\*f\*(e + 2\*f\*x)\*Log[F] + 2\*b^2\*d^2\*(e^2 + 6\*e\*f\*x + 6\*f^2\*x^2)\*Log[F]^2 - 2\*b^3\*d^3\*x\*(e^2 + 3\*e\*f\*x + 2\*f^2\*x^2)\*Log[F]^3 + b^4\*d^4\*x^2\*(e + f\*x)^2\*Log[F]^4))/(b^5\*d^5\*Log[F]^5)

**Maple** [A]

time = 0.06, size = 197, normalized size = 0.60

method	result
gospers	$\frac{(\ln(F)^4 b^4 d^4 f^2 x^4 + 2 \ln(F)^4 b^4 d^4 e f x^3 + \ln(F)^4 b^4 d^4 e^2 x^2 - 4 \ln(F)^3 b^3 d^3 f^2 x^3 - 6 \ln(F)^3 b^3 d^3 e f x^2 - 2 \ln(F)^3 b^3 d^3 e^2 x + 12 \ln(F)^2 b^2 d^2 f^2 x^2 + 24 \ln(F)^2 b^2 d^2 e f x + 12 \ln(F)^2 b^2 d^2 e^2 x - 4 \ln(F) b^2 d^2 f^2 x^2 - 12 \ln(F) b^2 d^2 e f x - 6 \ln(F) b^2 d^2 e^2 x + 2 b^2 d^2 f^2 x + 2 b^2 d^2 e f x + b^2 d^2 e^2 x - b^2 d^2) \ln(F)^5 b^5 d^5}{b^5 d^5 \ln(F)^5}$



**Sympy** [A]

time = 0.10, size = 260, normalized size = 0.79

$$\begin{cases} \frac{F^{a+4(c+d)}(b^4d^4e^2x^2\log(F)^4+2b^4d^4efx^3\log(F)^4+b^4d^4f^2x^4\log(F)^4-2b^4d^4e^2x\log(F)^3-6b^3d^4efx^2\log(F)^3-4b^3d^4f^2x^3\log(F)^3+2b^3d^4e^2\log(F)^2+12b^2d^4efx\log(F)^2+12b^2d^4f^2x^2\log(F)^2-12bde^2f\log(F)-24bd^2e^2x\log(F)+24f^2)}{b^4d^4\log(F)^4} & \text{for } b^4d^4\log(F)^4 \neq 0 \\ \frac{e^2x^3}{3} + \frac{efx^4}{2} + \frac{f^2x^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c))\*x\*\*2\*(f\*x+e)\*\*2,x)

[Out] Piecewise((F\*\*(a + b\*(c + d\*x))\*(b\*\*4\*d\*\*4\*e\*\*2\*x\*\*2\*log(F)\*\*4 + 2\*b\*\*4\*d\*\*4\*e\*f\*x\*\*3\*log(F)\*\*4 + b\*\*4\*d\*\*4\*f\*\*2\*x\*\*4\*log(F)\*\*4 - 2\*b\*\*3\*d\*\*3\*e\*\*2\*x\*log(F)\*\*3 - 6\*b\*\*3\*d\*\*3\*e\*f\*x\*\*2\*log(F)\*\*3 - 4\*b\*\*3\*d\*\*3\*f\*\*2\*x\*\*3\*log(F)\*\*3 + 2\*b\*\*2\*d\*\*2\*e\*\*2\*log(F)\*\*2 + 12\*b\*\*2\*d\*\*2\*e\*f\*x\*log(F)\*\*2 + 12\*b\*\*2\*d\*\*2\*f\*\*2\*x\*\*2\*log(F)\*\*2 - 12\*b\*d\*e\*f\*log(F) - 24\*b\*d\*f\*\*2\*x\*log(F) + 24\*f\*\*2)/(b\*\*5\*d\*\*5\*log(F)\*\*5), Ne(b\*\*5\*d\*\*5\*log(F)\*\*5, 0)), (e\*\*2\*x\*\*3/3 + e\*f\*x\*\*4/2 + f\*\*2\*x\*\*5/5, True))

**Giac** [C] Result contains complex when optimal does not.

time = 3.86, size = 7057, normalized size = 21.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c))\*x^2\*(f\*x+e)^2,x, algorithm="giac")

[Out] (((3\*pi^2\*b^3\*d^3\*log(abs(F))\*sgn(F) - 3\*pi^2\*b^3\*d^3\*log(abs(F)) + 2\*b^3\*d^3\*log(abs(F))^3)\*(pi^2\*b^2\*d^2\*x^2\*sgn(F) - pi^2\*b^2\*d^2\*x^2 + 2\*b^2\*d^2\*x^2\*log(abs(F))^2 - 4\*b\*d\*x\*log(abs(F)) + 4)/((pi^3\*b^3\*d^3\*sgn(F) - 3\*pi\*b^3\*d^3\*log(abs(F))^2)^2 + (3\*pi^2\*b^3\*d^3\*log(abs(F))\*sgn(F) - 3\*pi^2\*b^3\*d^3\*log(abs(F)) + 2\*b^3\*d^3\*log(abs(F))^3)^2) - 2\*(pi^3\*b^3\*d^3\*sgn(F) - 3\*pi\*b^3\*d^3\*log(abs(F))^2)\*sgn(F) - pi^3\*b^3\*d^3 + 3\*pi\*b^3\*d^3\*log(abs(F))^2)\*(pi\*b^2\*d^2\*x^2\*log(abs(F))\*sgn(F) - pi\*b^2\*d^2\*x^2\*log(abs(F)) - pi\*b\*d\*x\*sgn(F) + pi\*b\*d\*x)/((pi^3\*b^3\*d^3\*sgn(F) - 3\*pi\*b^3\*d^3\*log(abs(F))^2)\*sgn(F) - pi^3\*b^3\*d^3 + 3\*pi\*b^3\*d^3\*log(abs(F))^2)^2 + (3\*pi^2\*b^3\*d^3\*log(abs(F))\*sgn(F) - 3\*pi^2\*b^3\*d^3\*log(abs(F)) + 2\*b^3\*d^3\*log(abs(F))^3)^2)\*cos(-1/2\*pi\*b\*d\*x\*sgn(F) + 1/2\*pi\*b\*d\*x - 1/2\*pi\*b\*c\*sgn(F) + 1/2\*pi\*b\*c - 1/2\*pi\*a\*sgn(F) + 1/2\*pi\*a) + ((pi^3\*b^3\*d^3\*sgn(F) - 3\*pi\*b^3\*d^3\*log(abs(F))^2)\*sgn(F) - pi^3\*b^3\*d^3 + 3\*pi\*b^3\*d^3\*log(abs(F))^2)\*(pi^2\*b^2\*d^2\*x^2\*sgn(F) - pi^2\*b^2\*d^2\*x^2 + 2\*b^2\*d^2\*x^2\*log(abs(F))^2 - 4\*b\*d\*x\*log(abs(F)) + 4)/((pi^3\*b^3\*d^3\*sgn(F) - 3\*pi\*b^3\*d^3\*log(abs(F))^2)\*sgn(F) - pi^3\*b^3\*d^3 + 3\*pi\*b^3\*d^3\*log(abs(F))^2)^2 + (3\*pi^2\*b^3\*d^3\*log(abs(F))\*sgn(F) - 3\*pi^2\*b^3\*d^3\*log(abs(F)) + 2\*b^3\*d^3\*log(abs(F))^3)^2) + 2\*(3\*pi^2\*b^3\*d^3\*log(abs(F))\*sgn(F) - 3\*pi^2\*b^3\*d^3\*log(abs(F)) + 2\*b^3\*d^3\*log(abs(F))^3)\*(pi\*b^2\*d^2\*x^2\*log(abs(F))\*sgn(F) - pi\*b^2\*d^2\*x^2\*log(abs(F)) - pi\*b\*d\*x\*sgn(F) + pi\*b\*d\*x)/((pi

$$\begin{aligned}
& ^3b^3d^3\text{sgn}(F) - 3\pi b^3d^3\log(\text{abs}(F))^2\text{sgn}(F) - \pi^3b^3d^3 + 3\pi \\
& b^3d^3\log(\text{abs}(F))^2)^2 + (3\pi^2b^3d^3\log(\text{abs}(F))\text{sgn}(F) - 3\pi^2b^3 \\
& d^3\log(\text{abs}(F)) + 2b^3d^3\log(\text{abs}(F))^3)^2)\sin(-1/2\pi b d x \text{sgn}(F) + \\
& 1/2\pi b d x - 1/2\pi b c \text{sgn}(F) + 1/2\pi b c - 1/2\pi a \text{sgn}(F) + 1/2\pi a) \\
& )e^{(b d x \log(\text{abs}(F)) + b c \log(\text{abs}(F)) + a \log(\text{abs}(F)) + 2) - 2I * ((-I\pi \\
& ^2b^2d^2x^2\text{sgn}(F) + 2\pi b^2d^2x^2\log(\text{abs}(F))\text{sgn}(F) + I\pi^2b^2d^ \\
& 2x^2 - 2\pi b^2d^2x^2\log(\text{abs}(F)) - 2I b^2d^2x^2\log(\text{abs}(F))^2 - 2\pi \\
& b d x \text{sgn}(F) + 2\pi b d x + 4I b d x \log(\text{abs}(F)) - 4I) e^{(1/2I\pi b d x \\
& * \text{sgn}(F) - 1/2I\pi b d x + 1/2I\pi b c \text{sgn}(F) - 1/2I\pi b c + 1/2I\pi a * \\
& \text{sgn}(F) - 1/2I\pi a) / (-4I\pi^3b^3d^3\text{sgn}(F) + 12\pi^2b^3d^3\log(\text{abs}(F) \\
& )\text{sgn}(F) + 12I\pi b^3d^3\log(\text{abs}(F))^2\text{sgn}(F) + 4I\pi^3b^3d^3 - 12\pi^2 \\
& b^3d^3\log(\text{abs}(F)) - 12I\pi b^3d^3\log(\text{abs}(F))^2 + 8b^3d^3\log(\text{abs}(F) \\
& ))^3) - (-I\pi^2b^2d^2x^2\text{sgn}(F) - 2\pi b^2d^2x^2\log(\text{abs}(F))\text{sgn}(F) + \\
& I\pi^2b^2d^2x^2 + 2\pi b^2d^2x^2\log(\text{abs}(F)) - 2I b^2d^2x^2\log(\text{abs}(F) \\
& )^2 + 2\pi b d x \text{sgn}(F) - 2\pi b d x + 4I b d x \log(\text{abs}(F)) - 4I) e^{( \\
& -1/2I\pi b d x \text{sgn}(F) + 1/2I\pi b d x - 1/2I\pi b c \text{sgn}(F) + 1/2I\pi b c \\
& - 1/2I\pi a \text{sgn}(F) + 1/2I\pi a) / (4I\pi^3b^3d^3\text{sgn}(F) + 12\pi^2b^3d^3 \\
& \log(\text{abs}(F))\text{sgn}(F) - 12I\pi b^3d^3\log(\text{abs}(F))^2\text{sgn}(F) - 4I\pi^3b^3 \\
& d^3 - 12\pi^2b^3d^3\log(\text{abs}(F)) + 12I\pi b^3d^3\log(\text{abs}(F))^2 + 8b^3 \\
& d^3\log(\text{abs}(F))^3)} e^{(b d x \log(\text{abs}(F)) + b c \log(\text{abs}(F)) + a \log(\text{abs}(F)) \\
& + 2) - 2 * (((3\pi^2b^3d^3f x^3\log(\text{abs}(F))\text{sgn}(F) - 3\pi^2b^3d^3f x^3 \\
& * \log(\text{abs}(F)) + 2b^3d^3f x^3\log(\text{abs}(F))^3 - 3\pi^2b^2d^2f x^2\text{sgn}(F) \\
& + 3\pi^2b^2d^2f x^2 - 6b^2d^2f x^2\log(\text{abs}(F))^2 + 12b d f x \log(\text{abs}( \\
& F)) - 12f) * (\pi^4b^4d^4\text{sgn}(F) - 6\pi^2b^4d^4\log(\text{abs}(F))^2\text{sgn}(F) - \pi \\
& ^4b^4d^4 + 6\pi^2b^4d^4\log(\text{abs}(F))^2 - 2b^4d^4\log(\text{abs}(F))^4) / ((\pi^4 \\
& b^4d^4\text{sgn}(F) - 6\pi^2b^4d^4\log(\text{abs}(F))^2\text{sgn}(F) - \pi^4b^4d^4 + 6\pi \\
& ^2b^4d^4\log(\text{abs}(F))^2 - 2b^4d^4\log(\text{abs}(F))^4)^2 + 16 * (\pi^3b^4d^4 \log(\text{abs}(F))\text{sgn}(F) - \pi b^4d^4 \\
& \log(\text{abs}(F))^3\text{sgn}(F) - \pi^3b^4d^4\log(\text{abs}(F)) + \pi b^4d^4\log(\text{abs}(F)) \\
& + \pi b^4d^4\log(\text{abs}(F))^3)^2) - 4 * (\pi^3b^3d^3f x^3\text{sgn}(F) - 3\pi b^3d^3 \\
& f x^3\log(\text{abs}(F))^2\text{sgn}(F) - \pi^3b^3d^3f x^3 + 3\pi b^3d^3f x^3 * \log(\text{abs}(F))^2 \\
& + 6\pi b^2d^2f x^2\log(\text{abs}(F))\text{sgn}(F) - 6\pi b^2d^2f x^2 * \log(\text{abs}(F)) - 6\pi b \\
& d f x \text{sgn}(F) + 6\pi b d f x) * (\pi^3b^4d^4\log(\text{abs}(F)) \\
& * \text{sgn}(F) - \pi b^4d^4\log(\text{abs}(F))^3\text{sgn}(F) - \pi^3b^4d^4\log(\text{abs}(F)) + \pi b \\
& ^4d^4\log(\text{abs}(F))^3) / ((\pi^4b^4d^4\text{sgn}(F) - 6\pi^2b^4d^4\log(\text{abs}(F))^2 * \\
& \text{sgn}(F) - \pi^4b^4d^4 + 6\pi^2b^4d^4\log(\text{abs}(F))^2 - 2b^4d^4\log(\text{abs}(F) \\
& )^4)^2 + 16 * (\pi^3b^4d^4\log(\text{abs}(F))\text{sgn}(F) - \pi b^4d^4\log(\text{abs}(F))^3 \text{sgn}( \\
& F) - \pi^3b^4d^4\log(\text{abs}(F)) + \pi b^4d^4\log(\text{abs}(F))^3)^2) * \cos(-1/2\pi b \\
& d x \text{sgn}(F) + 1/2\pi b d x - 1/2\pi b c \text{sgn}(F) + 1/2\pi b c - 1/2\pi a \text{sgn}( \\
& F) + 1/2\pi a) - ((\pi^3b^3d^3f x^3\text{sgn}(F) - 3\pi b^3d^3f x^3\log(\text{abs}(F) \\
& )^2\text{sgn}(F) - \pi^3b^3d^3f x^3 + 3\pi b^3d^3f x^3\log(\text{abs}(F))^2 + 6\pi \\
& b^2d^2f x^2\log(\text{abs}(F))\text{sgn}(F) - 6\pi b^2d^2f x^2\log(\text{abs}(F)) - 6\pi b \\
& d f x \text{sgn}(F) + 6\pi b d f x) * (\pi^4b^4d^4\text{sgn}(F) - 6\pi^2b^4d^4\log(\text{abs}( \\
& F))^2\text{sgn}(F) - \pi^4b^4d^4 + 6\pi^2b^4d^4\log(\text{abs}(F))^2 - 2b^4d^4\log \\
& (\text{abs}(F))^4) / ((\pi^4b^4d^4\text{sgn}(F) - 6\pi^2b^4d^4\log(\text{abs}(F))^2\text{sgn}(F) - \pi \\
& ^4b^4d^4 + 6\pi^2b^4d^4\log(\text{abs}(F))^2 - 2b^4d^4\log(\text{abs}(F))^4)^2 + 1
\end{aligned}$$

$6*(\pi^3*b^4*d^4*\log(\text{abs}(F))*\text{sgn}(F) - \pi*b^4*d^4*\log(\text{abs}(F))^3*\text{sgn}(F) - \pi^3*b^4*d^4*\log(\text{abs}(F)) + \pi*b^4*d^4*\log(\text{abs}(F))^3...$

**Mupad [B]**

time = 3.56, size = 196, normalized size = 0.60

$$\frac{F^{5+3+d*x} (b^4 d^4 e^2 x^2 \ln(F)^4 + 2 b^4 d^4 e f x^2 \ln(F)^4 + b^4 d^4 f^2 x^4 \ln(F)^4 - 2 b^4 d^4 e^2 x \ln(F)^3 - 6 b^4 d^4 e f x^2 \ln(F)^3 - 4 b^4 d^4 f^2 x^3 \ln(F)^3 + 2 b^4 d^4 e^2 \ln(F)^2 + 12 b^4 d^4 e f x \ln(F)^2 + 12 b^4 d^4 f^2 x^2 \ln(F)^2 - 12 b d e f \ln(F) - 24 b d f^2 x \ln(F) + 24 f^2)}{b^5 d^5 \ln(F)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a + b*(c + d*x))*x^2*(e + f*x)^2,x)`

[Out] `(F^(a + b*c + b*d*x)*(24*f^2 + 2*b^2*d^2*e^2*log(F)^2 - 24*b*d*f^2*x*log(F) - 2*b^3*d^3*e^2*x*log(F)^3 + b^4*d^4*e^2*x^2*log(F)^4 + 12*b^2*d^2*f^2*x^2*log(F)^2 - 4*b^3*d^3*f^2*x^3*log(F)^3 + b^4*d^4*f^2*x^4*log(F)^4 - 12*b*d*e*f*log(F) + 12*b^2*d^2*e*f*x*log(F)^2 - 6*b^3*d^3*e*f*x^2*log(F)^3 + 2*b^4*d^4*e*f*x^3*log(F)^4))/(b^5*d^5*log(F)^5)`

### 3.67 $\int F^{a+b(c+dx)} x(e+fx)^2 dx$

**Optimal.** Leaf size=242

$$-\frac{6f^2 F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} + \frac{4ef F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} + \frac{6f^2 F^{a+bc+bdx} x}{b^3 d^3 \log^3(F)} - \frac{e^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} - \frac{4ef F^{a+bc+bdx} x}{b^2 d^2 \log^2(F)} - \frac{3f^2 F^{a+bc+bdx} x^2}{b^2 d^2 \log^2(F)} + \frac{e^2 F^{a+bc+bdx}}{bd \log(F)}$$

[Out]  $-6*f^2*F^{(b*d*x+b*c+a)}/b^4/d^4/\ln(F)^4+4*e*f*F^{(b*d*x+b*c+a)}/b^3/d^3/\ln(F)^3+6*f^2*F^{(b*d*x+b*c+a)*x}/b^3/d^3/\ln(F)^3-e^2*F^{(b*d*x+b*c+a)}/b^2/d^2/\ln(F)^2-4*e*f*F^{(b*d*x+b*c+a)*x}/b^2/d^2/\ln(F)^2-3*f^2*F^{(b*d*x+b*c+a)*x^2}/b^2/d^2/\ln(F)^2+e^2*F^{(b*d*x+b*c+a)*x}/b/d/\ln(F)+2*e*f*F^{(b*d*x+b*c+a)*x^2}/b/d/\ln(F)+f^2*F^{(b*d*x+b*c+a)*x^3}/b/d/\ln(F)$

**Rubi [A]**

time = 0.25, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {2227, 2207, 2225}

$$-\frac{6f^2 F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} + \frac{4ef F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} + \frac{6f^2 x F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} - \frac{e^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} - \frac{4ef x F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} - \frac{3f^2 x^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + \frac{e^2 x F^{a+bc+bdx}}{bd \log(F)} + \frac{2ef x^2 F^{a+bc+bdx}}{bd \log(F)} + \frac{f^2 x^3 F^{a+bc+bdx}}{bd \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x))\*x\*(e + f\*x)^2,x]

[Out]  $(-6*f^2*F^{(a + b*c + b*d*x)})/(b^4*d^4*Log[F]^4) + (4*e*f*F^{(a + b*c + b*d*x)})/(b^3*d^3*Log[F]^3) + (6*f^2*F^{(a + b*c + b*d*x)*x})/(b^3*d^3*Log[F]^3) - (e^2*F^{(a + b*c + b*d*x)})/(b^2*d^2*Log[F]^2) - (4*e*f*F^{(a + b*c + b*d*x)*x})/(b^2*d^2*Log[F]^2) - (3*f^2*F^{(a + b*c + b*d*x)*x^2})/(b^2*d^2*Log[F]^2) + (e^2*F^{(a + b*c + b*d*x)*x})/(b*d*Log[F]) + (2*e*f*F^{(a + b*c + b*d*x)*x^2})/(b*d*Log[F]) + (f^2*F^{(a + b*c + b*d*x)*x^3})/(b*d*Log[F])$

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x]
/; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol]
:> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2227

```
Int[(F_)^((c_.)*(v_))*(u_), x_Symbol]
:> Int[ExpandIntegrand[F^(c*ExpandToSum[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v,
```

x] && !TrueQ[\$UseGamma]

Rubi steps

$$\begin{aligned}
 \int F^{a+b(c+dx)} x(e+fx)^2 dx &= \int (e^2 F^{a+bc+bdx} x + 2ef F^{a+bc+bdx} x^2 + f^2 F^{a+bc+bdx} x^3) dx \\
 &= e^2 \int F^{a+bc+bdx} x dx + (2ef) \int F^{a+bc+bdx} x^2 dx + f^2 \int F^{a+bc+bdx} x^3 dx \\
 &= \frac{e^2 F^{a+bc+bdx} x}{bd \log(F)} + \frac{2ef F^{a+bc+bdx} x^2}{bd \log(F)} + \frac{f^2 F^{a+bc+bdx} x^3}{bd \log(F)} - \frac{e^2 \int F^{a+bc+bdx} dx}{bd \log(F)} - \frac{(4ef) \int F^{a+bc+bdx} x dx}{bd \log(F)} - \frac{f^2 \int F^{a+bc+bdx} x^2 dx}{bd \log(F)} \\
 &= -\frac{e^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} - \frac{4ef F^{a+bc+bdx} x}{b^2 d^2 \log^2(F)} - \frac{3f^2 F^{a+bc+bdx} x^2}{b^2 d^2 \log^2(F)} + \frac{e^2 F^{a+bc+bdx} x}{bd \log(F)} + \frac{2ef F^{a+bc+bdx} x^2}{bd \log(F)} + \frac{f^2 F^{a+bc+bdx} x^3}{bd \log(F)} \\
 &= \frac{4ef F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} + \frac{6f^2 F^{a+bc+bdx} x}{b^3 d^3 \log^3(F)} - \frac{e^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} - \frac{4ef F^{a+bc+bdx} x}{b^2 d^2 \log^2(F)} - \frac{3f^2 F^{a+bc+bdx} x^2}{b^2 d^2 \log^2(F)} \\
 &= -\frac{6f^2 F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} + \frac{4ef F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} + \frac{6f^2 F^{a+bc+bdx} x}{b^3 d^3 \log^3(F)} - \frac{e^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} - \frac{4ef F^{a+bc+bdx} x}{b^2 d^2 \log^2(F)} - \frac{3f^2 F^{a+bc+bdx} x^2}{b^2 d^2 \log^2(F)}
 \end{aligned}$$

Mathematica [A]

time = 0.20, size = 91, normalized size = 0.38

$$\frac{F^{a+b(c+dx)}(-6f^2 + 2bdf(2e + 3fx) \log(F) - b^2 d^2(e^2 + 4efx + 3f^2 x^2) \log^2(F) + b^3 d^3 x(e + fx)^2 \log^3(F))}{b^4 d^4 \log^4(F)}$$

Antiderivative was successfully verified.

[In] Integrate[F^(a + b\*(c + d\*x))\*x\*(e + f\*x)^2,x]

[Out] (F^(a + b\*(c + d\*x))\*(-6\*f^2 + 2\*b\*d\*f\*(2\*e + 3\*f\*x)\*Log[F] - b^2\*d^2\*(e^2 + 4\*e\*f\*x + 3\*f^2\*x^2)\*Log[F]^2 + b^3\*d^3\*x\*(e + f\*x)^2\*Log[F]^3))/(b^4\*d^4 \*Log[F]^4)

Maple [A]

time = 0.06, size = 144, normalized size = 0.60

method	result
gospers	$\frac{(\ln(F)^3 b^3 d^3 f^2 x^3 + 2 \ln(F)^3 b^3 d^3 e f x^2 + \ln(F)^3 b^3 d^3 e^2 x - 3 \ln(F)^2 b^2 d^2 f^2 x^2 - 4 \ln(F)^2 b^2 d^2 e f x - \ln(F)^2 b^2 d^2 e^2 + 6 \ln(F) b d f^2 x + 4 e f \ln(F))}{\ln(F)^4 b^4 d^4}$
risch	$\frac{(\ln(F)^3 b^3 d^3 f^2 x^3 + 2 \ln(F)^3 b^3 d^3 e f x^2 + \ln(F)^3 b^3 d^3 e^2 x - 3 \ln(F)^2 b^2 d^2 f^2 x^2 - 4 \ln(F)^2 b^2 d^2 e f x - \ln(F)^2 b^2 d^2 e^2 + 6 \ln(F) b d f^2 x + 4 e f \ln(F))}{\ln(F)^4 b^4 d^4}$
meijerg	$\frac{F^{cb+a} f^2 \left( 6 - \frac{(-4b^3 d^3 x^3 \ln(F)^3 + 12b^2 d^2 x^2 \ln(F)^2 - 24bdx \ln(F) + 24) e^{bdx \ln(F)}}{4} \right)}{\ln(F)^4 b^4 d^4} - \frac{2F^{cb+a} f e \left( 2 - \frac{(3b^2 d^2 x^2 \ln(F)^2 - 6bdx \ln(F) + 6) e^{bdx \ln(F)}}{3} \right)}{b^3 d^3 \ln(F)^3}$

norman	$\frac{f^2 x^3 e^{(a+b(dx+c)) \ln(F)}}{bd \ln(F)} + \frac{(\ln(F)^2 b^2 d^2 e^2 - 4ef \ln(F) bd + 6f^2) x e^{(a+b(dx+c)) \ln(F)}}{\ln(F)^3 b^3 d^3} + \frac{f(2 \ln(F) bde - 3f) x^2 e^{(a+b(dx+c)) \ln(F)}}{\ln(F)^2 b^2 d^2} - \dots$
--------	-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c))*x*(f*x+e)^2,x,method=_RETURNVERBOSE)`

[Out]  $(\ln(F)^3 b^3 d^3 f^2 x^3 + 2 \ln(F)^3 b^3 d^3 e f x^2 + \ln(F)^3 b^3 d^3 e^2 x - 3 \ln(F)^2 b^2 d^2 f^2 x^2 - 4 \ln(F)^2 b^2 d^2 e f x - \ln(F)^2 b^2 d^2 e^2 + 6 \ln(F) b^2 d f^2 x + 4 e f \ln(F) b^2 d - 6 f^2) F^{(b^2 d x^2 + b^2 c + a)} / \ln(F)^4 b^4 d^4$

**Maxima** [A]

time = 0.43, size = 198, normalized size = 0.82

$$\frac{(F^{bc+a} b dx \log(F) - F^{bc+a}) e^{(bdx \log(F)+2)}}{b^2 d^2 \log(F)^2} + \frac{2(F^{bc+a} b^2 d^2 x^2 \log(F)^2 - 2 F^{bc+a} b dx \log(F) + 2 F^{bc+a}) f e^{(bdx \log(F)+1)}}{b^3 d^3 \log(F)^3} + \frac{(F^{bc+a} b^3 d^3 x^3 \log(F)^3 - 3 F^{bc+a} b^2 d^2 x^2 \log(F)^2 + 6 F^{bc+a} b dx \log(F) - 6 F^{bc+a}) F^{bdx f^2}}{b^4 d^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c))*x*(f*x+e)^2,x, algorithm="maxima")`

[Out]  $(F^{(b^2 c + a)} b^2 d x \log(F) - F^{(b^2 c + a)}) e^{(b^2 d x \log(F) + 2)} / (b^2 d^2 \log(F)^2) + 2(F^{(b^2 c + a)} b^2 d^2 x^2 \log(F)^2 - 2 F^{(b^2 c + a)} b^2 d x \log(F) + 2 F^{(b^2 c + a)}) f e^{(b^2 d x \log(F) + 1)} / (b^3 d^3 \log(F)^3) + (F^{(b^2 c + a)} b^3 d^3 x^3 \log(F)^3 - 3 F^{(b^2 c + a)} b^2 d^2 x^2 \log(F)^2 + 6 F^{(b^2 c + a)} b^2 d x \log(F) - 6 F^{(b^2 c + a)}) F^{(b^2 d x)} f^2 / (b^4 d^4 \log(F)^4)$

**Fricas** [A]

time = 0.39, size = 133, normalized size = 0.55

$$\frac{((b^3 d^3 f^2 x^3 + 2 b^3 d^3 f x^2 e + b^3 d^3 x e^2) \log(F)^3 - (3 b^2 d^2 f^2 x^2 + 4 b^2 d^2 f x e + b^2 d^2 e^2) \log(F)^2 - 6 f^2 + 2(3 b d f^2 x + 2 b d f e) \log(F)) F^{b d x + b c + a}}{b^4 d^4 \log(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c))*x*(f*x+e)^2,x, algorithm="fricas")`

[Out]  $((b^3 d^3 f^2 x^3 + 2 b^3 d^3 f x^2 e + b^3 d^3 x e^2) \log(F)^3 - (3 b^2 d^2 f^2 x^2 + 4 b^2 d^2 f x e + b^2 d^2 e^2) \log(F)^2 - 6 f^2 + 2(3 b^2 d f^2 x + 2 b^2 d f e) \log(F)) F^{(b^2 d x + b^2 c + a)} / (b^4 d^4 \log(F)^4)$

**Sympy** [A]

time = 0.08, size = 199, normalized size = 0.82

$$\begin{cases} \frac{F^{a+b(c+dx)} (b^3 d^3 e^2 x \log(F)^3 + 2 b^3 d^3 e f x^2 \log(F)^3 + b^3 d^3 f^2 x^3 \log(F)^3 - b^2 d^2 e^2 \log(F)^2 - 4 b^2 d^2 e f x \log(F)^2 - 3 b^2 d^2 f^2 x^2 \log(F)^2 + 4 b d e f \log(F) + 6 b d f^2 x \log(F) - 6 f^2)}{b^4 d^4 \log(F)^4} & \text{for } b^4 d^4 \log(F)^4 \neq 0 \\ \frac{e^2 x^2}{2} + \frac{2 e f x^3}{3} + \frac{f^2 x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c))*x*(f*x+e)**2,x)`



```
[Out] Piecewise((F**(a + b*(c + d*x))*(b**3*d**3*e**2*x*log(F)**3 + 2*b**3*d**3*e
*f*x**2*log(F)**3 + b**3*d**3*f**2*x**3*log(F)**3 - b**2*d**2*e**2*log(F)**
2 - 4*b**2*d**2*e*f*x*log(F)**2 - 3*b**2*d**2*f**2*x**2*log(F)**2 + 4*b*d*e
*f*log(F) + 6*b*d*f**2*x*log(F) - 6*f**2)/(b**4*d**4*log(F)**4), Ne(b**4*d*
**4*log(F)**4, 0)), (e**2*x**2/2 + 2*e*f*x**3/3 + f**2*x**4/4, True))
```

**Giac** [C] Result contains complex when optimal does not.  
time = 2.19, size = 4688, normalized size = 19.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(a+b*(d*x+c))*x*(f*x+e)^2,x, algorithm="giac")
```

```
[Out] (2*((pi*b^2*d^2*log(abs(F))*sgn(F) - pi*b^2*d^2*log(abs(F)))*(pi*b*d*x*sgn(
F) - pi*b*d*x)/((pi^2*b^2*d^2*sgn(F) - pi^2*b^2*d^2 + 2*b^2*d^2*log(abs(F))
^2)^2 + 4*(pi*b^2*d^2*log(abs(F))*sgn(F) - pi*b^2*d^2*log(abs(F)))^2) + (pi
^2*b^2*d^2*sgn(F) - pi^2*b^2*d^2 + 2*b^2*d^2*log(abs(F))^2)*(b*d*x*log(abs(
F)) - 1)/((pi^2*b^2*d^2*sgn(F) - pi^2*b^2*d^2 + 2*b^2*d^2*log(abs(F))^2)^2
+ 4*(pi*b^2*d^2*log(abs(F))*sgn(F) - pi*b^2*d^2*log(abs(F)))^2))*cos(-1/2*p
i*b*d*x*sgn(F) + 1/2*pi*b*d*x - 1/2*pi*b*c*sgn(F) + 1/2*pi*b*c - 1/2*pi*a*s
gn(F) + 1/2*pi*a) + ((pi^2*b^2*d^2*sgn(F) - pi^2*b^2*d^2 + 2*b^2*d^2*log(ab
s(F))^2)*(pi*b*d*x*sgn(F) - pi*b*d*x)/((pi^2*b^2*d^2*sgn(F) - pi^2*b^2*d^2
+ 2*b^2*d^2*log(abs(F))^2)^2 + 4*(pi*b^2*d^2*log(abs(F))*sgn(F) - pi*b^2*d^
2*log(abs(F)))^2) - 4*(pi*b^2*d^2*log(abs(F))*sgn(F) - pi*b^2*d^2*log(abs(F
))))*(b*d*x*log(abs(F)) - 1)/((pi^2*b^2*d^2*sgn(F) - pi^2*b^2*d^2 + 2*b^2*d^
2*log(abs(F))^2)^2 + 4*(pi*b^2*d^2*log(abs(F))*sgn(F) - pi*b^2*d^2*log(abs(
F)))^2))*sin(-1/2*pi*b*d*x*sgn(F) + 1/2*pi*b*d*x - 1/2*pi*b*c*sgn(F) + 1/2*
pi*b*c - 1/2*pi*a*sgn(F) + 1/2*pi*a))*e^(b*d*x*log(abs(F)) + b*c*log(abs(F)
) + a*log(abs(F)) + 2) - 1/2*I*((pi*b*d*x*sgn(F) - pi*b*d*x - 2*I*b*d*x*log
(abs(F)) + 2*I)*e^(1/2*I*pi*b*d*x*sgn(F) - 1/2*I*pi*b*d*x + 1/2*I*pi*b*c*sg
n(F) - 1/2*I*pi*b*c + 1/2*I*pi*a*sgn(F) - 1/2*I*pi*a)/(pi^2*b^2*d^2*sgn(F)
+ 2*I*pi*b^2*d^2*log(abs(F))*sgn(F) - pi^2*b^2*d^2 - 2*I*pi*b^2*d^2*log(abs
(F)) + 2*b^2*d^2*log(abs(F))^2) + (pi*b*d*x*sgn(F) - pi*b*d*x + 2*I*b*d*x*l
og(abs(F)) - 2*I)*e^(-1/2*I*pi*b*d*x*sgn(F) + 1/2*I*pi*b*d*x - 1/2*I*pi*b*c
*sgn(F) + 1/2*I*pi*b*c - 1/2*I*pi*a*sgn(F) + 1/2*I*pi*a)/(pi^2*b^2*d^2*sgn(
F) - 2*I*pi*b^2*d^2*log(abs(F))*sgn(F) - pi^2*b^2*d^2 + 2*I*pi*b^2*d^2*log(
abs(F)) + 2*b^2*d^2*log(abs(F))^2))*e^(b*d*x*log(abs(F)) + b*c*log(abs(F))
+ a*log(abs(F)) + 2) + 2*((pi^2*b^2*d^2*f*x^2*sgn(F) - pi^2*b^2*d^2*f*x^2
+ 2*b^2*d^2*f*x^2*log(abs(F))^2 - 4*b*d*f*x*log(abs(F)) + 4*f)*(3*pi^2*b^3*
d^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*d^3*log(abs(F)) + 2*b^3*d^3*log(abs(F))
^3)/((pi^3*b^3*d^3*sgn(F) - 3*pi*b^3*d^3*log(abs(F))^2*sgn(F) - pi^3*b^3*d^
3 + 3*pi*b^3*d^3*log(abs(F))^2)^2 + (3*pi^2*b^3*d^3*log(abs(F))*sgn(F) - 3*
pi^2*b^3*d^3*log(abs(F)) + 2*b^3*d^3*log(abs(F))^3)^2) - 2*(pi^3*b^3*d^3*sg
n(F) - 3*pi*b^3*d^3*log(abs(F))^2*sgn(F) - pi^3*b^3*d^3 + 3*pi*b^3*d^3*log(
```

$$\begin{aligned} & \text{abs}(F))^2) * (\pi * b^2 * d^2 * f * x^2 * \log(\text{abs}(F)) * \text{sgn}(F) - \pi * b^2 * d^2 * f * x^2 * \log(\text{abs}(F))) - \pi * b * d * f * x * \text{sgn}(F) + \pi * b * d * f * x / ((\pi^3 * b^3 * d^3 * \text{sgn}(F) - 3 * \pi * b^3 * d^3 * \log(\text{abs}(F))^2 * \text{sgn}(F) - \pi^3 * b^3 * d^3 + 3 * \pi * b^3 * d^3 * \log(\text{abs}(F))^2)^2 + (3 * \pi^2 * b^3 * d^3 * \log(\text{abs}(F)) * \text{sgn}(F) - 3 * \pi^2 * b^3 * d^3 * \log(\text{abs}(F)) + 2 * b^3 * d^3 * \log(\text{abs}(F))^3)^2)) * \cos(-1/2 * \pi * b * d * x * \text{sgn}(F) + 1/2 * \pi * b * d * x - 1/2 * \pi * b * c * \text{sgn}(F) + 1/2 * \pi * b * c - 1/2 * \pi * a * \text{sgn}(F) + 1/2 * \pi * a) + ((\pi^3 * b^3 * d^3 * \text{sgn}(F) - 3 * \pi * b^3 * d^3 * \log(\text{abs}(F))^2 * \text{sgn}(F) - \pi^3 * b^3 * d^3 + 3 * \pi * b^3 * d^3 * \log(\text{abs}(F))^2) * (\pi^2 * b^2 * d^2 * f * x^2 * \text{sgn}(F) - \pi^2 * b^2 * d^2 * f * x^2 + 2 * b^2 * d^2 * f * x^2 * \log(\text{abs}(F))^2 - 4 * b * d * f * x * \log(\text{abs}(F)) + 4 * f) / ((\pi^3 * b^3 * d^3 * \text{sgn}(F) - 3 * \pi * b^3 * d^3 * \log(\text{abs}(F))^2 * \text{sgn}(F) - \pi^3 * b^3 * d^3 + 3 * \pi * b^3 * d^3 * \log(\text{abs}(F))^2)^2 + (3 * \pi^2 * b^3 * d^3 * \log(\text{abs}(F)) * \text{sgn}(F) - 3 * \pi^2 * b^3 * d^3 * \log(\text{abs}(F)) + 2 * b^3 * d^3 * \log(\text{abs}(F))^3)^2) + 2 * (3 * \pi^2 * b^3 * d^3 * \log(\text{abs}(F)) * \text{sgn}(F) - 3 * \pi^2 * b^3 * d^3 * \log(\text{abs}(F)) + 2 * b^3 * d^3 * \log(\text{abs}(F))^3) * (\pi * b^2 * d^2 * f * x^2 * \log(\text{abs}(F)) * \text{sgn}(F) - \pi * b^2 * d^2 * f * x^2 * \log(\text{abs}(F)) - \pi * b * d * f * x * \text{sgn}(F) + \pi * b * d * f * x) / ((\pi^3 * b^3 * d^3 * \text{sgn}(F) - 3 * \pi * b^3 * d^3 * \log(\text{abs}(F))^2 * \text{sgn}(F) - \pi^3 * b^3 * d^3 + 3 * \pi * b^3 * d^3 * \log(\text{abs}(F))^2)^2 + (3 * \pi^2 * b^3 * d^3 * \log(\text{abs}(F)) * \text{sgn}(F) - 3 * \pi^2 * b^3 * d^3 * \log(\text{abs}(F)) + 2 * b^3 * d^3 * \log(\text{abs}(F))^3)^2)) * \sin(-1/2 * \pi * b * d * x * \text{sgn}(F) + 1/2 * \pi * b * d * x - 1/2 * \pi * b * c * \text{sgn}(F) + 1/2 * \pi * b * c - 1/2 * \pi * a * \text{sgn}(F) + 1/2 * \pi * a) * e^{(b * d * x * \log(\text{abs}(F)) + b * c * \log(\text{abs}(F)) + a * \log(\text{abs}(F)) + 1) - 4 * I * ((-I * \pi^2 * b^2 * d^2 * f * x^2 * \text{sgn}(F) + 2 * \pi * b^2 * d^2 * f * x^2 * \log(\text{abs}(F)) * \text{sgn}(F) + I * \pi^2 * b^2 * d^2 * f * x^2 - 2 * \pi * b^2 * d^2 * f * x^2 * \log(\text{abs}(F)) - 2 * I * b^2 * d^2 * f * x^2 * \log(\text{abs}(F))^2 - 2 * \pi * b * d * f * x * \text{sgn}(F) + 2 * \pi * b * d * f * x + 4 * I * b * d * f * x * \log(\text{abs}(F)) - 4 * I * f) * e^{(1/2 * I * \pi * b * d * x * \text{sgn}(F) - 1/2 * I * \pi * b * d * x + 1/2 * I * \pi * b * c * \text{sgn}(F) - 1/2 * I * \pi * b * c + 1/2 * I * \pi * a * \text{sgn}(F) - 1/2 * I * \pi * a) / (-4 * I * \pi^3 * b^3 * d^3 * \text{sgn}(F) + 12 * \pi^2 * b^3 * d^3 * \log(\text{abs}(F)) * \text{sgn}(F) + 12 * I * \pi * b^3 * d^3 * \log(\text{abs}(F))^2 * \text{sgn}(F) + 4 * I * \pi^3 * b^3 * d^3 - 12 * \pi^2 * b^3 * d^3 * \log(\text{abs}(F)) - 12 * I * \pi * b^3 * d^3 * \log(\text{abs}(F))^2 + 8 * b^3 * d^3 * \log(\text{abs}(F))^3) - (-I * \pi^2 * b^2 * d^2 * f * x^2 * \text{sgn}(F) - 2 * \pi * b^2 * d^2 * f * x^2 * \log(\text{abs}(F)) * \text{sgn}(F) + I * \pi^2 * b^2 * d^2 * f * x^2 + 2 * \pi * b^2 * d^2 * f * x^2 * \log(\text{abs}(F)) - 2 * I * b^2 * d^2 * f * x^2 * \log(\text{abs}(F))^2 + 2 * \pi * b * d * f * x * \text{sgn}(F) - 2 * \pi * b * d * f * x + 4 * I * b * d * f * x * \log(\text{abs}(F)) - 4 * I * f) * e^{(-1/2 * I * \pi * b * d * x * \text{sgn}(F) + 1/2 * I * \pi * b * d * x - 1/2 * I * \pi * b * c * \text{sgn}(F) + 1/2 * I * \pi * b * c - 1/2 * I * \pi * a * \text{sgn}(F) + 1/2 * I * \pi * a) / (4 * I * \pi^3 * b^3 * d^3 * \text{sgn}(F) + 12 * \pi^2 * b^3 * d^3 * \log(\text{abs}(F)) * \text{sgn}(F) - 12 * I * \pi * b^3 * d^3 * \log(\text{abs}(F))^2 * \text{sgn}(F) - 4 * I * \pi^3 * b^3 * d^3 - 12 * \pi^2 * b^3 * d^3 * \log(\text{abs}(F)) + 12 * I * \pi * b^3 * d^3 * \log(\text{abs}(F))^2 + 8 * b^3 * d^3 * \log(\text{abs}(F))^3)} * e^{(b * d * x * \log(\text{abs}(F)) + b * c * \log(\text{abs}(F)) + a * \log(\text{abs}(F)) + 1) - ((3 * \pi^2 * b^3 * d^3 * f^2 * x^3 * \log(\text{abs}(F)) * \text{sgn}(F) - 3 * \pi^2 * b^3 * d^3 * f^2 * x^3 * \log(\text{abs}(F)) + 2 * b^3 * d^3 * \dots} \end{aligned}$$

**Mupad [B]**

time = 3.52, size = 143, normalized size = 0.59

$$\frac{F^{a+bc+bdx} (b^3 d^3 e^2 x \ln(F)^3 + 2 b^3 d^3 e f x^2 \ln(F)^3 + b^3 d^3 f^2 x^3 \ln(F)^3 - b^2 d^2 e^2 \ln(F)^2 - 4 b^2 d^2 e f x \ln(F)^2 - 3 b^2 d^2 f^2 x^2 \ln(F)^2 + 4 b d e f \ln(F) + 6 b d f^2 x \ln(F) - 6 f^2)}{b^4 d^4 \ln(F)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a + b\*(c + d\*x))\*x\*(e + f\*x)^2,x)

```
[Out] (F^(a + b*c + b*d*x)*(6*b*d*f^2*x*log(F) - b^2*d^2*e^2*log(F)^2 - 6*f^2 + b
^3*d^3*e^2*x*log(F)^3 - 3*b^2*d^2*f^2*x^2*log(F)^2 + b^3*d^3*f^2*x^3*log(F)
^3 + 4*b*d*e*f*log(F) - 4*b^2*d^2*e*f*x*log(F)^2 + 2*b^3*d^3*e*f*x^2*log(F)
^3))/(b^4*d^4*log(F)^4)
```

### 3.68 $\int F^{a+b(c+dx)}(e+fx)^2 dx$

Optimal. Leaf size=85

$$\frac{2f^2 F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} - \frac{2f F^{a+bc+bdx}(e+fx)}{b^2 d^2 \log^2(F)} + \frac{F^{a+bc+bdx}(e+fx)^2}{bd \log(F)}$$

[Out]  $2f^2 F^{(b*d*x+b*c+a)}/b^3/d^3/\ln(F)^3 - 2f F^{(b*d*x+b*c+a)}*(f*x+e)/b^2/d^2/\ln(F)^2 + F^{(b*d*x+b*c+a)}*(f*x+e)^2/b/d/\ln(F)$

Rubi [A]

time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2218, 2207, 2225}

$$\frac{2f^2 F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} - \frac{2f(e+fx)F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + \frac{(e+fx)^2 F^{a+bc+bdx}}{bd \log(F)}$$

Antiderivative was successfully verified.

[In] Int[F^(a + b\*(c + d\*x))\*(e + f\*x)^2, x]

[Out]  $(2f^2 F^{(a + b*c + b*d*x)})/(b^3 d^3 \text{Log}[F]^3) - (2f F^{(a + b*c + b*d*x)}*(e + f*x))/(b^2 d^2 \text{Log}[F]^2) + (F^{(a + b*c + b*d*x)}*(e + f*x)^2)/(b*d \text{Log}[F])$

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2218

```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*(u_)^(m_.), x_Symbol] :> Int[NormalizePowerOfLinear[u, x]^m*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p, x] /; FreeQ[{F, a, b, g, n, p}, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && !(LinearMatchQ[v, x] && PowerOfLinearMatchQ[u, x]) && IntegerQ[m]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int F^{a+b(c+dx)}(e+fx)^2 dx &= \int F^{a+bc+bdx}(e+fx)^2 dx \\
&= \frac{F^{a+bc+bdx}(e+fx)^2}{bd \log(F)} - \frac{(2f) \int F^{a+bc+bdx}(e+fx) dx}{bd \log(F)} \\
&= -\frac{2f F^{a+bc+bdx}(e+fx)}{b^2 d^2 \log^2(F)} + \frac{F^{a+bc+bdx}(e+fx)^2}{bd \log(F)} + \frac{(2f^2) \int F^{a+bc+bdx} dx}{b^2 d^2 \log^2(F)} \\
&= \frac{2f^2 F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} - \frac{2f F^{a+bc+bdx}(e+fx)}{b^2 d^2 \log^2(F)} + \frac{F^{a+bc+bdx}(e+fx)^2}{bd \log(F)}
\end{aligned}$$

**Mathematica [A]**

time = 0.17, size = 58, normalized size = 0.68

$$\frac{F^{a+b(c+dx)}(2f^2 - 2bdf(e+fx)\log(F) + b^2 d^2(e+fx)^2 \log^2(F))}{b^3 d^3 \log^3(F)}$$

Antiderivative was successfully verified.

`[In] Integrate[F^(a + b*(c + d*x))*(e + f*x)^2,x]`

```
[Out] (F^(a + b*(c + d*x))*(2*f^2 - 2*b*d*f*(e + f*x)*Log[F] + b^2*d^2*(e + f*x)^2*Log[F]^2))/(b^3*d^3*Log[F]^3)
```

**Maple [A]**

time = 0.05, size = 93, normalized size = 1.09

method	result
gospers	$\frac{(\ln(F)^2 b^2 d^2 f^2 x^2 + 2 \ln(F)^2 b^2 d^2 e f x + \ln(F)^2 b^2 d^2 e^2 - 2 \ln(F) b d f^2 x - 2 e f \ln(F) b d + 2 f^2) F^{b d x + c b + a}}{b^3 d^3 \ln(F)^3}$
risch	$\frac{(\ln(F)^2 b^2 d^2 f^2 x^2 + 2 \ln(F)^2 b^2 d^2 e f x + \ln(F)^2 b^2 d^2 e^2 - 2 \ln(F) b d f^2 x - 2 e f \ln(F) b d + 2 f^2) F^{b d x + c b + a}}{b^3 d^3 \ln(F)^3}$
norman	$\frac{(\ln(F)^2 b^2 d^2 e^2 - 2 e f \ln(F) b d + 2 f^2) e^{(a+b(dx+c)) \ln(F)}}{b^3 d^3 \ln(F)^3} + \frac{f^2 x^2 e^{(a+b(dx+c)) \ln(F)}}{b d \ln(F)} + \frac{2 f (\ln(F) b d e - f) x e^{(a+b(dx+c)) \ln(F)}}{b^2 d^2 \ln(F)^2}$
meijerg	$-\frac{F^{c b + a} f^2 \left( 2 - \frac{(3 b^2 d^2 x^2 \ln(F)^2 - 6 b d x \ln(F) + 6) e^{b d x \ln(F)}}{3} \right)}{b^3 d^3 \ln(F)^3} + \frac{2 F^{c b + a} f e \left( 1 - \frac{(-2 b d x \ln(F) + 2) e^{b d x \ln(F)}}{2} \right)}{b^2 d^2 \ln(F)^2} - \frac{F^{c b + a} e^2 (1 - e^{b d x \ln(F)})}{b d \ln(F)}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(F^(a+b*(d*x+c))*(f*x+e)^2,x,method=_RETURNVERBOSE)`

```
[Out] (ln(F)^2*b^2*d^2*f^2*x^2+2*ln(F)^2*b^2*d^2*e*f*x+ln(F)^2*b^2*d^2*e^2-2*ln(F)*b*d*f^2*x-2*e*f*ln(F)*b*d+2*f^2)*F^(b*d*x+b*c+a)/b^3/d^3/ln(F)^3
```

**Maxima [A]**

time = 0.44, size = 135, normalized size = 1.59

$$\frac{F^{bdx+bc+a}e^2}{bd \log(F)} + \frac{2(F^{bc+a}bdx \log(F) - F^{bc+a})fe^{(bdx \log(F)+1)}}{b^2d^2 \log(F)^2} + \frac{(F^{bc+a}b^2d^2x^2 \log(F)^2 - 2F^{bc+a}bdx \log(F) + 2F^{bc+a})F^{bdx}f^2}{b^3d^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(F^(a+b\*(d\*x+c))\*(f\*x+e)^2,x, algorithm="maxima")

**[Out]** F^(b\*d\*x + b\*c + a)\*e^2/(b\*d\*log(F)) + 2\*(F^(b\*c + a)\*b\*d\*x\*log(F) - F^(b\*c + a))\*f\*e^(b\*d\*x\*log(F) + 1)/(b^2\*d^2\*log(F)^2) + (F^(b\*c + a)\*b^2\*d^2\*x^2\*log(F)^2 - 2\*F^(b\*c + a)\*b\*d\*x\*log(F) + 2\*F^(b\*c + a))\*F^(b\*d\*x)\*f^2/(b^3\*d^3\*log(F)^3)

**Fricas [A]**

time = 0.37, size = 86, normalized size = 1.01

$$\frac{((b^2d^2f^2x^2 + 2b^2d^2fxe + b^2d^2e^2) \log(F)^2 + 2f^2 - 2(bdf^2x + bdf e) \log(F))F^{bdx+bc+a}}{b^3d^3 \log(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(F^(a+b\*(d\*x+c))\*(f\*x+e)^2,x, algorithm="fricas")

**[Out]** ((b^2\*d^2\*f^2\*x^2 + 2\*b^2\*d^2\*f\*x\*e + b^2\*d^2\*e^2)\*log(F)^2 + 2\*f^2 - 2\*(b\*d\*f^2\*x + b\*d\*f\*e)\*log(F))\*F^(b\*d\*x + b\*c + a)/(b^3\*d^3\*log(F)^3)

**Sympy [A]**

time = 0.07, size = 134, normalized size = 1.58

$$\begin{cases} \frac{F^{a+b(c+dx)}(b^2d^2e^2 \log(F)^2 + 2b^2d^2efx \log(F)^2 + b^2d^2f^2x^2 \log(F)^2 - 2bdef \log(F) - 2bdf^2x \log(F) + 2f^2)}{b^3d^3 \log(F)^3} & \text{for } b^3d^3 \log(F)^3 \neq 0 \\ e^2x + efx^2 + \frac{f^2x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(F\*\*(a+b\*(d\*x+c))\*(f\*x+e)\*\*2,x)

**[Out]** Piecewise((F\*\*(a + b\*(c + d\*x))\*(b\*\*2\*d\*\*2\*e\*\*2\*log(F)\*\*2 + 2\*b\*\*2\*d\*\*2\*e\*f\*x\*log(F)\*\*2 + b\*\*2\*d\*\*2\*f\*\*2\*x\*\*2\*log(F)\*\*2 - 2\*b\*d\*e\*f\*log(F) - 2\*b\*d\*f\*\*2\*x\*log(F) + 2\*f\*\*2)/(b\*\*3\*d\*\*3\*log(F)\*\*3), Ne(b\*\*3\*d\*\*3\*log(F)\*\*3, 0)), (e\*\*2\*x + e\*f\*x\*\*2 + f\*\*2\*x\*\*3/3, True))

**Giac [C]** Result contains complex when optimal does not.

time = 3.04, size = 2743, normalized size = 32.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c))\*(f\*x+e)^2,x, algorithm="giac")

[Out] 
$$2*(2*b*d*\cos(-1/2*\pi*b*d*x*\operatorname{sgn}(F) + 1/2*\pi*b*d*x - 1/2*\pi*b*c*\operatorname{sgn}(F) + 1/2*\pi*b*c - 1/2*\pi*a*\operatorname{sgn}(F) + 1/2*\pi*a)*\log(\operatorname{abs}(F)))/(4*b^2*d^2*\log(\operatorname{abs}(F))^2 + (\pi*b*d*\operatorname{sgn}(F) - \pi*b*d)^2) - (\pi*b*d*\operatorname{sgn}(F) - \pi*b*d)*\sin(-1/2*\pi*b*d*x*\operatorname{sgn}(F) + 1/2*\pi*b*d*x - 1/2*\pi*b*c*\operatorname{sgn}(F) + 1/2*\pi*b*c - 1/2*\pi*a*\operatorname{sgn}(F) + 1/2*\pi*a)/(4*b^2*d^2*\log(\operatorname{abs}(F))^2 + (\pi*b*d*\operatorname{sgn}(F) - \pi*b*d)^2)*e^{(b*d*x*\log(\operatorname{abs}(F)) + b*c*\log(\operatorname{abs}(F)) + a*\log(\operatorname{abs}(F)) + 2)} + I*(I*e^{(1/2*I*\pi*b*d*x*\operatorname{sgn}(F) - 1/2*I*\pi*b*d*x + 1/2*I*\pi*b*c*\operatorname{sgn}(F) - 1/2*I*\pi*b*c + 1/2*I*\pi*a*\operatorname{sgn}(F) - 1/2*I*\pi*a)/(I*\pi*b*d*\operatorname{sgn}(F) - I*\pi*b*d + 2*b*d*\log(\operatorname{abs}(F)))} - I*e^{(-1/2*I*\pi*b*d*x*\operatorname{sgn}(F) + 1/2*I*\pi*b*d*x - 1/2*I*\pi*b*c*\operatorname{sgn}(F) + 1/2*I*\pi*b*c - 1/2*I*\pi*a*\operatorname{sgn}(F) + 1/2*I*\pi*a)/(-I*\pi*b*d*\operatorname{sgn}(F) + I*\pi*b*d + 2*b*d*\log(\operatorname{abs}(F)))})*e^{(b*d*x*\log(\operatorname{abs}(F)) + b*c*\log(\operatorname{abs}(F)) + a*\log(\operatorname{abs}(F)) + 2)} + 2*(2*((\pi*b^2*d^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - \pi*b^2*d^2*\log(\operatorname{abs}(F)))*(\pi*b*d*f*x*\operatorname{sgn}(F) - \pi*b*d*f*x)/((\pi^2*b^2*d^2*\operatorname{sgn}(F) - \pi^2*b^2*d^2 + 2*b^2*d^2*\log(\operatorname{abs}(F)))^2)^2 + 4*(\pi*b^2*d^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - \pi*b^2*d^2*\log(\operatorname{abs}(F)))^2) + (\pi^2*b^2*d^2*\operatorname{sgn}(F) - \pi^2*b^2*d^2 + 2*b^2*d^2*\log(\operatorname{abs}(F)))^2*(b*d*f*x*\log(\operatorname{abs}(F)) - f)/((\pi^2*b^2*d^2*\operatorname{sgn}(F) - \pi^2*b^2*d^2 + 2*b^2*d^2*\log(\operatorname{abs}(F)))^2)^2 + 4*(\pi*b^2*d^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - \pi*b^2*d^2*\log(\operatorname{abs}(F)))^2)*\cos(-1/2*\pi*b*d*x*\operatorname{sgn}(F) + 1/2*\pi*b*d*x - 1/2*\pi*b*c*\operatorname{sgn}(F) + 1/2*\pi*b*c - 1/2*\pi*a*\operatorname{sgn}(F) + 1/2*\pi*a) + ((\pi^2*b^2*d^2*\operatorname{sgn}(F) - \pi^2*b^2*d^2 + 2*b^2*d^2*\log(\operatorname{abs}(F)))^2*(\pi*b*d*f*x*\operatorname{sgn}(F) - \pi*b*d*f*x)/((\pi^2*b^2*d^2*\operatorname{sgn}(F) - \pi^2*b^2*d^2 + 2*b^2*d^2*\log(\operatorname{abs}(F)))^2)^2 + 4*(\pi*b^2*d^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - \pi*b^2*d^2*\log(\operatorname{abs}(F)))^2) - 4*(\pi*b^2*d^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - \pi*b^2*d^2*\log(\operatorname{abs}(F)))^2*(b*d*f*x*\log(\operatorname{abs}(F)) - f)/((\pi^2*b^2*d^2*\operatorname{sgn}(F) - \pi^2*b^2*d^2 + 2*b^2*d^2*\log(\operatorname{abs}(F)))^2)^2 + 4*(\pi*b^2*d^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - \pi*b^2*d^2*\log(\operatorname{abs}(F)))^2)*\sin(-1/2*\pi*b*d*x*\operatorname{sgn}(F) + 1/2*\pi*b*d*x - 1/2*\pi*b*c*\operatorname{sgn}(F) + 1/2*\pi*b*c - 1/2*\pi*a*\operatorname{sgn}(F) + 1/2*\pi*a))*e^{(b*d*x*\log(\operatorname{abs}(F)) + b*c*\log(\operatorname{abs}(F)) + a*\log(\operatorname{abs}(F)) + 1)} - I*((\pi*b*d*f*x*\operatorname{sgn}(F) - \pi*b*d*f*x - 2*I*b*d*f*x*\log(\operatorname{abs}(F)) + 2*I*f)*e^{(1/2*I*\pi*b*d*x*\operatorname{sgn}(F) - 1/2*I*\pi*b*d*x + 1/2*I*\pi*b*c*\operatorname{sgn}(F) - 1/2*I*\pi*b*c + 1/2*I*\pi*a*\operatorname{sgn}(F) - 1/2*I*\pi*a)/(\pi^2*b^2*d^2*\operatorname{sgn}(F) + 2*I*\pi*b^2*d^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - \pi^2*b^2*d^2 - 2*I*\pi*b^2*d^2*\log(\operatorname{abs}(F)) + 2*b^2*d^2*\log(\operatorname{abs}(F))^2)} + (\pi*b*d*f*x*\operatorname{sgn}(F) - \pi*b*d*f*x + 2*I*b*d*f*x*\log(\operatorname{abs}(F)) - 2*I*f)*e^{(-1/2*I*\pi*b*d*x*\operatorname{sgn}(F) + 1/2*I*\pi*b*d*x - 1/2*I*\pi*b*c*\operatorname{sgn}(F) + 1/2*I*\pi*b*c - 1/2*I*\pi*a*\operatorname{sgn}(F) + 1/2*I*\pi*a)/(\pi^2*b^2*d^2*\operatorname{sgn}(F) - 2*I*\pi*b^2*d^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - \pi^2*b^2*d^2 + 2*I*\pi*b^2*d^2*\log(\operatorname{abs}(F)) + 2*b^2*d^2*\log(\operatorname{abs}(F))^2)})*e^{(b*d*x*\log(\operatorname{abs}(F)) + b*c*\log(\operatorname{abs}(F)) + a*\log(\operatorname{abs}(F)) + 1)} - ((2*(\pi*b^2*d^2*f^2*x^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - \pi*b^2*d^2*f^2*x^2*\log(\operatorname{abs}(F))) - \pi*b*d*f^2*x*\operatorname{sgn}(F) + \pi*b*d*f^2*x*(\pi^3*b^3*d^3*\operatorname{sgn}(F) - 3*\pi*b^3*d^3*\log(\operatorname{abs}(F))^2*\operatorname{sgn}(F) - \pi^3*b^3*d^3 + 3*\pi*b^3*d^3*\log(\operatorname{abs}(F))^2)/((\pi^3*b^3*d^3*\operatorname{sgn}(F) - 3*\pi*b^3*d^3*\log(\operatorname{abs}(F))^2*\operatorname{sgn}(F) - \pi^3*b^3*d^3 + 3*\pi*b^3*d^3*\log(\operatorname{abs}(F))^2)^2 + (3*\pi^2*b^3*d^3*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 3*\pi^2*b^3*d^3*\log(\operatorname{abs}(F)) + 2*b^3*d^3*\log(\operatorname{abs}(F))^3)^2) - (\pi^2*b^2*d^2*f^2*x^2*\operatorname{sgn}(F) - \pi^2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*f^2*x^2*\log(\operatorname{abs}(F)))^2 - 4*b*d*f^2*x*\log(\operatorname{abs}(F)) + 4*f^2)*(3*\pi^2*b^3$$

```

*d^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*d^3*log(abs(F)) + 2*b^3*d^3*log(abs(F)
)^3)/((pi^3*b^3*d^3*sgn(F) - 3*pi*b^3*d^3*log(abs(F))^2*sgn(F) - pi^3*b^3*d
^3 + 3*pi*b^3*d^3*log(abs(F))^2)^2 + (3*pi^2*b^3*d^3*log(abs(F))*sgn(F) - 3
*pi^2*b^3*d^3*log(abs(F)) + 2*b^3*d^3*log(abs(F))^3)^2))*cos(-1/2*pi*b*d*x*
sgn(F) + 1/2*pi*b*d*x - 1/2*pi*b*c*sgn(F) + 1/2*pi*b*c - 1/2*pi*a*sgn(F) +
1/2*pi*a) - ((pi^2*b^2*d^2*f^2*x^2*sgn(F) - pi^2*b^2*d^2*f^2*x^2 + 2*b^2*d^
2*f^2*x^2*log(abs(F))^2 - 4*b*d*f^2*x*log(abs(F)) + 4*f^2)*(pi^3*b^3*d^3*sg
n(F) - 3*pi*b^3*d^3*log(abs(F))^2*sgn(F) - pi^3*b^3*d^3 + 3*pi*b^3*d^3*log(
abs(F))^2)/((pi^3*b^3*d^3*sgn(F) - 3*pi*b^3*d^3*log(abs(F))^2*sgn(F) - pi^3
*b^3*d^3 + 3*pi*b^3*d^3*log(abs(F))^2)^2 + (3*pi^2*b^3*d^3*log(abs(F))*sgn(
F) - 3*pi^2*b^3*d^3*log(abs(F)) + 2*b^3*d^3*log(abs(F))^3)^2) + 2*(pi*b^2*d
^2*f^2*x^2*log(abs(F))*sgn(F) - pi*b^2*d^2*f^2*x^2*log(abs(F)) - pi*b*d*f^2
*x*sgn(F) + pi*b*d*f^2*x)*(3*pi^2*b^3*d^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*d
^3*log(abs(F)) + 2*b^3*d^3*log(abs(F))^3)/((pi^3*b^3*d^3*sgn(F) - 3*pi*b^3*
d^3*log(abs(F))^2*sgn(F) - pi^3*b^3*d^3 + 3*pi*b^3*d^3*log(abs(F))^2)^2 + (
3*pi^2*b^3*d^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*d^3*log(abs(F)) + 2*b^3*d^3*
log(abs(F))^3)^2))*sin(-1/2*pi*b*d*x*sgn(F) + 1/2*pi*b*d*x - 1/2*pi*b*c*sgn
(F) + 1/2*pi*b*c - 1/2*pi*a*sgn(F) + 1/2*pi*a))*e^(b*d*x*log(abs(F)) + b*c*
log(abs(F)) + a*log(abs(F))) - 2*I*((-I*pi^2*b^2*d^2*f^2*x^2*sgn(F) + 2*pi*
b^2*d^2*f^2*x^2*log(abs(F))*sgn(F) + I*pi^2*b^2*d^2*f^2*x^2 - 2*pi*b^2*d^2*
f^2*x^2*log(abs(F)) - 2*I*b^2*d^2*f^2*x^2*log(abs(F))^2 - 2*pi*b*d*f^2*x*sg
n(F) + 2*pi*b*d*f^2*x + 4*I*b*d*f^2*x*log(abs(F)) - 4*I*f^2)*e^(1/2*I*pi*b*
d*x*sgn(F) - 1/2*I*pi*b*d*x + 1/2*I*pi*b*c*sgn(F) - 1/2*I*pi*b*c + 1/2*I*pi
*a*sgn(F) - 1/2*I*pi*a)/(-4*I*pi^3*b^3*d^3*sgn(...

```

**Mupad [B]**

time = 3.44, size = 92, normalized size = 1.08

$$\frac{F^{a+bc+bdx} (b^2 d^2 e^2 \ln(F)^2 + 2b^2 d^2 e f x \ln(F)^2 + b^2 d^2 f^2 x^2 \ln(F)^2 - 2b d e f \ln(F) - 2b d f^2 x \ln(F) + 2 f^2)}{b^3 d^3 \ln(F)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(a + b*(c + d*x))*(e + f*x)^2,x)
```

```
[Out] (F^(a + b*c + b*d*x)*(2*f^2 + b^2*d^2*e^2*log(F)^2 - 2*b*d*f^2*x*log(F) + b
^2*d^2*f^2*x^2*log(F)^2 - 2*b*d*e*f*log(F) + 2*b^2*d^2*e*f*x*log(F)^2))/(b^
3*d^3*log(F)^3)
```



$$3.69 \quad \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x} dx$$

Optimal. Leaf size=96

$$e^2 F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{f^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + \frac{2ef F^{a+bc+bdx}}{bd \log(F)} + \frac{f^2 F^{a+bc+bdx} x}{bd \log(F)}$$

[Out]  $e^{2*F^{(b*c+a)}*Ei(b*d*x*\ln(F))-f^{2*F^{(b*d*x+b*c+a)}/b^{2/d^{2}/\ln(F)^{2+2*e*f*F^{(b*d*x+b*c+a)}/b/d/\ln(F)+f^{2*F^{(b*d*x+b*c+a)}*x/b/d/\ln(F)}$

**Rubi** [A]

time = 0.17, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2230, 2225, 2209, 2207}

$$-\frac{f^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + e^2 F^{a+bc} \text{Ei}(bdx \log(F)) + \frac{2ef F^{a+bc+bdx}}{bd \log(F)} + \frac{f^2 x F^{a+bc+bdx}}{bd \log(F)}$$

Antiderivative was successfully verified.

[In] Int[(F^(a + b\*(c + d\*x))\*(e + f\*x)^2)/x,x]

[Out]  $e^{2*F^{(a + b*c)}*ExpIntegralEi[b*d*x*Log[F]] - (f^{2*F^{(a + b*c + b*d*x)}}/(b^{2*d^{2}*Log[F]^2} + (2*e*f*F^{(a + b*c + b*d*x)}}/(b*d*Log[F]) + (f^{2*F^{(a + b*c + b*d*x)}*x))/(b*d*Log[F])$

Rule 2207

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(c + d\*x)^m\*((b\*F^(g\*(e + f\*x)))^n/(f\*g\*n\*Log[F])), x] - Dist[d\*(m/(f\*g\*n\*Log[F])), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

Rule 2209

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - c\*(f/d)))/d)\*ExpIntegralEi[f\*g\*(c + d\*x)\*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

Rule 2225

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 2230

```
Int[(F_)^((c_)*(v_))*(u_)^(m_)*(w_), x_Symbol] := Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x] /; FreeQ[{F,
c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] && !TrueQ[$UseGamma]
```

Rubi steps

$$\begin{aligned}
\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x} dx &= \int \left( 2efF^{a+bc+bdx} + \frac{e^2F^{a+bc+bdx}}{x} + f^2F^{a+bc+bdx}x \right) dx \\
&= e^2 \int \frac{F^{a+bc+bdx}}{x} dx + (2ef) \int F^{a+bc+bdx} dx + f^2 \int F^{a+bc+bdx}x dx \\
&= e^2 F^{a+bc} \text{Ei}(bdx \log(F)) + \frac{2efF^{a+bc+bdx}}{bd \log(F)} + \frac{f^2 F^{a+bc+bdx}x}{bd \log(F)} - \frac{f^2 \int F^{a+bc+bdx} dx}{bd \log(F)} \\
&= e^2 F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{f^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + \frac{2efF^{a+bc+bdx}}{bd \log(F)} + \frac{f^2 F^{a+bc+bdx}x}{bd \log(F)}
\end{aligned}$$

**Mathematica [A]**

time = 0.23, size = 54, normalized size = 0.56

$$F^{a+bc} \left( e^2 \text{Ei}(bdx \log(F)) + \frac{fF^{bdx}(-f + bd(2e + fx) \log(F))}{b^2 d^2 \log^2(F)} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(F^(a + b*(c + d*x))*(e + f*x)^2)/x,x]
```

```
[Out] F^(a + b*c)*(e^2*ExpIntegralEi[b*d*x*Log[F]] + (f*F^(b*d*x)*(-f + b*d*(2*e
+ f*x)*Log[F]))/(b^2*d^2*Log[F]^2))
```

**Maple [A]**

time = 0.08, size = 118, normalized size = 1.23

method	result
meijerg	$\frac{F^{cb+a} f^2 \left( 1 - \frac{(-2bdx \ln(F)+2)e^{bdx \ln(F)}}{2} \right)}{b^2 d^2 \ln(F)^2} - \frac{2F^{cb+a} f e^{(1-e^{bdx \ln(F)})}}{bd \ln(F)} + F^{cb+a} e^2 (-\ln(-bdx \ln(F)) - \text{expIntegral} ($
risch	$-e^2 F^{cb} F^a \text{expIntegral} (1, cb \ln(F) + \ln(F) a - bdx \ln(F) - (cb + a) \ln(F)) + \frac{f^2 F^{bdx} F^{cb+a} x}{db \ln(F)} - \frac{f^2 F^{$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(a+b*(d*x+c))*(f*x+e)^2/x,x,method=_RETURNVERBOSE)
```

[Out]  $\frac{1}{b^2 d^2 \ln(F)^2} F^{bc+a} f^2 (1 - \frac{1}{2} (-2 b d x \ln(F) + 2) \exp(b d x \ln(F))) - 2 F^{bc+a} f e / b d \ln(F) (1 - \exp(b d x \ln(F))) + F^{bc+a} e^2 (-\ln(-b d x \ln(F)) - \text{Ei}(1, -b d x \ln(F)) + \ln(x) + \ln(-b d) + \ln(\ln(F)))$

**Maxima** [A]

time = 0.50, size = 87, normalized size = 0.91

$$F^{bc+a} \text{Ei}(b d x \log(F)) e^2 + \frac{2 F^{bc+a} f e}{b d \log(F)} + \frac{(F^{bc+a} b d x \log(F) - F^{bc+a}) F^{b d x} f^2}{b^2 d^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x,x, algorithm="maxima")`

[Out]  $F^{bc+a} \text{Ei}(b d x \log(F)) e^2 + 2 F^{bc+a} f e / (b d \log(F)) + (F^{bc+a} b d x \log(F) - F^{bc+a}) F^{b d x} f^2 / (b^2 d^2 \log(F)^2)$

**Fricas** [A]

time = 0.37, size = 75, normalized size = 0.78

$$\frac{F^{bc+a} b^2 d^2 \text{Ei}(b d x \log(F)) e^2 \log(F)^2 - (f^2 - (b d f^2 x + 2 b d f e) \log(F)) F^{b d x + b c + a}}{b^2 d^2 \log(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x,x, algorithm="fricas")`

[Out]  $(F^{bc+a} b^2 d^2 \text{Ei}(b d x \log(F)) e^2 \log(F)^2 - (f^2 - (b d f^2 x + 2 b d f e) \log(F)) F^{b d x + b c + a}) / (b^2 d^2 \log(F)^2)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{a+b(c+dx)} (e + f x)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c))*(f*x+e)**2/x,x)`

[Out] `Integral(F**(a + b*(c + d*x))*(e + f*x)**2/x, x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x,x, algorithm="giac")`

[Out] integrate((f\*x + e)^2\*F^((d\*x + c)\*b + a)/x, x)

**Mupad [B]**

time = 3.58, size = 80, normalized size = 0.83

$$\frac{F^{a+bc} (b^2 d^2 e^2 \operatorname{ei}(bdx \ln(F)) \ln(F)^2 - F^{bdx} f^2 + F^{bdx} b d f^2 x \ln(F) + 2 F^{bdx} b d e f \ln(F))}{b^2 d^2 \ln(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((F^(a + b\*(c + d\*x))\*(e + f\*x)^2)/x,x)

[Out] (F^(a + b\*c)\*(b^2\*d^2\*e^2\*ei(b\*d\*x\*log(F))\*log(F)^2 - F^(b\*d\*x)\*f^2 + F^(b\*d\*x)\*b\*d\*f^2\*x\*log(F) + 2\*F^(b\*d\*x)\*b\*d\*e\*f\*log(F))/(b^2\*d^2\*log(F)^2)

$$3.70 \quad \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^2} dx$$

**Optimal.** Leaf size=85

$$-\frac{e^2 F^{a+bc+bdx}}{x} + 2ef F^{a+bc} \text{Ei}(bdx \log(F)) + \frac{f^2 F^{a+bc+bdx}}{bd \log(F)} + bde^2 F^{a+bc} \text{Ei}(bdx \log(F)) \log(F)$$

[Out]  $-e^2 F^{(b*d*x+b*c+a)}/x + 2*e*f*F^{(b*c+a)}*Ei(b*d*x*\ln(F)) + f^2*F^{(b*d*x+b*c+a)}/b/d/\ln(F) + b*d*e^2*F^{(b*c+a)}*Ei(b*d*x*\ln(F))*\ln(F)$

**Rubi [A]**

time = 0.19, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2230, 2225, 2208, 2209}

$$bde^2 \log(F) F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{e^2 F^{a+bc+bdx}}{x} + 2ef F^{a+bc} \text{Ei}(bdx \log(F)) + \frac{f^2 F^{a+bc+bdx}}{bd \log(F)}$$

Antiderivative was successfully verified.

[In] Int[(F^(a + b\*(c + d\*x))\*(e + f\*x)^2)/x^2,x]

[Out]  $-(e^2 F^{(a + b*c + b*d*x)})/x + 2*e*f*F^{(a + b*c)}*ExpIntegralEi[b*d*x*Log[F]] + (f^2*F^{(a + b*c + b*d*x)})/(b*d*Log[F]) + b*d*e^2*F^{(a + b*c)}*ExpIntegralEi[b*d*x*Log[F]]*Log[F]$

Rule 2208

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)\*((b\*F^(g\*(e + f\*x)))^n/(d\*(m + 1))), x] - Dist[f\*g\*n\*(Log[F]/(d\*(m + 1))), Int[(c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

Rule 2209

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - c\*(f/d)))/d)\*ExpIntegralEi[f\*g\*(c + d\*x)\*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

Rule 2225

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 2230

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] := Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F,
c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] && !TrueQ[$UseGamma]
```

Rubi steps

$$\begin{aligned} \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^2} dx &= \int \left( f^2 F^{a+bc+bdx} + \frac{e^2 F^{a+bc+bdx}}{x^2} + \frac{2ef F^{a+bc+bdx}}{x} \right) dx \\ &= e^2 \int \frac{F^{a+bc+bdx}}{x^2} dx + (2ef) \int \frac{F^{a+bc+bdx}}{x} dx + f^2 \int F^{a+bc+bdx} dx \\ &= -\frac{e^2 F^{a+bc+bdx}}{x} + 2ef F^{a+bc} \text{Ei}(bdx \log(F)) + \frac{f^2 F^{a+bc+bdx}}{bd \log(F)} + (bde^2 \log(F)) \int \frac{F^{a+bc+bdx}}{x} dx \\ &= -\frac{e^2 F^{a+bc+bdx}}{x} + 2ef F^{a+bc} \text{Ei}(bdx \log(F)) + \frac{f^2 F^{a+bc+bdx}}{bd \log(F)} + bde^2 F^{a+bc} \text{Ei}(bdx \log(F)) \end{aligned}$$

**Mathematica [A]**

time = 0.22, size = 58, normalized size = 0.68

$$F^{a+bc} \left( F^{bdx} \left( -\frac{e^2}{x} + \frac{f^2}{bd \log(F)} \right) + e \text{Ei}(bdx \log(F)) (2f + bde \log(F)) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(F^(a + b*(c + d*x))*(e + f*x)^2)/x^2,x]
```

```
[Out] F^(a + b*c)*(F^(b*d*x)*(-(e^2/x) + f^2/(b*d*Log[F]))) + e*ExpIntegralEi[b*d*x*Log[F]]*(2*f + b*d*e*Log[F])
```

**Maple [A]**

time = 0.08, size = 135, normalized size = 1.59

method	result
risch	$-\ln(F) b d e^2 F^{cb} F^a \text{expIntegral}(1, cb \ln(F) + \ln(F) a - bdx \ln(F) - (cb + a) \ln(F)) + \frac{f^2 F^{bdx} F^{cb}}{\ln(F) bd}$
meijerg	$-\frac{F^{cb+a} f^2 (1 - e^{bdx \ln(F)})}{bd \ln(F)} + 2F^{cb+a} f e (-\ln(-bdx \ln(F)) - \text{expIntegral}(1, -bdx \ln(F)) + \ln(x) + \ln(-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(a+b*(d*x+c))*(f*x+e)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(F)*b*d*e^2*F^(c*b)*F^a*Ei(1,c*b*ln(F)+ln(F)*a-b*d*x*ln(F)-(b*c+a)*ln(F)
)+1/ln(F)/b/d*f^2*F^(b*d*x)*F^(b*c+a)-e^2*F^(b*d*x)*F^(b*c+a)/x-2*e*f*F^(c*
b)*F^a*Ei(1,c*b*ln(F)+ln(F)*a-b*d*x*ln(F)-(b*c+a)*ln(F))
```

**Maxima [A]**

time = 0.35, size = 68, normalized size = 0.80

$$F^{bc+a} b d e^2 \Gamma(-1, -b d x \log(F)) \log(F) + 2 F^{bc+a} f \operatorname{Ei}(b d x \log(F)) e + \frac{F^{b d x + b c + a} f^2}{b d \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c))\*(f\*x+e)^2/x^2,x, algorithm="maxima")

[Out] F^(b\*c + a)\*b\*d\*e^2\*gamma(-1, -b\*d\*x\*log(F))\*log(F) + 2\*F^(b\*c + a)\*f\*Ei(b\*d\*x\*log(F))\*e + F^(b\*d\*x + b\*c + a)\*f^2/(b\*d\*log(F))

**Fricas [A]**

time = 0.37, size = 82, normalized size = 0.96

$$\frac{(b^2 d^2 x e^2 \log(F)^2 + 2 b d f x e \log(F)) F^{bc+a} \operatorname{Ei}(b d x \log(F)) - (b d e^2 \log(F) - f^2 x) F^{b d x + b c + a}}{b d x \log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c))\*(f\*x+e)^2/x^2,x, algorithm="fricas")

[Out] ((b^2\*d^2\*x\*e^2\*log(F)^2 + 2\*b\*d\*f\*x\*e\*log(F))\*F^(b\*c + a)\*Ei(b\*d\*x\*log(F)) - (b\*d\*e^2\*log(F) - f^2\*x)\*F^(b\*d\*x + b\*c + a))/(b\*d\*x\*log(F))

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c))\*(f\*x+e)\*\*2/x\*\*2,x)

[Out] Integral(F\*\*(a + b\*(c + d\*x))\*(e + f\*x)\*\*2/x\*\*2, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c))\*(f\*x+e)^2/x^2,x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*F^((d\*x + c)\*b + a)/x^2, x)

**Mupad [B]**

time = 3.59, size = 89, normalized size = 1.05

$$2 F^{a+bc} e f \operatorname{ei}(bdx \ln(F)) - \frac{F^{bdx} F^{a+bc} e^2}{x} + \frac{F^{a+bc+bdx} f^2}{bd \ln(F)} - F^{a+bc} b d e^2 \ln(F) \operatorname{expint}(-bdx \ln(F))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((F^(a + b\*(c + d\*x))\*(e + f\*x)^2)/x^2,x)

[Out] 2\*F^(a + b\*c)\*e\*f\*ei(b\*d\*x\*log(F)) - (F^(b\*d\*x)\*F^(a + b\*c)\*e^2)/x + (F^(a + b\*c + b\*d\*x)\*f^2)/(b\*d\*log(F)) - F^(a + b\*c)\*b\*d\*e^2\*log(F)\*expint(-b\*d\*x\*log(F))



$$3.71 \quad \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^3} dx$$

**Optimal.** Leaf size=136

$$-\frac{e^2 F^{a+bc+bdx}}{2x^2} - \frac{2ef F^{a+bc+bdx}}{x} + f^2 F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{bde^2 F^{a+bc+bdx} \log(F)}{2x} + 2bdef F^{a+bc} \text{Ei}(bdx \log(F))$$

[Out]  $-1/2 * e^2 * F^{(b*d*x+b*c+a)}/x^2 - 2 * e * f * F^{(b*d*x+b*c+a)}/x + f^2 * F^{(b*c+a)} * \text{Ei}(b*d*x * \ln(F)) - 1/2 * b*d * e^2 * F^{(b*d*x+b*c+a)} * \ln(F) / x + 2 * b*d * e * f * F^{(b*c+a)} * \text{Ei}(b*d*x * \ln(F)) * \ln(F) + 1/2 * b^2 * d^2 * e^2 * F^{(b*c+a)} * \text{Ei}(b*d*x * \ln(F)) * \ln(F)^2$

**Rubi [A]**

time = 0.24, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {2230, 2208, 2209}

$$\frac{1}{2} b^2 d^2 e^2 \log^2(F) F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{e^2 F^{a+bc+bdx}}{2x^2} - \frac{bde^2 \log(F) F^{a+bc+bdx}}{2x} + 2bdef \log(F) F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{2ef F^{a+bc+bdx}}{x} + f^2 F^{a+bc} \text{Ei}(bdx \log(F))$$

Antiderivative was successfully verified.

[In] Int[(F^(a + b\*(c + d\*x))\*(e + f\*x)^2)/x^3,x]

[Out]  $-1/2 * (e^2 * F^{(a + b*c + b*d*x)})/x^2 - (2 * e * f * F^{(a + b*c + b*d*x)})/x + f^2 * F^{(a + b*c)} * \text{ExpIntegralEi}[b*d*x * \text{Log}[F]] - (b*d * e^2 * F^{(a + b*c + b*d*x)} * \text{Log}[F]) / (2 * x) + 2 * b*d * e * f * F^{(a + b*c)} * \text{ExpIntegralEi}[b*d*x * \text{Log}[F]] * \text{Log}[F] + (b^2 * d^2 * e^2 * F^{(a + b*c)} * \text{ExpIntegralEi}[b*d*x * \text{Log}[F]] * \text{Log}[F]^2) / 2$

Rule 2208

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)\*((b \* F^(g\*(e + f\*x)))^n/(d\*(m + 1))), x] - Dist[f\*g\*n\*(Log[F]/(d\*(m + 1))), Int[(c + d\*x)^(m + 1)\*(b \* F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

Rule 2209

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - c\*(f/d)))/d)\*ExpIntegralEi[f\*g\*(c + d\*x)\*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

Rule 2230

Int[(F\_)^((c\_.)\*(v\_))\*(u\_)^(m\_.)\*(w\_), x\_Symbol] :> Int[ExpandIntegrand[F^(c\*ExpandToSum[v, x]), w\*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && IntegerQ[m] && !TrueQ[\$UseGamma]

### Rubi steps

$$\begin{aligned}
\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^3} dx &= \int \left( \frac{e^2 F^{a+bc+bdx}}{x^3} + \frac{2ef F^{a+bc+bdx}}{x^2} + \frac{f^2 F^{a+bc+bdx}}{x} \right) dx \\
&= e^2 \int \frac{F^{a+bc+bdx}}{x^3} dx + (2ef) \int \frac{F^{a+bc+bdx}}{x^2} dx + f^2 \int \frac{F^{a+bc+bdx}}{x} dx \\
&= -\frac{e^2 F^{a+bc+bdx}}{2x^2} - \frac{2ef F^{a+bc+bdx}}{x} + f^2 F^{a+bc} \text{Ei}(bdx \log(F)) + \frac{1}{2} (bde^2 \log(F)) \int \frac{F^{a+bc+bdx}}{x} dx \\
&= -\frac{e^2 F^{a+bc+bdx}}{2x^2} - \frac{2ef F^{a+bc+bdx}}{x} + f^2 F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{bde^2 F^{a+bc+bdx} \log(F)}{2x} \\
&= -\frac{e^2 F^{a+bc+bdx}}{2x^2} - \frac{2ef F^{a+bc+bdx}}{x} + f^2 F^{a+bc} \text{Ei}(bdx \log(F)) - \frac{bde^2 F^{a+bc+bdx} \log(F)}{2x}
\end{aligned}$$

### Mathematica [A]

time = 0.21, size = 76, normalized size = 0.56

$$\frac{F^{a+bc} (-eF^{bdx}(e+4fx+bdex \log(F)) + x^2 \text{Ei}(bdx \log(F)) (2f^2 + 4bdef \log(F) + b^2 d^2 e^2 \log^2(F)))}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(F^(a + b\*(c + d\*x))\*(e + f\*x)^2)/x^3,x]

[Out] (F^(a + b\*c)\*(-(e\*F^(b\*d\*x))\*(e + 4\*f\*x + b\*d\*e\*x\*Log[F])) + x^2\*ExpIntegralEi[b\*d\*x\*Log[F]]\*(2\*f^2 + 4\*b\*d\*e\*f\*Log[F] + b^2\*d^2\*e^2\*Log[F]^2))/(2\*x^2)

### Maple [A]

time = 0.09, size = 204, normalized size = 1.50

method	result
risch	$-\frac{2ef F^{bdx} F^{cb+a}}{x} - 2bd \ln(F) ef F^{cb} F^a \text{expIntegral}(1, cb \ln(F) + \ln(F) a - bdx \ln(F) - (cb + a) \ln(F))$
meijerg	$F^{cb+a} f^2 (-\ln(-bdx \ln(F)) - \text{expIntegral}(1, -bdx \ln(F)) + \ln(x) + \ln(-bd) + \ln(\ln(F))) - 2b$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(a+b\*(d\*x+c))\*(f\*x+e)^2/x^3,x,method=\_RETURNVERBOSE)

[Out]  $-2*e*f*F^{(b*d*x)}*F^{(b*c+a)}/x-2*b*d*\ln(F)*e*f*F^{(c*b)}*F^a*\text{Ei}(1,c*b*\ln(F)+\ln(F)*a-b*d*x*\ln(F)-(b*c+a)*\ln(F))-1/2*b^2*d^2*\ln(F)^2*e^2*F^{(c*b)}*F^a*\text{Ei}(1,c*b*\ln(F)+\ln(F)*a-b*d*x*\ln(F)-(b*c+a)*\ln(F))-f^2*F^{(c*b)}*F^a*\text{Ei}(1,c*b*\ln(F)+\ln(F)*a-b*d*x*\ln(F)-(b*c+a)*\ln(F))-1/2*e^2*F^{(b*d*x)}*F^{(b*c+a)}/x^2-1/2*b*d*\ln(F)*e^2*F^{(b*d*x)}*F^{(b*c+a)}/x$

**Maxima [A]**

time = 0.36, size = 74, normalized size = 0.54

$$-F^{bc+a}b^2d^2e^2\Gamma(-2, -bdx \log(F)) \log(F)^2 + 2F^{bc+a}bdf e\Gamma(-1, -bdx \log(F)) \log(F) + F^{bc+a}f^2\text{Ei}(bdx \log(F))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c))\*(f\*x+e)^2/x^3,x, algorithm="maxima")

[Out]  $-F^{(b*c + a)*b^2*d^2*e^2*\text{gamma}(-2, -b*d*x*\log(F))*\log(F)^2 + 2*F^{(b*c + a)*b*d*f*e*\text{gamma}(-1, -b*d*x*\log(F))*\log(F) + F^{(b*c + a)*f^2*\text{Ei}(b*d*x*\log(F))}$

**Fricas [A]**

time = 0.37, size = 88, normalized size = 0.65

$$\frac{(b^2d^2x^2e^2\log(F)^2 + 4bdfx^2e\log(F) + 2f^2x^2)F^{bc+a}\text{Ei}(bdx \log(F)) - (bdxe^2\log(F) + 4fxe + e^2)F^{bdx+bc+a}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c))\*(f\*x+e)^2/x^3,x, algorithm="fricas")

[Out]  $1/2*((b^2*d^2*x^2*e^2*\log(F)^2 + 4*b*d*f*x^2*e*\log(F) + 2*f^2*x^2)*F^{(b*c + a)*\text{Ei}(b*d*x*\log(F))} - (b*d*x*e^2*\log(F) + 4*f*x*e + e^2)*F^{(b*d*x + b*c + a)})/x^2$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(a+b\*(d\*x+c))\*(f\*x+e)\*\*2/x\*\*3,x)

[Out] Integral(F\*\*(a + b\*(c + d\*x))\*(e + f\*x)\*\*2/x\*\*3, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c))\*(f\*x+e)^2/x^3,x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*F^((d\*x + c)\*b + a)/x^3, x)

**Mupad [B]**

time = 3.60, size = 133, normalized size = 0.98

$$F^{a+bc}f^2\text{ei}(bdx \ln(F)) - \frac{2F^{bdx}F^{a+bc}ef}{x} - F^{a+bc}b^2d^2e^2\ln(F)^2 \left( \frac{\text{expint}(-bdx \ln(F))}{2} + F^{bdx} \left( \frac{1}{2bdx \ln(F)} + \frac{1}{2b^2d^2x^2 \ln(F)^2} \right) \right) - 2F^{a+bc}bdef \ln(F) \text{expint}(-bdx \ln(F))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((F^(a + b*(c + d*x))*(e + f*x)^2)/x^3,x)`

[Out]  $F^{(a + b*c)} * f^2 * \text{ei}(b*d*x*\log(F)) - (2 * F^{(b*d*x)} * F^{(a + b*c)} * e * f) / x - F^{(a + b*c)} * b^2 * d^2 * e^2 * \log(F)^2 * (\text{expint}(-b*d*x*\log(F)) / 2 + F^{(b*d*x)} * (1 / (2 * b*d*x * \log(F)) + 1 / (2 * b^2 * d^2 * x^2 * \log(F)^2))) - 2 * F^{(a + b*c)} * b * d * e * f * \log(F) * \text{expint}(-b*d*x*\log(F))$

$$3.72 \quad \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^4} dx$$

**Optimal.** Leaf size=217

$$\frac{e^2 F^{a+bc+bdx}}{3x^3} - \frac{ef F^{a+bc+bdx}}{x^2} - \frac{f^2 F^{a+bc+bdx}}{x} - \frac{bde^2 F^{a+bc+bdx} \log(F)}{6x^2} - \frac{bdef F^{a+bc+bdx} \log(F)}{x} + bdf^2 F^{a+bc} \text{Ei}(bdx \log(F))$$

[Out]  $-1/3*e^2*F^{(b*d*x+b*c+a)}/x^3 - e*f*F^{(b*d*x+b*c+a)}/x^2 - f^2*F^{(b*d*x+b*c+a)}/x - 1/6*b*d*e^2*F^{(b*d*x+b*c+a)}*\ln(F)/x^2 - b*d*e*f*F^{(b*d*x+b*c+a)}*\ln(F)/x + b*d*f^2*F^{(b*c+a)}*\text{Ei}(b*d*x*\ln(F))*\ln(F) - 1/6*b^2*d^2*e^2*F^{(b*d*x+b*c+a)}*\ln(F)^2/x + b^2*d^2*e*f*F^{(b*c+a)}*\text{Ei}(b*d*x*\ln(F))*\ln(F)^2 + 1/6*b^3*d^3*e^2*F^{(b*c+a)}*\text{Ei}(b*d*x*\ln(F))*\ln(F)^3$

**Rubi [A]**

time = 0.31, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {2230, 2208, 2209}

$$\frac{1}{6}b^3d^3e^2\log^3(F)F^{a+bc}\text{Ei}(bdx\log(F)) - \frac{b^2d^2e^2\log^2(F)F^{a+bc+bdx}}{6x} + b^2d^2ef\log^2(F)F^{a+bc}\text{Ei}(bdx\log(F)) - \frac{e^2F^{a+bc+bdx}}{3x^3} - \frac{bde^2\log(F)F^{a+bc+bdx}}{6x^2} - \frac{efF^{a+bc+bdx}}{x^2} - \frac{bdef\log(F)F^{a+bc+bdx}}{x} + bdf^2\log(F)F^{a+bc}\text{Ei}(bdx\log(F)) - \frac{f^2F^{a+bc+bdx}}{x}$$

Antiderivative was successfully verified.

[In] Int[(F^(a + b\*(c + d\*x))\*(e + f\*x)^2)/x^4,x]

[Out]  $-1/3*(e^2*F^{(a + b*c + b*d*x)})/x^3 - (e*f*F^{(a + b*c + b*d*x)})/x^2 - (f^2*F^{(a + b*c + b*d*x)})/x - (b*d*e^2*F^{(a + b*c + b*d*x)}*\text{Log}[F])/(6*x^2) - (b*d*e*f*F^{(a + b*c + b*d*x)}*\text{Log}[F])/x + b*d*f^2*F^{(a + b*c)}*\text{ExpIntegralEi}[b*d*x*\text{Log}[F]]*\text{Log}[F] - (b^2*d^2*e^2*F^{(a + b*c + b*d*x)}*\text{Log}[F]^2)/(6*x) + b^2*d^2*e*f*F^{(a + b*c)}*\text{ExpIntegralEi}[b*d*x*\text{Log}[F]]*\text{Log}[F]^2 + (b^3*d^3*e^2*F^{(a + b*c)}*\text{ExpIntegralEi}[b*d*x*\text{Log}[F]]*\text{Log}[F]^3)/6$

**Rule 2208**

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(c + d\*x)^(m + 1)\*((b\*F^(g\*(e + f\*x)))^n/(d\*(m + 1))), x] - Dist[f\*g\*n\*(Log[F]/(d\*(m + 1))), Int[(c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

**Rule 2209**

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - c\*(f/d)))/d)\*ExpIntegralEi[f\*g\*(c + d\*x)\*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

**Rule 2230**

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] := Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F,
c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] && !TrueQ[$UseGamma]
```

Rubi steps

$$\begin{aligned}
\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^4} dx &= \int \left( \frac{e^2 F^{a+bc+bdx}}{x^4} + \frac{2ef F^{a+bc+bdx}}{x^3} + \frac{f^2 F^{a+bc+bdx}}{x^2} \right) dx \\
&= e^2 \int \frac{F^{a+bc+bdx}}{x^4} dx + (2ef) \int \frac{F^{a+bc+bdx}}{x^3} dx + f^2 \int \frac{F^{a+bc+bdx}}{x^2} dx \\
&= -\frac{e^2 F^{a+bc+bdx}}{3x^3} - \frac{ef F^{a+bc+bdx}}{x^2} - \frac{f^2 F^{a+bc+bdx}}{x} + \frac{1}{3} (bde^2 \log(F)) \int \frac{F^{a+bc+bdx}}{x^3} dx \\
&= -\frac{e^2 F^{a+bc+bdx}}{3x^3} - \frac{ef F^{a+bc+bdx}}{x^2} - \frac{f^2 F^{a+bc+bdx}}{x} - \frac{bde^2 F^{a+bc+bdx} \log(F)}{6x^2} - \frac{bdef F^{a+bc+bdx}}{6x^2} \\
&= -\frac{e^2 F^{a+bc+bdx}}{3x^3} - \frac{ef F^{a+bc+bdx}}{x^2} - \frac{f^2 F^{a+bc+bdx}}{x} - \frac{bde^2 F^{a+bc+bdx} \log(F)}{6x^2} - \frac{bdef F^{a+bc+bdx}}{6x^2} \\
&= -\frac{e^2 F^{a+bc+bdx}}{3x^3} - \frac{ef F^{a+bc+bdx}}{x^2} - \frac{f^2 F^{a+bc+bdx}}{x} - \frac{bde^2 F^{a+bc+bdx} \log(F)}{6x^2} - \frac{bdef F^{a+bc+bdx}}{6x^2}
\end{aligned}$$

**Mathematica [A]**

time = 0.25, size = 116, normalized size = 0.53

$$\frac{F^{a+bc} (bdx^3 \text{Ei}(bdx \log(F)) \log(F) (6f^2 + 6bdef \log(F) + b^2 d^2 e^2 \log^2(F)) - F^{bdx} (2(e^2 + 3efx + 3f^2 x^2) + bdx(e + 6fx) \log(F) + b^2 d^2 e^2 x^2 \log^2(F)))}{6x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(F^(a + b\*(c + d\*x))\*(e + f\*x)^2)/x^4,x]

[Out] (F^(a + b\*c)\*(b\*d\*x^3\*ExpIntegralEi[b\*d\*x\*Log[F]]\*Log[F]\*(6\*f^2 + 6\*b\*d\*e\*f\*Log[F] + b^2\*d^2\*e^2\*Log[F]^2) - F^(b\*d\*x)\*(2\*(e^2 + 3\*e\*f\*x + 3\*f^2\*x^2) + b\*d\*e\*x\*(e + 6\*f\*x)\*Log[F] + b^2\*d^2\*e^2\*x^2\*Log[F]^2))/(6\*x^3)

**Maple [A]**

time = 0.10, size = 290, normalized size = 1.34

method	result
risch	$-\frac{\ln(F)^3 b^3 d^3 e^2 F^{cb} F^a \exp\text{Integral}(1, cb \ln(F) + \ln(F)a - bdx \ln(F) - (cb+a) \ln(F))}{6} - \frac{ef F^{bdx} F^{cb+a}}{x^2} - \frac{\ln(F) bdef F^{bdx} F^{cb+a}}{x} - \ln$
meijerg	$-bd \ln(F) F^{cb+a} f^2 \left( -\frac{2+2bdx \ln(F)}{2bdx \ln(F)} + \frac{e^{bdx \ln(F)}}{bdx \ln(F)} + \ln(-bdx \ln(F)) + \exp\text{Integral}(1, -bdx \ln(F)) + 1 \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c))*(f*x+e)^2/x^4,x,method=_RETURNVERBOSE)`

[Out] 
$$-1/6*\ln(F)^3*b^3*d^3*e^2*F^{(c*b)*F^a*Ei(1,c*b*\ln(F)+\ln(F)*a-b*d*x*\ln(F)-(b*c+a)*\ln(F))-e*f*F^{(b*d*x)*F^{(b*c+a)}/x^2-\ln(F)*b*d*e*f*F^{(b*d*x)*F^{(b*c+a)}/x-\ln(F)^2*b^2*d^2*e*f*F^{(c*b)*F^a*Ei(1,c*b*\ln(F)+\ln(F)*a-b*d*x*\ln(F)-(b*c+a)*\ln(F))}-1/3*e^2*F^{(b*d*x)*F^{(b*c+a)}/x^3-1/6*\ln(F)*b*d*e^2*F^{(b*d*x)*F^{(b*c+a)}/x^2-1/6*\ln(F)^2*b^2*d^2*e^2*F^{(b*d*x)*F^{(b*c+a)}/x-f^2*F^{(b*d*x)*F^{(b*c+a)}/x-\ln(F)*b*d*f^2*F^{(c*b)*F^a*Ei(1,c*b*\ln(F)+\ln(F)*a-b*d*x*\ln(F)-(b*c+a)*\ln(F))}$$

**Maxima** [A]

time = 0.45, size = 85, normalized size = 0.39

$$F^{bc+a}b^3d^3e^2\Gamma(-3,-bdx\log(F))\log(F)^3 - 2F^{bc+a}b^2d^2fe\Gamma(-2,-bdx\log(F))\log(F)^2 + F^{bc+a}bdf^2\Gamma(-1,-bdx\log(F))\log(F)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^4,x, algorithm="maxima")`

[Out] 
$$F^{(b*c + a)*b^3*d^3*e^2*\gamma(-3, -b*d*x*\log(F))*\log(F)^3 - 2*F^{(b*c + a)*b^2*d^2*f*e*\gamma(-2, -b*d*x*\log(F))*\log(F)^2 + F^{(b*c + a)*b*d*f^2*\gamma(-1, -b*d*x*\log(F))*\log(F)}$$

**Fricas** [A]

time = 0.35, size = 136, normalized size = 0.63

$$\frac{(b^3d^3x^3e^2\log(F)^3 + 6b^2d^2fx^3e\log(F)^2 + 6bdf^2x^3\log(F))F^{bc+a}Ei(bdx\log(F)) - (b^2d^2x^2e^2\log(F)^2 + 6f^2x^2 + 6fxe + (6bdfx^2e + bdx^2e)\log(F) + 2e^2)F^{bdx+bc+a}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^4,x, algorithm="fricas")`

[Out] 
$$1/6*((b^3*d^3*x^3*e^2*\log(F)^3 + 6*b^2*d^2*f*x^3*e*\log(F)^2 + 6*b*d*f^2*x^3*\log(F))*F^{(b*c + a)*Ei(b*d*x*\log(F))} - (b^2*d^2*x^2*e^2*\log(F)^2 + 6*f^2*x^2 + 6*f*x*e + (6*b*d*f*x^2*e + b*d*x*e^2)*\log(F) + 2*e^2)*F^{(b*d*x + b*c + a)})/x^3$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F**(a+b*(d*x+c))*(f*x+e)**2/x**4,x)`

[Out] Integral(F\*\*(a + b\*(c + d\*x))\*(e + f\*x)\*\*2/x\*\*4, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(a+b\*(d\*x+c))\*(f\*x+e)^2/x^4,x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*F^((d\*x + c)\*b + a)/x^4, x)

**Mupad [B]**

time = 3.66, size = 202, normalized size = 0.93

$$-\frac{F^{bdx} F^{a+bc} f^2}{x} - F^{a+bc} b^3 d^3 e^2 \ln(F)^3 \left( F^{bdx} \left( \frac{1}{6bdx \ln(F)} + \frac{1}{6b^2 d^2 x^2 \ln(F)^2} + \frac{1}{3b^3 d^3 x^3 \ln(F)^3} \right) + \frac{\operatorname{expint}(-bdx \ln(F))}{6} \right) - F^{a+bc} b d f^2 \ln(F) \operatorname{expint}(-bdx \ln(F)) - 2 F^{a+bc} b^2 d^2 e f \ln(F)^2 \left( \frac{\operatorname{expint}(-bdx \ln(F))}{2} + F^{bdx} \left( \frac{1}{2bdx \ln(F)} + \frac{1}{2b^2 d^2 x^2 \ln(F)^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((F^(a + b\*(c + d\*x))\*(e + f\*x)^2)/x^4,x)

[Out] - (F^(b\*d\*x)\*F^(a + b\*c)\*f^2)/x - F^(a + b\*c)\*b^3\*d^3\*e^2\*log(F)^3\*(F^(b\*d\*x)\*(1/(6\*b\*d\*x\*log(F)) + 1/(6\*b^2\*d^2\*x^2\*log(F)^2) + 1/(3\*b^3\*d^3\*x^3\*log(F)^3)) + expint(-b\*d\*x\*log(F))/6) - F^(a + b\*c)\*b\*d\*f^2\*log(F)\*expint(-b\*d\*x\*log(F)) - 2\*F^(a + b\*c)\*b^2\*d^2\*e\*f\*log(F)^2\*(expint(-b\*d\*x\*log(F))/2 + F^(b\*d\*x)\*(1/(2\*b\*d\*x\*log(F)) + 1/(2\*b^2\*d^2\*x^2\*log(F)^2)))



$$3.73 \quad \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^5} dx$$

**Optimal.** Leaf size=321

$$\frac{e^2 F^{a+bc+bdx}}{4x^4} - \frac{2ef F^{a+bc+bdx}}{3x^3} - \frac{f^2 F^{a+bc+bdx}}{2x^2} - \frac{bde^2 F^{a+bc+bdx} \log(F)}{12x^3} - \frac{bdef F^{a+bc+bdx} \log(F)}{3x^2} - \frac{bdf^2 F^{a+bc+bdx}}{2x}$$

[Out]  $-1/4 * e^2 * F^{(b*d*x+b*c+a)} / x^4 - 2/3 * e * f * F^{(b*d*x+b*c+a)} / x^3 - 1/2 * f^2 * F^{(b*d*x+b*c+a)} / x^2 - 1/12 * b * d * e^2 * F^{(b*d*x+b*c+a)} * \ln(F) / x^3 - 1/3 * b * d * e * f * F^{(b*d*x+b*c+a)} * \ln(F) / x^2 - 1/2 * b * d * f^2 * F^{(b*d*x+b*c+a)} * \ln(F) / x - 1/24 * b^2 * d^2 * e^2 * F^{(b*d*x+b*c+a)} * \ln(F)^2 / x + 1/2 * b^2 * d^2 * f^2 * F^{(b*c+a)} * \text{Ei}(b*d*x*\ln(F)) * \ln(F)^2 - 1/24 * b^3 * d^3 * e^2 * F^{(b*d*x+b*c+a)} * \ln(F)^3 / x + 1/3 * b^3 * d^3 * e * f * F^{(b*c+a)} * \text{Ei}(b*d*x*\ln(F)) * \ln(F)^3 + 1/24 * b^4 * d^4 * e^2 * F^{(b*c+a)} * \text{Ei}(b*d*x*\ln(F)) * \ln(F)^4$

**Rubi** [A]

time = 0.40, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {2230, 2208, 2209}

$$\frac{1}{24} b^4 d^4 e^2 \log(F) F^{a+b(c+dx)} \text{Ei}(bdx \log(F)) - \frac{b^4 d^4 e^2 \log(F) F^{a+b(c+dx)}}{24x} + \frac{1}{3} b^4 d^4 e f \log(F) F^{a+b(c+dx)} \text{Ei}(bdx \log(F)) - \frac{b^4 d^4 e^3 \log(F) F^{a+b(c+dx)}}{24x^2} - \frac{b^4 d^4 e f \log(F) F^{a+b(c+dx)}}{3x} + \frac{1}{2} b^4 d^4 f^2 \log(F) F^{a+b(c+dx)} \text{Ei}(bdx \log(F)) - \frac{e^2 F^{a+b(c+dx)}}{4x^4} - \frac{bde^2 \log(F) F^{a+b(c+dx)}}{12x^3} - \frac{2ef F^{a+b(c+dx)}}{3x^3} - \frac{bdef \log(F) F^{a+b(c+dx)}}{3x^2} - \frac{f^2 F^{a+b(c+dx)}}{2x^2} - \frac{bdf^2 \log(F) F^{a+b(c+dx)}}{2x}$$

Antiderivative was successfully verified.

[In] Int[(F^(a + b\*(c + d\*x))\*(e + f\*x)^2)/x^5, x]

[Out]  $-1/4 * (e^2 * F^{(a + b*c + b*d*x)}) / x^4 - (2 * e * f * F^{(a + b*c + b*d*x)}) / (3 * x^3) - (f^2 * F^{(a + b*c + b*d*x)}) / (2 * x^2) - (b * d * e^2 * F^{(a + b*c + b*d*x)} * \text{Log}[F]) / (12 * x^3) - (b * d * e * f * F^{(a + b*c + b*d*x)} * \text{Log}[F]) / (3 * x^2) - (b * d * f^2 * F^{(a + b*c + b*d*x)} * \text{Log}[F]) / (2 * x) - (b^2 * d^2 * e^2 * F^{(a + b*c + b*d*x)} * \text{Log}[F]^2) / (24 * x^2) - (b^2 * d^2 * e * f * F^{(a + b*c + b*d*x)} * \text{Log}[F]^2) / (3 * x) + (b^2 * d^2 * f^2 * F^{(a + b*c)} * \text{ExpIntegralEi}[b*d*x*\text{Log}[F]] * \text{Log}[F]^2) / 2 - (b^3 * d^3 * e^2 * F^{(a + b*c + b*d*x)} * \text{Log}[F]^3) / (24 * x) + (b^3 * d^3 * e * f * F^{(a + b*c)} * \text{ExpIntegralEi}[b*d*x*\text{Log}[F]] * \text{Log}[F]^3) / 3 + (b^4 * d^4 * e^2 * F^{(a + b*c)} * \text{ExpIntegralEi}[b*d*x*\text{Log}[F]] * \text{Log}[F]^4) / 24$

Rule 2208

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(c + d\*x)^(m + 1)\*((b \* F^(g\*(e + f\*x)))^n/(d\*(m + 1))), x] - Dist[f \* g \* n \* (Log[F]/(d\*(m + 1))), Int[(c + d\*x)^(m + 1)\*(b \* F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntEgerQ[2\*m] && !TrueQ[\$UseGamma]

Rule 2209

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - c\*(f/d)))/d) \* ExpIntegralEi[f \* g \* (c + d\*x) \* (Log[F]/d)], x] /; F

```
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

### Rule 2230

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] := Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x] /; FreeQ[{F,
c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] && !TrueQ[$UseGamma]
```

### Rubi steps

$$\begin{aligned}
\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^5} dx &= \int \left( \frac{e^2 F^{a+bc+bdx}}{x^5} + \frac{2ef F^{a+bc+bdx}}{x^4} + \frac{f^2 F^{a+bc+bdx}}{x^3} \right) dx \\
&= e^2 \int \frac{F^{a+bc+bdx}}{x^5} dx + (2ef) \int \frac{F^{a+bc+bdx}}{x^4} dx + f^2 \int \frac{F^{a+bc+bdx}}{x^3} dx \\
&= -\frac{e^2 F^{a+bc+bdx}}{4x^4} - \frac{2ef F^{a+bc+bdx}}{3x^3} - \frac{f^2 F^{a+bc+bdx}}{2x^2} + \frac{1}{4} (bde^2 \log(F)) \int \frac{F^{a+bc+bdx}}{x^4} dx \\
&= -\frac{e^2 F^{a+bc+bdx}}{4x^4} - \frac{2ef F^{a+bc+bdx}}{3x^3} - \frac{f^2 F^{a+bc+bdx}}{2x^2} - \frac{bde^2 F^{a+bc+bdx} \log(F)}{12x^3} - \frac{bdef F^{a+bc+bdx} \log^2(F)}{24x^4} \\
&= -\frac{e^2 F^{a+bc+bdx}}{4x^4} - \frac{2ef F^{a+bc+bdx}}{3x^3} - \frac{f^2 F^{a+bc+bdx}}{2x^2} - \frac{bde^2 F^{a+bc+bdx} \log(F)}{12x^3} - \frac{bdef F^{a+bc+bdx} \log^2(F)}{24x^4} \\
&= -\frac{e^2 F^{a+bc+bdx}}{4x^4} - \frac{2ef F^{a+bc+bdx}}{3x^3} - \frac{f^2 F^{a+bc+bdx}}{2x^2} - \frac{bde^2 F^{a+bc+bdx} \log(F)}{12x^3} - \frac{bdef F^{a+bc+bdx} \log^2(F)}{24x^4} \\
&= -\frac{e^2 F^{a+bc+bdx}}{4x^4} - \frac{2ef F^{a+bc+bdx}}{3x^3} - \frac{f^2 F^{a+bc+bdx}}{2x^2} - \frac{bde^2 F^{a+bc+bdx} \log(F)}{12x^3} - \frac{bdef F^{a+bc+bdx} \log^2(F)}{24x^4}
\end{aligned}$$

### Mathematica [A]

time = 0.30, size = 156, normalized size = 0.49

$$\frac{F^{a+bc} (b^2 d^2 x^4 \text{Ei}(bdx \log(F)) \log^2(F) (12f^2 + 8bdef \log(F) + b^2 d^2 e^2 \log^2(F)) - F^{bdx} (2(3e^2 + 8efx + 6f^2 x^2) + 2bdx(e^2 + 4efx + 6f^2 x^2) \log(F) + b^2 d^2 ex^2(e + 8fx) \log^2(F) + b^3 d^3 e^2 x^3 \log^3(F)))}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(F^(a + b*(c + d*x))*(e + f*x)^2)/x^5, x]
```

```
[Out] (F^(a + b*c)*(b^2*d^2*x^4*ExpIntegralEi[b*d*x*Log[F]]*Log[F]^2*(12*f^2 + 8*
b*d*e*f*Log[F] + b^2*d^2*e^2*Log[F]^2) - F^(b*d*x)*(2*(3*e^2 + 8*e*f*x + 6*
f^2*x^2) + 2*b*d*x*(e^2 + 4*e*f*x + 6*f^2*x^2)*Log[F] + b^2*d^2*e*x^2*(e +
8*f*x)*Log[F]^2 + b^3*d^3*e^2*x^3*Log[F]^3)))/(24*x^4)
```

### Maple [A]

time = 0.10, size = 382, normalized size = 1.19

method	result
risch	$\frac{\ln(F)^4 b^4 d^4 e^2 F^{cb} F^a \exp\text{Integral}(1, cb \ln(F) + \ln(F)a - bdx \ln(F) - (cb+a) \ln(F))}{24} - \frac{2ef F^{bdx} F^{cb+a}}{3x^3} - \frac{\ln(F)bdef F^{bdx} F^{cb+a}}{3x^2}$
meijerg	$\ln(F)^2 b^2 d^2 F^{cb+a} f^2 \left( \frac{9b^2 d^2 x^2 \ln(F)^2 + 12bdx \ln(F) + 6}{12b^2 d^2 x^2 \ln(F)^2} - \frac{(3bdx \ln(F) + 3)e^{bdx \ln(F)}}{6b^2 d^2 x^2 \ln(F)^2} - \frac{\ln(-bdx \ln(F))}{2} - \frac{\exp\text{Integral}(1, -}{2} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(a+b*(d*x+c))*(f*x+e)^2/x^5,x,method=_RETURNVERBOSE)`

[Out] 
$$\begin{aligned} & -1/24*\ln(F)^4*b^4*d^4*e^2*F^{(c*b)*F^a*Ei(1,c*b*\ln(F)+\ln(F)*a-b*d*x*\ln(F)-(b \\ & *c+a)*\ln(F))-2/3*e*f*F^{(b*d*x)*F^{(b*c+a)}/x^3-1/3*\ln(F)*b*d*e*f*F^{(b*d*x)*F^{ \\ & (b*c+a)}/x^2-1/3*\ln(F)^2*b^2*d^2*e*f*F^{(b*d*x)*F^{(b*c+a)}/x-1/3*\ln(F)^3*b^3*d \\ & ^3*e*f*F^{(c*b)*F^a*Ei(1,c*b*\ln(F)+\ln(F)*a-b*d*x*\ln(F)-(b*c+a)*\ln(F))-1/2*\ln \\ & (F)^2*b^2*d^2*f^2*F^{(c*b)*F^a*Ei(1,c*b*\ln(F)+\ln(F)*a-b*d*x*\ln(F)-(b*c+a)*\ln \\ & (F))-1/2*f^2*F^{(b*d*x)*F^{(b*c+a)}/x^2-1/2*\ln(F)*b*d*f^2*F^{(b*d*x)*F^{(b*c+a)}/ \\ & x-1/12*\ln(F)*b*d*e^2*F^{(b*d*x)*F^{(b*c+a)}/x^3-1/24*\ln(F)^2*b^2*d^2*e^2*F^{(b* \\ & d*x)*F^{(b*c+a)}/x^2-1/24*\ln(F)^3*b^3*d^3*e^2*F^{(b*d*x)*F^{(b*c+a)}/x-1/4*e^2*F \\ & ^{(b*d*x)*F^{(b*c+a)}/x^4} \end{aligned}$$

**Maxima [A]**

time = 0.41, size = 93, normalized size = 0.29

$$-F^{bc+a} b^4 d^4 e^2 \Gamma(-4, -bdx \log(F)) \log(F)^4 + 2 F^{bc+a} b^3 d^3 f e \Gamma(-3, -bdx \log(F)) \log(F)^3 - F^{bc+a} b^2 d^2 f^2 \Gamma(-2, -bdx \log(F)) \log(F)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^5,x, algorithm="maxima")`

[Out] 
$$\begin{aligned} & -F^{(b*c + a)*b^4*d^4*e^2*\gamma(-4, -b*d*x*\log(F))*\log(F)^4 + 2*F^{(b*c + a)* \\ & b^3*d^3*f*e*\gamma(-3, -b*d*x*\log(F))*\log(F)^3 - F^{(b*c + a)*b^2*d^2*f^2*\gamma \\ & (-2, -b*d*x*\log(F))*\log(F)^2 \end{aligned}$$

**Fricas [A]**

time = 0.50, size = 185, normalized size = 0.58

$$\frac{(b^4 d^4 x^4 e^2 \log(F)^4 + 8 b^3 d^3 f x^3 e \log(F)^3 + 12 b^2 d^2 f^2 x^2 \log(F)^2) F^{bc+a} Ei(bdx \log(F)) - (b^3 d^3 x^3 e^2 \log(F)^3 + 12 f^2 x^2 + 16 f x e + (8 b^2 d^2 f x^2 e + b^2 d^2 x^2 e^2) \log(F)^2 + 2(6 b d f^2 x^2 + 4 b d f x^2 e + b d x e^2) \log(F) + 6 e^2) F^{bdx+bc+a}}{24 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^5,x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & 1/24*((b^4*d^4*x^4*e^2*\log(F)^4 + 8*b^3*d^3*f*x^3*e*\log(F)^3 + 12*b^2*d^2*f \\ & ^2*x^2*\log(F)^2)*F^{(b*c + a)*Ei(b*d*x*\log(F))} - (b^3*d^3*x^3*e^2*\log(F)^3 + \\ & 12*f^2*x^2 + 16*f*x*e + (8*b^2*d^2*f*x^2*e + b^2*d^2*x^2*e^2)*\log(F)^2 + 2 \\ & *(6*b*d*f^2*x^3 + 4*b*d*f*x^2*e + b*d*x*e^2)*\log(F) + 6*e^2)*F^{(b*d*x + b*c \\ & + a)}/x^4 \end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(F\*\*(a+b\*(d\*x+c))\*(f\*x+e)\*\*2/x\*\*5,x)**[Out]** Integral(F\*\*(a + b\*(c + d\*x))\*(e + f\*x)\*\*2/x\*\*5, x)**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(F^(a+b\*(d\*x+c))\*(f\*x+e)^2/x^5,x, algorithm="giac")**[Out]** integrate((f\*x + e)^2\*F^((d\*x + c)\*b + a)/x^5, x)**Mupad [B]**

time = 3.68, size = 258, normalized size = 0.80

$$-F^{a+b(c+dx)} e^2 f^2 \ln(F)^2 \left( \frac{\operatorname{expint}(-bdx \ln(F))}{2} + F^{a+b(c+dx)} \left( \frac{1}{2bdx \ln(F)^2} + \frac{1}{2b^2 d^2 x^2 \ln(F)^2} \right) \right) - F^{a+b(c+dx)} e^2 f \ln(F)^2 \left( F^{a+b(c+dx)} \left( \frac{1}{24bdx \ln(F)^2} + \frac{1}{24b^2 d^2 x^2 \ln(F)^2} + \frac{1}{12b^3 d^3 x^3 \ln(F)^2} + \frac{1}{4b^4 d^4 x^4 \ln(F)^2} \right) + \frac{\operatorname{expint}(-bdx \ln(F))}{24} \right) - 2F^{a+b(c+dx)} e f \ln(F)^2 \left( F^{a+b(c+dx)} \left( \frac{1}{6bdx \ln(F)^2} + \frac{1}{6b^2 d^2 x^2 \ln(F)^2} + \frac{1}{3b^3 d^3 x^3 \ln(F)^2} \right) + \frac{\operatorname{expint}(-bdx \ln(F))}{6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int((F^(a + b\*(c + d\*x))\*(e + f\*x)^2)/x^5,x)

**[Out]**  $-F^{a+b(c+dx)} b^2 d^2 f^2 \log(F)^2 (\operatorname{expint}(-b d x \log(F))/2 + F^{b d x} (1/(2 b d x \log(F)) + 1/(2 b^2 d^2 x^2 \log(F)^2))) - F^{a+b(c+dx)} b^4 d^4 e^2 \log(F)^4 (F^{b d x} (1/(24 b d x \log(F)) + 1/(24 b^2 d^2 x^2 \log(F)^2) + 1/(12 b^3 d^3 x^3 \log(F)^3) + 1/(4 b^4 d^4 x^4 \log(F)^4)) + \operatorname{expint}(-b d x \log(F))/24) - 2 F^{a+b(c+dx)} b^3 d^3 e f \log(F)^3 (F^{b d x} (1/(6 b d x \log(F)) + 1/(6 b^2 d^2 x^2 \log(F)^2) + 1/(3 b^3 d^3 x^3 \log(F)^3)) + \operatorname{expint}(-b d x \log(F))/6)$

### 3.74 $\int e^{-a-bx}(a+bx)^4(c+dx)^3 dx$

**Optimal.** Leaf size=754

$$\frac{5040d^3e^{-a-bx}}{b^4} - \frac{2160d^2(bc-ad)e^{-a-bx}}{b^4} - \frac{360d(bc-ad)^2e^{-a-bx}}{b^4} - \frac{24(bc-ad)^3e^{-a-bx}}{b^4} - \frac{5040d^3e^{-a-bx}(a+bx)^4}{b^4}$$

[Out]  $-(a*d+b*c)^3*\exp(-b*x-a)*(b*x+a)^4/b^4-d^3*\exp(-b*x-a)*(b*x+a)^7/b^4-5040*d^3*\exp(-b*x-a)/b^4-24*(a*d+b*c)^3*\exp(-b*x-a)/b^4-2160*d^2*(-a*d+b*c)*\exp(-b*x-a)/b^4-360*d*(a*d+b*c)^2*\exp(-b*x-a)/b^4-5040*d^3*\exp(-b*x-a)*(b*x+a)/b^4-24*(a*d+b*c)^3*\exp(-b*x-a)*(b*x+a)/b^4-2520*d^3*\exp(-b*x-a)*(b*x+a)^2/b^4-12*(a*d+b*c)^3*\exp(-b*x-a)*(b*x+a)^2/b^4-840*d^3*\exp(-b*x-a)*(b*x+a)^3/b^4-4*(a*d+b*c)^3*\exp(-b*x-a)*(b*x+a)^3/b^4-210*d^3*\exp(-b*x-a)*(b*x+a)^4/b^4-42*d^3*\exp(-b*x-a)*(b*x+a)^5/b^4-7*d^3*\exp(-b*x-a)*(b*x+a)^6/b^4-2160*d^2*(-a*d+b*c)*\exp(-b*x-a)*(b*x+a)/b^4-360*d*(a*d+b*c)^2*\exp(-b*x-a)*(b*x+a)/b^4-1080*d^2*(-a*d+b*c)*\exp(-b*x-a)*(b*x+a)^2/b^4-180*d*(a*d+b*c)^2*\exp(-b*x-a)*(b*x+a)^2/b^4-360*d^2*(-a*d+b*c)*\exp(-b*x-a)*(b*x+a)^3/b^4-60*d*(a*d+b*c)^2*\exp(-b*x-a)*(b*x+a)^3/b^4-90*d^2*(-a*d+b*c)*\exp(-b*x-a)*(b*x+a)^4/b^4-15*d*(a*d+b*c)^2*\exp(-b*x-a)*(b*x+a)^4/b^4-18*d^2*(-a*d+b*c)*\exp(-b*x-a)*(b*x+a)^5/b^4-3*d*(a*d+b*c)^2*\exp(-b*x-a)*(b*x+a)^5/b^4-3*d^2*(-a*d+b*c)*\exp(-b*x-a)*(b*x+a)^6/b^4$

**Rubi [A]**

time = 0.65, antiderivative size = 754, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$ , Rules used = {2227, 2207, 2225}

Antiderivative was successfully verified.

[In] Int[E<sup>-(a + b\*x)</sup>\*(a + b\*x)<sup>4</sup>\*(c + d\*x)<sup>3</sup>, x]

[Out]  $(-5040*d^3*E^{-(a+b*x)})/b^4 - (2160*d^2*(b*c-a*d)*E^{-(a+b*x)})/b^4 - (360*d*(b*c-a*d)^2*E^{-(a+b*x)})/b^4 - (24*(b*c-a*d)^3*E^{-(a+b*x)})/b^4 - (5040*d^3*E^{-(a+b*x)}*(a+b*x))/b^4 - (2160*d^2*(b*c-a*d)*E^{-(a+b*x)}*(a+b*x))/b^4 - (360*d*(b*c-a*d)^2*E^{-(a+b*x)}*(a+b*x))/b^4 - (24*(b*c-a*d)^3*E^{-(a+b*x)}*(a+b*x))/b^4 - (2520*d^3*E^{-(a+b*x)}*(a+b*x)^2)/b^4 - (1080*d^2*(b*c-a*d)*E^{-(a+b*x)}*(a+b*x)^2)/b^4 - (180*d*(b*c-a*d)^2*E^{-(a+b*x)}*(a+b*x)^2)/b^4 - (12*(b*c-a*d)^3*E^{-(a+b*x)}*(a+b*x)^2)/b^4 - (840*d^3*E^{-(a+b*x)}*(a+b*x)^3)/b^4 - (360*d^2*(b*c-a*d)*E^{-(a+b*x)}*(a+b*x)^3)/b^4 - (60*d*(b*c-a*d)^2*E^{-(a+b*x)}*(a+b*x)^3)/b^4 - (4*(b*c-a*d)^3*E^{-(a+b*x)}*(a+b*x)^3)/b^4 - (210*d^3*E^{-(a+b*x)}*(a+b*x)^4)/b^4 - (90*d^2*(b*c-a*d)*E^{-(a+b*x)}*(a+b*x)^4)/b^4 - (15*d*(b*c-a*d)^2*E^{-(a+b*x)}*(a+b*x)^4)/b^4 - ((b*c-a*d)^3*E^{-(a+b*x)}*(a+b*x)^4)/b^4 - (42*d^3*E^{-(a+b*x)}*(a+b*x)^5)/b^4 - (18*d^2$

$$2*(b*c - a*d)*E^{-a - b*x}*(a + b*x)^5/b^4 - (3*d*(b*c - a*d)^2*E^{-a - b*x}*(a + b*x)^5)/b^4 - (7*d^3*E^{-a - b*x}*(a + b*x)^6)/b^4 - (3*d^2*(b*c - a*d)*E^{-a - b*x}*(a + b*x)^6)/b^4 - (d^3*E^{-a - b*x}*(a + b*x)^7)/b^4$$
Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2227

```
Int[(F_)^((c_.)*(v_))*(u_), x_Symbol] := Int[ExpandIntegrand[F^(c*ExpandToSum[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v, x] && !TrueQ[$UseGamma]
```

Rubi steps

$$\begin{aligned}
\int e^{-a-bx}(a+bx)^4(c+dx)^3 dx &= \int \left( \frac{(bc-ad)^3 e^{-a-bx}(a+bx)^4}{b^3} + \frac{3d(bc-ad)^2 e^{-a-bx}(a+bx)^5}{b^3} + \frac{3d^2(bc-ad)}{b^3} \right. \\
&= \frac{d^3 \int e^{-a-bx}(a+bx)^7 dx}{b^3} + \frac{(3d^2(bc-ad)) \int e^{-a-bx}(a+bx)^6 dx}{b^3} + \frac{(3d(bc-ad)) \int e^{-a-bx}(a+bx)^5 dx}{b^3} \\
&= -\frac{(bc-ad)^3 e^{-a-bx}(a+bx)^4}{b^4} - \frac{3d(bc-ad)^2 e^{-a-bx}(a+bx)^5}{b^4} - \frac{3d^2(bc-ad) e^{-a-bx}(a+bx)^6}{b^4} \\
&= -\frac{4(bc-ad)^3 e^{-a-bx}(a+bx)^3}{b^4} - \frac{15d(bc-ad)^2 e^{-a-bx}(a+bx)^4}{b^4} - \frac{(bc-ad)^3 e^{-a-bx}(a+bx)^5}{b^4} \\
&= -\frac{12(bc-ad)^3 e^{-a-bx}(a+bx)^2}{b^4} - \frac{60d(bc-ad)^2 e^{-a-bx}(a+bx)^3}{b^4} - \frac{4(bc-ad)^3 e^{-a-bx}(a+bx)^4}{b^4} \\
&= -\frac{24(bc-ad)^3 e^{-a-bx}(a+bx)}{b^4} - \frac{180d(bc-ad)^2 e^{-a-bx}(a+bx)^2}{b^4} - \frac{12(bc-ad)^3 e^{-a-bx}(a+bx)^3}{b^4} \\
&= -\frac{24(bc-ad)^3 e^{-a-bx}}{b^4} - \frac{360d(bc-ad)^2 e^{-a-bx}(a+bx)}{b^4} - \frac{24(bc-ad)^3 e^{-a-bx}(a+bx)^2}{b^4} \\
&= -\frac{360d(bc-ad)^2 e^{-a-bx}}{b^4} - \frac{24(bc-ad)^3 e^{-a-bx}}{b^4} - \frac{2160d^2(bc-ad) e^{-a-bx}(a+bx)}{b^4} \\
&= -\frac{2160d^2(bc-ad) e^{-a-bx}}{b^4} - \frac{360d(bc-ad)^2 e^{-a-bx}}{b^4} - \frac{24(bc-ad)^3 e^{-a-bx}}{b^4} \\
&= -\frac{5040d^3 e^{-a-bx}}{b^4} - \frac{2160d^2(bc-ad) e^{-a-bx}}{b^4} - \frac{360d(bc-ad)^2 e^{-a-bx}}{b^4} - \frac{24(bc-ad)^3 e^{-a-bx}}{b^4}
\end{aligned}$$

**Mathematica [A]**

time = 1.88, size = 458, normalized size = 0.61

Antiderivative was successfully verified.

`[In] Integrate[E^(-a - b*x)*(a + b*x)^4*(c + d*x)^3,x]`

```

[Out] (E^(-a - b*x)*(-6*(840 + 480*a + 120*a^2 + 16*a^3 + a^4)*d^3 - b^7*x^4*(c +
d*x)^3 - b^6*x^3*(c + d*x)^2*(4*(1 + a)*c + (7 + 4*a)*d*x) - 6*b*d^2*((360
+ 240*a + 72*a^2 + 12*a^3 + a^4)*c + (840 + 480*a + 120*a^2 + 16*a^3 + a^4
)*d*x) - 6*b^5*x^2*(c + d*x)*((2 + 2*a + a^2)*c^2 + 2*(4 + 3*a + a^2)*c*d*x
+ (7 + 4*a + a^2)*d^2*x^2) - 3*b^2*d*((120 + 96*a + 36*a^2 + 8*a^3 + a^4)*
c^2 + 2*(360 + 240*a + 72*a^2 + 12*a^3 + a^4)*c*d*x + (840 + 480*a + 120*a^
2 + 16*a^3 + a^4)*d^2*x^2) - 2*b^4*x*(2*(6 + 6*a + 3*a^2 + a^3)*c^3 + 3*(30
+ 24*a + 9*a^2 + 2*a^3)*c^2*d*x + 6*(30 + 20*a + 6*a^2 + a^3)*c*d^2*x^2 +
(105 + 60*a + 15*a^2 + 2*a^3)*d^3*x^3) - b^3*((24 + 24*a + 12*a^2 + 4*a^3 +
a^4)*c^3 + 3*(120 + 96*a + 36*a^2 + 8*a^3 + a^4)*c^2*d*x + 3*(360 + 240*a
+ 72*a^2 + 12*a^3 + a^4)*c*d^2*x^2 + (840 + 480*a + 120*a^2 + 16*a^3 + a^4)
*d^3*x^3))/b^4

```

**Maple [A]**

time = 0.08, size = 1240, normalized size = 1.64 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(-b*x-a)*(b*x+a)^4*(d*x+c)^3,x,method=_RETURNVERBOSE)`

[Out] 
$$\begin{aligned} & -1/b*(c^3*((-b*x-a)^4*exp(-b*x-a)-4*exp(-b*x-a)*(-b*x-a)^3+12*(-b*x-a)^2*exp(-b*x-a)-24*(-b*x-a)*exp(-b*x-a)+24*exp(-b*x-a))-d^3/b^3*((-b*x-a)^7*exp(-b*x-a)-7*exp(-b*x-a)*(-b*x-a)^6+42*(-b*x-a)^5*exp(-b*x-a)-210*(-b*x-a)^4*exp(-b*x-a)+840*exp(-b*x-a)*(-b*x-a)^3-2520*(-b*x-a)^2*exp(-b*x-a)+5040*(-b*x-a)*exp(-b*x-a)-5040*exp(-b*x-a))-d^3/b^3*a^3*((-b*x-a)^4*exp(-b*x-a)-4*exp(-b*x-a)*(-b*x-a)^3+12*(-b*x-a)^2*exp(-b*x-a)-24*(-b*x-a)*exp(-b*x-a)+24*exp(-b*x-a))-3*d/b*c^2*((-b*x-a)^5*exp(-b*x-a)-5*(-b*x-a)^4*exp(-b*x-a)+20*exp(-b*x-a)*(-b*x-a)^3-60*(-b*x-a)^2*exp(-b*x-a)+120*(-b*x-a)*exp(-b*x-a)-120*exp(-b*x-a))-3*d^3/b^3*a^2*((-b*x-a)^5*exp(-b*x-a)-5*(-b*x-a)^4*exp(-b*x-a)+20*exp(-b*x-a)*(-b*x-a)^3-60*(-b*x-a)^2*exp(-b*x-a)+120*(-b*x-a)*exp(-b*x-a)-120*exp(-b*x-a))+3*d^2/b^2*c*(exp(-b*x-a)*(-b*x-a)^6-6*(-b*x-a)^5*exp(-b*x-a)+30*(-b*x-a)^4*exp(-b*x-a)-120*exp(-b*x-a)*(-b*x-a)^3+360*(-b*x-a)^2*exp(-b*x-a)-720*(-b*x-a)*exp(-b*x-a)+720*exp(-b*x-a))-3*d^3/b^3*a*(exp(-b*x-a)*(-b*x-a)^6-6*(-b*x-a)^5*exp(-b*x-a)+30*(-b*x-a)^4*exp(-b*x-a)-120*exp(-b*x-a)*(-b*x-a)^3+360*(-b*x-a)^2*exp(-b*x-a)-720*(-b*x-a)*exp(-b*x-a)+720*exp(-b*x-a))-3*d/b*a*c^2*((-b*x-a)^4*exp(-b*x-a)-4*exp(-b*x-a)*(-b*x-a)^3+12*(-b*x-a)^2*exp(-b*x-a)-24*(-b*x-a)*exp(-b*x-a)+24*exp(-b*x-a))+3*d^2/b^2*a^2*c*((-b*x-a)^4*exp(-b*x-a)-4*exp(-b*x-a)*(-b*x-a)^3+12*(-b*x-a)^2*exp(-b*x-a)-24*(-b*x-a)*exp(-b*x-a)+24*exp(-b*x-a))+6*d^2/b^2*a*c*((-b*x-a)^5*exp(-b*x-a)-5*(-b*x-a)^4*exp(-b*x-a)+20*exp(-b*x-a)*(-b*x-a)^3-60*(-b*x-a)^2*exp(-b*x-a)+120*(-b*x-a)*exp(-b*x-a)-120*exp(-b*x-a))) \end{aligned}$$

**Maxima [A]**

time = 0.34, size = 894, normalized size = 1.19

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^4*(d*x+c)^3,x, algorithm="maxima")`

[Out] 
$$\begin{aligned} & -4*(b*x + 1)*a^3*c^3*e^{(-b*x - a)}/b - a^4*c^3*e^{(-b*x - a)}/b - 3*(b*x + 1)*a^4*c^2*d*e^{(-b*x - a)}/b^2 - 6*(b^2*x^2 + 2*b*x + 2)*a^2*c^3*e^{(-b*x - a)}/b - 12*(b^2*x^2 + 2*b*x + 2)*a^3*c^2*d*e^{(-b*x - a)}/b^2 - 3*(b^2*x^2 + 2*b*x + 2)*a^4*c*d^2*e^{(-b*x - a)}/b^3 - 4*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a*c^3*e^{(-b*x - a)}/b - 18*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a^2*c^2*d*e^{(-b*x - a)}/b^2 - 12*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a^3*c*d^2*e^{(-b*x - a)}/b^3 - (b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a^4*d^3*e^{(-b*x - a)}/b^4 - (b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*c^3*e^{(-b*x - a)}/b - 12*(b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*a*c^2*d*e^{(-b*x - a)}/b^2 - 18*(b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*a^2*c*d^2*e^{(-b*x - a)}/b^3 - 4*(b^4*x \end{aligned}$$



$$\begin{aligned} &^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*a^3*d^3*e^{(-b*x - a)/b^4} - 3*(b^5*x^5 + 5*b^4*x^4 + 20*b^3*x^3 + 60*b^2*x^2 + 120*b*x + 120)*c^2*d*e^{(-b*x - a)/b^2} - 12*(b^5*x^5 + 5*b^4*x^4 + 20*b^3*x^3 + 60*b^2*x^2 + 120*b*x + 120)*a*c*d^2*e^{(-b*x - a)/b^3} - 6*(b^5*x^5 + 5*b^4*x^4 + 20*b^3*x^3 + 60*b^2*x^2 + 120*b*x + 120)*a^2*d^3*e^{(-b*x - a)/b^4} - 3*(b^6*x^6 + 6*b^5*x^5 + 30*b^4*x^4 + 120*b^3*x^3 + 360*b^2*x^2 + 720*b*x + 720)*c*d^2*e^{(-b*x - a)/b^3} - 4*(b^6*x^6 + 6*b^5*x^5 + 30*b^4*x^4 + 120*b^3*x^3 + 360*b^2*x^2 + 720*b*x + 720)*a*d^3*e^{(-b*x - a)/b^4} - (b^7*x^7 + 7*b^6*x^6 + 42*b^5*x^5 + 210*b^4*x^4 + 840*b^3*x^3 + 2520*b^2*x^2 + 5040*b*x + 5040)*d^3*e^{(-b*x - a)/b^4} \end{aligned}$$

**Fricas** [A]

time = 0.49, size = 544, normalized size = 0.72

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4\*(d\*x+c)^3,x, algorithm="fricas")

[Out]  $-(b^7*d^3*x^7 + (a^4 + 4*a^3 + 12*a^2 + 24*a + 24)*b^3*c^3 + (3*b^7*c*d^2 + (4*a + 7)*b^6*d^3)*x^6 + 3*(a^4 + 8*a^3 + 36*a^2 + 96*a + 120)*b^2*c^2*d + 3*(b^7*c^2*d + 2*(2*a + 3)*b^6*c*d^2 + 2*(a^2 + 4*a + 7)*b^5*d^3)*x^5 + 6*(a^4 + 12*a^3 + 72*a^2 + 240*a + 360)*b*c*d^2 + (b^7*c^3 + 3*(4*a + 5)*b^6*c^2*d + 6*(3*a^2 + 10*a + 15)*b^5*c*d^2 + 2*(2*a^3 + 15*a^2 + 60*a + 105)*b^4*d^3)*x^4 + 6*(a^4 + 16*a^3 + 120*a^2 + 480*a + 840)*d^3 + (4*(a + 1)*b^6*c^3 + 6*(3*a^2 + 8*a + 10)*b^5*c^2*d + 12*(a^3 + 6*a^2 + 20*a + 30)*b^4*c*d^2 + (a^4 + 16*a^3 + 120*a^2 + 480*a + 840)*b^3*d^3)*x^3 + 3*(2*(a^2 + 2*a + 2)*b^5*c^3 + 2*(2*a^3 + 9*a^2 + 24*a + 30)*b^4*c^2*d + (a^4 + 12*a^3 + 72*a^2 + 240*a + 360)*b^3*c*d^2 + (a^4 + 16*a^3 + 120*a^2 + 480*a + 840)*b^2*d^3)*x^2 + (4*(a^3 + 3*a^2 + 6*a + 6)*b^4*c^3 + 3*(a^4 + 8*a^3 + 36*a^2 + 96*a + 120)*b^3*c^2*d + 6*(a^4 + 12*a^3 + 72*a^2 + 240*a + 360)*b^2*c*d^2 + 6*(a^4 + 16*a^3 + 120*a^2 + 480*a + 840)*b*d^3)*x)*e^{(-b*x - a)/b^4}$

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 1445 vs.  $2(695) = 1390$ .

time = 0.34, size = 1445, normalized size = 1.92

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)\*\*4\*(d\*x+c)\*\*3,x)

[Out] Piecewise((( -a\*\*4\*b\*\*3\*c\*\*3 - 3\*a\*\*4\*b\*\*3\*c\*\*2\*d\*x - 3\*a\*\*4\*b\*\*3\*c\*d\*\*2\*x\*\*2 - a\*\*4\*b\*\*3\*d\*\*3\*x\*\*3 - 3\*a\*\*4\*b\*\*2\*c\*\*2\*d - 6\*a\*\*4\*b\*\*2\*c\*d\*\*2\*x - 3\*a\*\*4\*b\*\*2\*d\*\*3\*x\*\*2 - 6\*a\*\*4\*b\*c\*d\*\*2 - 6\*a\*\*4\*b\*d\*\*3\*x - 6\*a\*\*4\*d\*\*3 - 4\*a\*\*3\*b\*\*4\*c\*\*3\*x - 12\*a\*\*3\*b\*\*4\*c\*\*2\*d\*x\*\*2 - 12\*a\*\*3\*b\*\*4\*c\*d\*\*2\*x\*\*3 - 4\*a\*\*3

```

b**4*d**3*x**4 - 4*a**3*b**3*c**3 - 24*a**3*b**3*c**2*d*x - 36*a**3*b**3*c
*d**2*x**2 - 16*a**3*b**3*d**3*x**3 - 24*a**3*b**2*c**2*d - 72*a**3*b**2*c*
d**2*x - 48*a**3*b**2*d**3*x**2 - 72*a**3*b*c*d**2 - 96*a**3*b*d**3*x - 96*
a**3*d**3 - 6*a**2*b**5*c**3*x**2 - 18*a**2*b**5*c**2*d*x**3 - 18*a**2*b**5
*c*d**2*x**4 - 6*a**2*b**5*d**3*x**5 - 12*a**2*b**4*c**3*x - 54*a**2*b**4*c
**2*d*x**2 - 72*a**2*b**4*c*d**2*x**3 - 30*a**2*b**4*d**3*x**4 - 12*a**2*b*
*3*c**3 - 108*a**2*b**3*c**2*d*x - 216*a**2*b**3*c*d**2*x**2 - 120*a**2*b**
3*d**3*x**3 - 108*a**2*b**2*c**2*d - 432*a**2*b**2*c*d**2*x - 360*a**2*b**2
*d**3*x**2 - 432*a**2*b*c*d**2 - 720*a**2*b*d**3*x - 720*a**2*d**3 - 4*a*b*
*6*c**3*x**3 - 12*a*b**6*c**2*d*x**4 - 12*a*b**6*c*d**2*x**5 - 4*a*b**6*d**
3*x**6 - 12*a*b**5*c**3*x**2 - 48*a*b**5*c**2*d*x**3 - 60*a*b**5*c*d**2*x**
4 - 24*a*b**5*d**3*x**5 - 24*a*b**4*c**3*x - 144*a*b**4*c**2*d*x**2 - 240*a
*b**4*c*d**2*x**3 - 120*a*b**4*d**3*x**4 - 24*a*b**3*c**3 - 288*a*b**3*c**2
*d*x - 720*a*b**3*c*d**2*x**2 - 480*a*b**3*d**3*x**3 - 288*a*b**2*c**2*d -
1440*a*b**2*c*d**2*x - 1440*a*b**2*d**3*x**2 - 1440*a*b*c*d**2 - 2880*a*b*d
**3*x - 2880*a*d**3 - b**7*c**3*x**4 - 3*b**7*c**2*d*x**5 - 3*b**7*c*d**2*x
**6 - b**7*d**3*x**7 - 4*b**6*c**3*x**3 - 15*b**6*c**2*d*x**4 - 18*b**6*c*d
**2*x**5 - 7*b**6*d**3*x**6 - 12*b**5*c**3*x**2 - 60*b**5*c**2*d*x**3 - 90*
b**5*c*d**2*x**4 - 42*b**5*d**3*x**5 - 24*b**4*c**3*x - 180*b**4*c**2*d*x**
2 - 360*b**4*c*d**2*x**3 - 210*b**4*d**3*x**4 - 24*b**3*c**3 - 360*b**3*c**
2*d*x - 1080*b**3*c*d**2*x**2 - 840*b**3*d**3*x**3 - 360*b**2*c**2*d - 2160
*b**2*c*d**2*x - 2520*b**2*d**3*x**2 - 2160*b*c*d**2 - 5040*b*d**3*x - 5040
*d**3)*exp(-a - b*x)/b**4, Ne(b**4, 0)), (a**4*c**3*x + b**4*d**3*x**8/8 +
x**7*(4*a*b**3*d**3/7 + 3*b**4*c*d**2/7) + x**6*(a**2*b**2*d**3 + 2*a*b**3*
c*d**2 + b**4*c**2*d/2) + x**5*(4*a**3*b*d**3/5 + 18*a**2*b**2*c*d**2/5 + 1
2*a*b**3*c**2*d/5 + b**4*c**3/5) + x**4*(a**4*d**3/4 + 3*a**3*b*c*d**2 + 9*
a**2*b**2*c**2*d/2 + a*b**3*c**3) + x**3*(a**4*c*d**2 + 4*a**3*b*c**2*d + 2
*a**2*b**2*c**3) + x**2*(3*a**4*c**2*d/2 + 2*a**3*b*c**3), True))

```

**Giac [A]**

time = 2.08, size = 1096, normalized size = 1.45

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4\*(d\*x+c)^3,x, algorithm="giac")

[Out]  $-(b^{11}d^3x^7 + 3b^{11}cd^2x^6 + 4ab^{10}d^3x^6 + 3b^{11}c^2d^2x^5 + 12a^2ab^{10}cd^2x^5 + 6a^2b^9d^3x^5 + 7b^{10}d^3x^6 + b^{11}c^3x^4 + 12a^2ab^{10}c^2d^2x^4 + 18a^2b^9cd^2x^4 + 4a^3b^8d^3x^4 + 18b^{10}cd^2x^5 + 24a^2ab^9d^3x^5 + 4a^2ab^{10}c^3x^3 + 18a^2b^9c^2d^2x^3 + 12a^3b^8cd^2x^3 + a^4b^7d^3x^3 + 15b^{10}c^2d^2x^4 + 60a^2b^9cd^2x^4 + 30a^2b^8d^3x^4 + 42b^9d^3x^5 + 6a^2b^9c^3x^2 + 12a^3b^8c^2d^2x^2 + 3a^4b^7cd^2x^2 + 4b^{10}c^3x^3 + 48a^2b^9c^2d^2x^3 + 72a^2b^8c^2d^2x^3 + 16a^3b^7d^3x^3 + 90b^9cd^2x^4 + 120a^2b^8d^3x^4 +$

$$\begin{aligned}
& 4a^3b^8c^3x + 3a^4b^7c^2dx + 12ab^9c^3x^2 + 54a^2b^8c^2dx \\
& ^2 + 36a^3b^7c^2dx^2 + 3a^4b^6d^3x^2 + 60b^9c^2dx^3 + 240ab^8 \\
& c^2dx^3 + 120a^2b^7d^3x^3 + 210b^8d^3x^4 + a^4b^7c^3 + 12a^2b^8 \\
& c^3x + 24a^3b^7c^2dx + 6a^4b^6cd^2x + 12b^9c^3x^2 + 144a \\
& b^8c^2dx^2 + 216a^2b^7cd^2x^2 + 48a^3b^6d^3x^2 + 360b^8cd^2 \\
& x^3 + 480ab^7d^3x^3 + 4a^3b^7c^3 + 3a^4b^6c^2d + 24ab^8c^3x \\
& + 108a^2b^7c^2dx + 72a^3b^6cd^2x + 6a^4b^5d^3x + 180b^8c^2 \\
& dx^2 + 720ab^7cd^2x^2 + 360a^2b^6d^3x^2 + 840b^7d^3x^3 + 12a \\
& ^2b^7c^3 + 24a^3b^6c^2d + 6a^4b^5cd^2 + 24b^8c^3x + 288ab^7c^2 \\
& dx + 432a^2b^6cd^2x + 96a^3b^5d^3x + 1080b^7cd^2x^2 + 144 \\
& 0ab^6d^3x^2 + 24ab^7c^3 + 108a^2b^6c^2d + 72a^3b^5cd^2 + 6a \\
& ^4b^4d^3 + 360b^7c^2dx + 1440ab^6cd^2x + 720a^2b^5d^3x + 252 \\
& 0b^6d^3x^2 + 24b^7c^3 + 288ab^6c^2d + 432a^2b^5cd^2 + 96a^3b \\
& ^4d^3 + 2160b^6cd^2x + 2880ab^5d^3x + 360b^6c^2d + 1440ab^5cd \\
& ^2 + 720a^2b^4d^3 + 5040b^5d^3x + 2160b^5cd^2 + 2880ab^4d^3 + \\
& 5040b^4d^3)e^{(-bx - a)}/b^8
\end{aligned}$$

Mupad [B]

time = 3.82, size = 803, normalized size = 1.06

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\exp(-a - bx)(a + bx)^4(c + dx)^3, x)$

[Out]  $-x^3 \exp(-a - bx)(b^2(4ac^3 + 4c^3) + 360cd^2 + (480ad^3 + 840d^3 + 120a^2d^3 + 16a^3d^3 + a^4d^3)/b + b(60c^2d + 18a^2c^2d + 48ac^2d) + 72a^2cd^2 + 12a^3cd^2 + 240acd^2) - x^4 \exp(-a - bx)(120ad^3 + 210d^3 + 30a^2d^3 + 4a^3d^3 + b^3c^3 + 15b^2c^2d + 90b^2cd^2 + 60ab^2cd^2 + 12ab^2c^2d + 18a^2b^2cd^2) - (\exp(-a - bx)(2880ad^3 + 5040d^3 + 720a^2d^3 + 96a^3d^3 + 24b^3c^3 + 6a^4d^3 + 24ab^3c^3 + 360b^2c^2d + 12a^2b^3c^3 + 4a^3b^3c^3 + a^4b^3c^3 + 2160b^2cd^2 + 108a^2b^2c^2d + 24a^3b^2c^2d + 3a^4b^2c^2d + 1440ab^2cd^2 + 288ab^2c^2d + 432a^2b^2cd^2 + 72a^3b^2cd^2 + 6a^4b^2cd^2))/b^4 - x \exp(-a - bx)(4c^3(6a + 3a^2 + a^3 + 6) + (6d^3(480a + 120a^2 + 16a^3 + a^4 + 840))/b^3 + (3c^2d(96a + 36a^2 + 8a^3 + a^4 + 120))/b + (6cd^2(240a + 72a^2 + 12a^3 + a^4 + 360))/b^2) - (3x^2 \exp(-a - bx)(480ad^3 + 840d^3 + 120a^2d^3 + 16a^3d^3 + 4b^3c^3 + a^4d^3 + 4ab^3c^3 + 60b^2c^2d + 2a^2b^3c^3 + 360b^2cd^2 + 18a^2b^2c^2d + 4a^3b^2c^2d + 240ab^2cd^2 + 48ab^2c^2d + 72a^2b^2cd^2 + 12a^3b^2cd^2 + a^4b^2cd^2))/b^2 - b^3d^3x^7 \exp(-a - bx) - b^2d^2x^6 \exp(-a - bx)(7d + 4ad + 3b^2c) - 3b^2d^2x^5 \exp(-a - bx)(8ad^2 + 14d^2 + 2a^2d^2 + b^2c^2 + 6b^2cd + 4ab^2cd)$

### 3.75 $\int e^{-a-bx}(a+bx)^4(c+dx)^2 dx$

**Optimal.** Leaf size=495

$$\frac{720d^2e^{-a-bx}}{b^3} - \frac{240d(bc-ad)e^{-a-bx}}{b^3} - \frac{24(bc-ad)^2e^{-a-bx}}{b^3} - \frac{720d^2e^{-a-bx}(a+bx)}{b^3} - \frac{240d(bc-ad)e^{-a-bx}(a+bx)}{b^3}$$

[Out]  $-720*d^2*\exp(-b*x-a)/b^3-240*d*(-a*d+b*c)*\exp(-b*x-a)/b^3-24*(-a*d+b*c)^2*\exp(-b*x-a)/b^3-720*d^2*\exp(-b*x-a)*(b*x+a)/b^3-240*d*(-a*d+b*c)*\exp(-b*x-a)*(b*x+a)/b^3-24*(-a*d+b*c)^2*\exp(-b*x-a)*(b*x+a)/b^3-360*d^2*\exp(-b*x-a)*(b*x+a)^2/b^3-120*d*(-a*d+b*c)*\exp(-b*x-a)*(b*x+a)^2/b^3-12*(-a*d+b*c)^2*\exp(-b*x-a)*(b*x+a)^2/b^3-120*d^2*\exp(-b*x-a)*(b*x+a)^3/b^3-40*d*(-a*d+b*c)*\exp(-b*x-a)*(b*x+a)^3/b^3-4*(-a*d+b*c)^2*\exp(-b*x-a)*(b*x+a)^3/b^3-30*d^2*\exp(-b*x-a)*(b*x+a)^4/b^3-10*d*(-a*d+b*c)*\exp(-b*x-a)*(b*x+a)^4/b^3-(-a*d+b*c)^2*\exp(-b*x-a)*(b*x+a)^4/b^3-6*d^2*\exp(-b*x-a)*(b*x+a)^5/b^3-2*d*(-a*d+b*c)*\exp(-b*x-a)*(b*x+a)^5/b^3-d^2*\exp(-b*x-a)*(b*x+a)^6/b^3$

**Rubi [A]**

time = 0.45, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$ , Rules used = {2227, 2207, 2225}

Antiderivative was successfully verified.

[In] Int[E<sup>-(a + b\*x)</sup>\*(a + b\*x)<sup>4</sup>\*(c + d\*x)<sup>2</sup>, x]

[Out]  $(-720*d^2*E^{-(a+b*x)})/b^3 - (240*d*(b*c - a*d)*E^{-(a+b*x)})/b^3 - (24*(b*c - a*d)^2*E^{-(a+b*x)})/b^3 - (720*d^2*E^{-(a+b*x)}*(a+b*x))/b^3 - (240*d*(b*c - a*d)*E^{-(a+b*x)}*(a+b*x))/b^3 - (24*(b*c - a*d)^2*E^{-(a+b*x)}*(a+b*x))/b^3 - (360*d^2*E^{-(a+b*x)}*(a+b*x)^2)/b^3 - (120*d*(b*c - a*d)*E^{-(a+b*x)}*(a+b*x)^2)/b^3 - (12*(b*c - a*d)^2*E^{-(a+b*x)}*(a+b*x)^2)/b^3 - (120*d^2*E^{-(a+b*x)}*(a+b*x)^3)/b^3 - (40*d*(b*c - a*d)*E^{-(a+b*x)}*(a+b*x)^3)/b^3 - (4*(b*c - a*d)^2*E^{-(a+b*x)}*(a+b*x)^3)/b^3 - (30*d^2*E^{-(a+b*x)}*(a+b*x)^4)/b^3 - (10*d*(b*c - a*d)*E^{-(a+b*x)}*(a+b*x)^4)/b^3 - ((b*c - a*d)^2*E^{-(a+b*x)}*(a+b*x)^4)/b^3 - (6*d^2*E^{-(a+b*x)}*(a+b*x)^5)/b^3 - (2*d*(b*c - a*d)*E^{-(a+b*x)}*(a+b*x)^5)/b^3 - (d^2*E^{-(a+b*x)}*(a+b*x)^6)/b^3$

Rule 2207

Int[((b\_.)\*(F\_))<sup>n</sup>((g\_.)\*((e\_.) + (f\_.)\*(x\_)))<sup>m</sup>((c\_.) + (d\_.)\*(x\_))<sup>m</sup>), x\_Symbol] :> Simp[(c + d\*x)<sup>m</sup>((b\*F<sup>g\*(e + f\*x)</sup>)<sup>n</sup>/(f\*g\*n\*Log[F])), x] - Dist[d\*(m/(f\*g\*n\*Log[F])), Int[(c + d\*x)<sup>(m - 1)</sup>\*(b\*F<sup>g\*(e + f\*x)</sup>)<sup>n</sup>, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

## Rule 2225

```
Int[((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.), x_Symbol] := Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

## Rule 2227

```
Int[(F_)^((c_.)*(v_)*(u_)), x_Symbol] := Int[ExpandIntegrand[F^(c*ExpandToS
um[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v,
x] && !TrueQ[$UseGamma]
```

## Rubi steps

$$\begin{aligned}
\int e^{-a-bx}(a+bx)^4(c+dx)^2 dx &= \int \left( \frac{(bc-ad)^2 e^{-a-bx}(a+bx)^4}{b^2} + \frac{2d(bc-ad)e^{-a-bx}(a+bx)^5}{b^2} + \frac{d^2 e^{-a-bx}(a+bx)^6}{b^2} \right) dx \\
&= \frac{d^2 \int e^{-a-bx}(a+bx)^6 dx}{b^2} + \frac{(2d(bc-ad)) \int e^{-a-bx}(a+bx)^5 dx}{b^2} + \frac{(bc-ad)^2 \int e^{-a-bx}(a+bx)^4 dx}{b^2} \\
&= -\frac{(bc-ad)^2 e^{-a-bx}(a+bx)^4}{b^3} - \frac{2d(bc-ad)e^{-a-bx}(a+bx)^5}{b^3} - \frac{d^2 e^{-a-bx}(a+bx)^6}{b^3} \\
&= -\frac{4(bc-ad)^2 e^{-a-bx}(a+bx)^3}{b^3} - \frac{10d(bc-ad)e^{-a-bx}(a+bx)^4}{b^3} - \frac{(bc-ad)^2 e^{-a-bx}(a+bx)^4}{b^3} \\
&= -\frac{12(bc-ad)^2 e^{-a-bx}(a+bx)^2}{b^3} - \frac{40d(bc-ad)e^{-a-bx}(a+bx)^3}{b^3} - \frac{4(bc-ad)^2 e^{-a-bx}(a+bx)^2}{b^3} \\
&= -\frac{24(bc-ad)^2 e^{-a-bx}(a+bx)}{b^3} - \frac{120d(bc-ad)e^{-a-bx}(a+bx)^2}{b^3} - \frac{12(bc-ad)^2 e^{-a-bx}(a+bx)}{b^3} \\
&= -\frac{24(bc-ad)^2 e^{-a-bx}}{b^3} - \frac{240d(bc-ad)e^{-a-bx}(a+bx)}{b^3} - \frac{24(bc-ad)^2 e^{-a-bx}}{b^3} \\
&= -\frac{240d(bc-ad)e^{-a-bx}}{b^3} - \frac{24(bc-ad)^2 e^{-a-bx}}{b^3} - \frac{720d^2 e^{-a-bx}(a+bx)}{b^3} - \frac{24(bc-ad)^2 e^{-a-bx}}{b^3} \\
&= -\frac{720d^2 e^{-a-bx}}{b^3} - \frac{240d(bc-ad)e^{-a-bx}}{b^3} - \frac{24(bc-ad)^2 e^{-a-bx}}{b^3} - \frac{720d^2 e^{-a-bx}}{b^3}
\end{aligned}$$

**Mathematica [A]**

time = 1.98, size = 320, normalized size = 0.65

-----

Antiderivative was successfully verified.

```
[In] Integrate[E^(-a - b*x)*(a + b*x)^4*(c + d*x)^2,x]
```

```
[Out] (E^(-a - b*x))*(-2*(360 + 240*a + 72*a^2 + 12*a^3 + a^4)*d^2 - b^6*x^4*(c +
d*x)^2 - 2*b^5*x^3*(c + d*x)*(2*(1 + a)*c + (3 + 2*a)*d*x) - 2*b*d*((120 +
```

$$96*a + 36*a^2 + 8*a^3 + a^4)*c + (360 + 240*a + 72*a^2 + 12*a^3 + a^4)*d*x) - 2*b^4*x^2*(3*(2 + 2*a + a^2)*c^2 + 2*(10 + 8*a + 3*a^2)*c*d*x + (15 + 10*a + 3*a^2)*d^2*x^2) - 4*b^3*x*((6 + 6*a + 3*a^2 + a^3)*c^2 + (30 + 24*a + 9*a^2 + 2*a^3)*c*d*x + (30 + 20*a + 6*a^2 + a^3)*d^2*x^2) - b^2*((24 + 24*a + 12*a^2 + 4*a^3 + a^4)*c^2 + 2*(120 + 96*a + 36*a^2 + 8*a^3 + a^4)*c*d*x + (360 + 240*a + 72*a^2 + 12*a^3 + a^4)*d^2*x^2))/b^3$$

**Maple [A]**

time = 0.08, size = 694, normalized size = 1.40 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-b\*x-a)\*(b\*x+a)^4\*(d\*x+c)^2,x,method=\_RETURNVERBOSE)

[Out] 
$$\begin{aligned} & -1/b*(c^2*((-b*x-a)^4*exp(-b*x-a)-4*exp(-b*x-a)*(-b*x-a)^3+12*(-b*x-a)^2*exp(-b*x-a)-24*(-b*x-a)*exp(-b*x-a)+24*exp(-b*x-a))+d^2/b^2*(exp(-b*x-a)*(-b*x-a)^6-6*(-b*x-a)^5*exp(-b*x-a)+30*(-b*x-a)^4*exp(-b*x-a)-120*exp(-b*x-a)*(-b*x-a)^3+360*(-b*x-a)^2*exp(-b*x-a)-720*(-b*x-a)*exp(-b*x-a)+720*exp(-b*x-a)))+d^2/b^2*a^2*((-b*x-a)^4*exp(-b*x-a)-4*exp(-b*x-a)*(-b*x-a)^3+12*(-b*x-a)^2*exp(-b*x-a)-24*(-b*x-a)*exp(-b*x-a)+24*exp(-b*x-a))-2*d/b*c*((-b*x-a)^5*exp(-b*x-a)-5*(-b*x-a)^4*exp(-b*x-a)+20*exp(-b*x-a)*(-b*x-a)^3-60*(-b*x-a)^2*exp(-b*x-a)+120*(-b*x-a)*exp(-b*x-a)-120*exp(-b*x-a))+2*d^2/b^2*a*((-b*x-a)^5*exp(-b*x-a)-5*(-b*x-a)^4*exp(-b*x-a)+20*exp(-b*x-a)*(-b*x-a)^3-60*(-b*x-a)^2*exp(-b*x-a)+120*(-b*x-a)*exp(-b*x-a)-120*exp(-b*x-a))-2*d/b*a*c*((-b*x-a)^4*exp(-b*x-a)-4*exp(-b*x-a)*(-b*x-a)^3+12*(-b*x-a)^2*exp(-b*x-a)-24*(-b*x-a)*exp(-b*x-a)+24*exp(-b*x-a))) \end{aligned}$$

**Maxima [A]**

time = 0.35, size = 599, normalized size = 1.21

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4\*(d\*x+c)^2,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -4*(b*x + 1)*a^3*c^2*e^{(-b*x - a)}/b - a^4*c^2*e^{(-b*x - a)}/b - 2*(b*x + 1)*a^4*c*d*e^{(-b*x - a)}/b^2 - 6*(b^2*x^2 + 2*b*x + 2)*a^2*c^2*e^{(-b*x - a)}/b - 8*(b^2*x^2 + 2*b*x + 2)*a^3*c*d*e^{(-b*x - a)}/b^2 - (b^2*x^2 + 2*b*x + 2)*a^4*d^2*e^{(-b*x - a)}/b^3 - 4*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a*c^2*e^{(-b*x - a)}/b - 12*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a^2*c*d*e^{(-b*x - a)}/b^2 - 4*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a^3*d^2*e^{(-b*x - a)}/b^3 - (b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*c^2*e^{(-b*x - a)}/b - 8*(b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*a*c*d*e^{(-b*x - a)}/b^2 - 6*(b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*a^2*d^2*e^{(-b*x - a)}/b^3 - 2*(b^5*x^5 + 5*b^4*x^4 + 20*b^3*x^3 + 60*b^2*x^2 + 120*b*x + 120)*c*d*e^{(-b*x - a)}/b^2 - 4*(b^5*x^5 + 5*b^4*x^4 + 20*b^3*x^3 + 60*b^2*x^2 + 120*b*x + 120)*a*d^2*e^{(-b*x - a)}/b^3 - (b^6*x^6 + 6*b^5*x^5 + 30*b^4*x^4 + 120*b^3*x^3 + 360*b^2*x^2 + 720*b*x + 720)*d^2*e^{(-b*x - a)}/b^3 \end{aligned}$$

**Fricas [A]**

time = 0.36, size = 354, normalized size = 0.72

---

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(exp(-b\*x-a)\*(b\*x+a)^4\*(d\*x+c)^2,x, algorithm="fricas")

**[Out]**  $-(b^6*d^2*x^6 + 2*(b^6*c*d + (2*a + 3)*b^5*d^2)*x^5 + (a^4 + 4*a^3 + 12*a^2 + 24*a + 24)*b^2*c^2 + (b^6*c^2 + 2*(4*a + 5)*b^5*c*d + 2*(3*a^2 + 10*a + 15)*b^4*d^2)*x^4 + 2*(a^4 + 8*a^3 + 36*a^2 + 96*a + 120)*b*c*d + 4*((a + 1)*b^5*c^2 + (3*a^2 + 8*a + 10)*b^4*c*d + (a^3 + 6*a^2 + 20*a + 30)*b^3*d^2)*x^3 + 2*(a^4 + 12*a^3 + 72*a^2 + 240*a + 360)*d^2 + (6*(a^2 + 2*a + 2)*b^4*c^2 + 4*(2*a^3 + 9*a^2 + 24*a + 30)*b^3*c*d + (a^4 + 12*a^3 + 72*a^2 + 240*a + 360)*b^2*d^2)*x^2 + 2*(2*(a^3 + 3*a^2 + 6*a + 6)*b^3*c^2 + (a^4 + 8*a^3 + 36*a^2 + 96*a + 120)*b^2*c*d + (a^4 + 12*a^3 + 72*a^2 + 240*a + 360)*b*d^2)*x)*e^{(-b*x - a)/b^3}$

**Sympy [A]**

time = 0.21, size = 899, normalized size = 1.82

---

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(exp(-b\*x-a)\*(b\*x+a)\*\*4\*(d\*x+c)\*\*2,x)

**[Out]** Piecewise((( $-a**4*b**2*c**2 - 2*a**4*b**2*c*d*x - a**4*b**2*d**2*x**2 - 2*a**4*b*c*d - 2*a**4*b*d**2*x - 2*a**4*d**2 - 4*a**3*b**3*c**2*x - 8*a**3*b**3*c*d*x**2 - 4*a**3*b**3*d**2*x**3 - 4*a**3*b**2*c**2 - 16*a**3*b**2*c*d*x - 12*a**3*b**2*d**2*x**2 - 16*a**3*b*c*d - 24*a**3*b*d**2*x - 24*a**3*d**2 - 6*a**2*b**4*c**2*x**2 - 12*a**2*b**4*c*d*x**3 - 6*a**2*b**4*d**2*x**4 - 12*a**2*b**3*c**2*x - 36*a**2*b**3*c*d*x**2 - 24*a**2*b**3*d**2*x**3 - 12*a**2*b**2*c**2 - 72*a**2*b**2*c*d*x - 72*a**2*b**2*d**2*x**2 - 72*a**2*b*c*d - 144*a**2*b*d**2*x - 144*a**2*d**2 - 4*a*b**5*c**2*x**3 - 8*a*b**5*c*d*x**4 - 4*a*b**5*d**2*x**5 - 12*a*b**4*c**2*x**2 - 32*a*b**4*c*d*x**3 - 20*a*b**4*d**2*x**4 - 24*a*b**3*c**2*x - 96*a*b**3*c*d*x**2 - 80*a*b**3*d**2*x**3 - 24*a*b**2*c**2 - 192*a*b**2*c*d*x - 240*a*b**2*d**2*x**2 - 192*a*b*c*d - 480*a*b*d**2*x - 480*a*d**2 - b**6*c**2*x**4 - 2*b**6*c*d*x**5 - b**6*d**2*x**6 - 4*b**5*c**2*x**3 - 10*b**5*c*d*x**4 - 6*b**5*d**2*x**5 - 12*b**4*c**2*x**2 - 40*b**4*c*d*x**3 - 30*b**4*d**2*x**4 - 24*b**3*c**2*x - 120*b**3*c*d*x**2 - 120*b**3*d**2*x**3 - 24*b**2*c**2 - 240*b**2*c*d*x - 360*b**2*d**2*x**2 - 240*b*c*d - 720*b*d**2*x - 720*d**2$ )\*exp(-a - b\*x)/b\*\*3, Ne(b\*\*3, 0)), (a\*\*4\*c\*\*2\*x + b\*\*4\*d\*\*2\*x\*\*7/7 + x\*\*6\*(2\*a\*b\*\*3\*d\*\*2/3 + b\*\*4\*c\*d/3) + x\*\*5\*(6\*a\*\*2\*b\*\*2\*d\*\*2/5 + 8\*a\*b\*\*3\*c\*d/5 + b\*\*4\*c\*\*2/5) + x\*\*4\*(a\*\*3\*b\*d\*\*2 + 3\*a\*\*2\*b\*\*2\*c\*d + a\*b\*\*3\*c\*\*2) + x\*\*3\*(a\*\*4\*d\*\*2/3 + 8\*a\*\*3\*b\*c\*d/3 + 2\*a\*\*2\*b\*\*2\*c\*\*2) + x\*\*2\*(a\*\*4\*c\*d + 2\*a\*\*3\*b\*c\*\*2), True))

**Giac [A]**

time = 1.84, size = 674, normalized size = 1.36

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4\*(d\*x+c)^2,x, algorithm="giac")

[Out]  $-(b^{10}d^2x^6 + 2b^{10}c*d*x^5 + 4a*b^9d^2x^5 + b^{10}c^2x^4 + 8a*b^9*c*d*x^4 + 6a^2b^8d^2x^4 + 6b^9d^2x^5 + 4a*b^9c^2x^3 + 12a^2b^8*c*d*x^3 + 4a^3b^7d^2x^3 + 10b^9c*d*x^4 + 20a*b^8d^2x^4 + 6a^2b^8*c^2x^2 + 8a^3b^7*c*d*x^2 + a^4b^6d^2x^2 + 4b^9c^2x^3 + 32a*b^8*c*d*x^3 + 24a^2b^7d^2x^3 + 30b^8d^2x^4 + 4a^3b^7c^2x + 2a^4b^6*c*d*x + 12a*b^8c^2x^2 + 36a^2b^7*c*d*x^2 + 12a^3b^6d^2x^2 + 40b^8*c*d*x^3 + 80a*b^7d^2x^3 + a^4b^6c^2 + 12a^2b^7c^2x + 16a^3b^6*c*d*x + 2a^4b^5d^2x + 12b^8c^2x^2 + 96a*b^7*c*d*x^2 + 72a^2b^6d^2x^2 + 120b^7d^2x^3 + 4a^3b^6c^2 + 2a^4b^5*c*d + 24a*b^7c^2x + 72a^2b^6*c*d*x + 24a^3b^5d^2x + 120b^7c*d*x^2 + 240a*b^6d^2x^2 + 12a^2b^6c^2 + 16a^3b^5*c*d + 2a^4b^4d^2 + 24b^7c^2x + 192a*b^6*c*d*x + 144a^2b^5d^2x + 360b^6d^2x^2 + 24a*b^6c^2 + 72a^2b^5*c*d + 24a^3b^4d^2 + 240b^6*c*d*x + 480a*b^5d^2x + 24b^6c^2 + 192a*b^5*c*d + 144a^2b^4d^2 + 720b^5d^2x + 240b^5*c*d + 480a*b^4d^2 + 720*b^4d^2)*e^{(-b*x - a)}/b^7$

**Mupad [B]**

time = 3.66, size = 537, normalized size = 1.08

---

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(- a - b\*x)\*(a + b\*x)^4\*(c + d\*x)^2,x)

[Out]  $-x^2 \exp(-a - b*x) * (120*c*d + b*(12*a*c^2 + 12*c^2 + 6*a^2*c^2) + (240*a*d^2 + 360*d^2 + 72*a^2*d^2 + 12*a^3*d^2 + a^4*d^2)/b + 96*a*c*d + 36*a^2*c*d + 8*a^3*c*d) - x^3 \exp(-a - b*x) * (80*a*d^2 + 120*d^2 + 24*a^2*d^2 + 4*b^2*c^2 + 4*a^3*d^2 + 4*a*b^2*c^2 + 40*b*c*d + 12*a^2*b*c*d + 32*a*b*c*d) - (\exp(-a - b*x) * (480*a*d^2 + 720*d^2 + 144*a^2*d^2 + 24*b^2*c^2 + 24*a^3*d^2 + 2*a^4*d^2 + 24*a*b^2*c^2 + 240*b*c*d + 12*a^2*b^2*c^2 + 4*a^3*b^2*c^2 + a^4*b^2*c^2 + 72*a^2*b*c*d + 16*a^3*b*c*d + 2*a^4*b*c*d + 192*a*b*c*d))/b^3 - b^3*d^2*x^6*\exp(-a - b*x) - b*x^4*\exp(-a - b*x)*(20*a*d^2 + 30*d^2 + 6*a^2*d^2 + b^2*c^2 + 10*b*c*d + 8*a*b*c*d) - (2*x*\exp(-a - b*x)*(240*a*d^2 + 360*d^2 + 72*a^2*d^2 + 12*b^2*c^2 + 12*a^3*d^2 + a^4*d^2 + 12*a*b^2*c^2 + 120*b*c*d + 6*a^2*b^2*c^2 + 2*a^3*b^2*c^2 + 36*a^2*b*c*d + 8*a^3*b*c*d + a^4*b*c*d + 96*a*b*c*d))/b^2 - 2*b^2*d*x^5*\exp(-a - b*x)*(3*d + 2*a*d + b*c)$



### 3.76 $\int e^{-a-bx}(a+bx)^4(c+dx) dx$

**Optimal.** Leaf size=271

$$\frac{120de^{-a-bx}}{b^2} - \frac{24(bc-ad)e^{-a-bx}}{b^2} - \frac{120de^{-a-bx}(a+bx)}{b^2} - \frac{24(bc-ad)e^{-a-bx}(a+bx)}{b^2} - \frac{60de^{-a-bx}(a+bx)^2}{b^2}$$

[Out]  $-120*d*\exp(-b*x-a)/b^2-24*(-a*d+b*c)*\exp(-b*x-a)/b^2-120*d*\exp(-b*x-a)*(b*x+a)/b^2-24*(-a*d+b*c)*\exp(-b*x-a)*(b*x+a)/b^2-60*d*\exp(-b*x-a)*(b*x+a)^2/b^2-12*(-a*d+b*c)*\exp(-b*x-a)*(b*x+a)^2/b^2-20*d*\exp(-b*x-a)*(b*x+a)^3/b^2-4*(-a*d+b*c)*\exp(-b*x-a)*(b*x+a)^3/b^2-5*d*\exp(-b*x-a)*(b*x+a)^4/b^2-(-a*d+b*c)*\exp(-b*x-a)*(b*x+a)^4/b^2-d*\exp(-b*x-a)*(b*x+a)^5/b^2$

**Rubi** [A]

time = 0.24, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ ,

Rules used = {2227, 2207, 2225}

$$\frac{e^{-a-bx}(a+bx)^4(bc-ad)}{b^2} - \frac{4e^{-a-bx}(a+bx)^2(bc-ad)}{b^2} - \frac{12e^{-a-bx}(a+bx)(bc-ad)}{b^2} - \frac{24e^{-a-bx}(a+bx)(bc-ad)}{b^2} - \frac{24e^{-a-bx}(bc-ad)}{b^2} - \frac{de^{-a-bx}(a+bx)^5}{b^2} - \frac{5de^{-a-bx}(a+bx)^4}{b^2} - \frac{20de^{-a-bx}(a+bx)^3}{b^2} - \frac{60de^{-a-bx}(a+bx)^2}{b^2} - \frac{120de^{-a-bx}(a+bx)}{b^2} - \frac{120de^{-a-bx}}{b^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[E^{(-a - b*x)}*(a + b*x)^4*(c + d*x), x]$

[Out]  $(-120*d*E^{(-a - b*x)})/b^2 - (24*(b*c - a*d)*E^{(-a - b*x)})/b^2 - (120*d*E^{(-a - b*x)}*(a + b*x))/b^2 - (24*(b*c - a*d)*E^{(-a - b*x)}*(a + b*x))/b^2 - (60*d*E^{(-a - b*x)}*(a + b*x)^2)/b^2 - (12*(b*c - a*d)*E^{(-a - b*x)}*(a + b*x)^2)/b^2 - (20*d*E^{(-a - b*x)}*(a + b*x)^3)/b^2 - (4*(b*c - a*d)*E^{(-a - b*x)}*(a + b*x)^3)/b^2 - (5*d*E^{(-a - b*x)}*(a + b*x)^4)/b^2 - ((b*c - a*d)*E^{(-a - b*x)}*(a + b*x)^4)/b^2 - (d*E^{(-a - b*x)}*(a + b*x)^5)/b^2$

Rule 2207

$\text{Int}[(b_1*(F_1)^{(g_1*((e_1) + (f_1)*(x_1))))^{(n_1)}*((c_1) + (d_1)*(x_1))^{(m_1)}, x\_Symbol] :> \text{Simp}[(c + d*x)^m*((b*F^{(g*(e + f*x)))^n/(f*g*n*\text{Log}[F])), x] - \text{Dist}[d*(m/(f*g*n*\text{Log}[F])), \text{Int}[(c + d*x)^{(m-1)}*(b*F^{(g*(e + f*x)))^n}, x], x] /; \text{FreeQ}\{F, b, c, d, e, f, g, n\}, x\} \&\& \text{GtQ}[m, 0] \&\& \text{IntegerQ}[2*m] \&\& !\text{TrueQ}[\$UseGamma]$

Rule 2225

$\text{Int}[(F_1)^{(c_1*((a_1) + (b_1)*(x_1)))^{(n_1)}, x\_Symbol] :> \text{Simp}[(F^{(c*(a + b*x)))^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

Rule 2227

$\text{Int}[(F_1)^{(c_1*(v_1))*(u_1)}, x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[F^{(c*\text{ExpandToSum}[v, x])}, u, x], x] /; \text{FreeQ}\{F, c\}, x\} \&\& \text{PolynomialQ}[u, x] \&\& \text{LinearQ}[v,$

x] && !TrueQ[\$UseGamma]

Rubi steps

$$\begin{aligned}
 \int e^{-a-bx}(a+bx)^4(c+dx) dx &= \int \left( \frac{(bc-ad)e^{-a-bx}(a+bx)^4}{b} + \frac{de^{-a-bx}(a+bx)^5}{b} \right) dx \\
 &= \frac{d \int e^{-a-bx}(a+bx)^5 dx}{b} + \frac{(bc-ad) \int e^{-a-bx}(a+bx)^4 dx}{b} \\
 &= -\frac{(bc-ad)e^{-a-bx}(a+bx)^4}{b^2} - \frac{de^{-a-bx}(a+bx)^5}{b^2} + \frac{(5d) \int e^{-a-bx}(a+bx)^4 dx}{b} \\
 &= -\frac{4(bc-ad)e^{-a-bx}(a+bx)^3}{b^2} - \frac{5de^{-a-bx}(a+bx)^4}{b^2} - \frac{(bc-ad)e^{-a-bx}(a+bx)^4}{b^2} \\
 &= -\frac{12(bc-ad)e^{-a-bx}(a+bx)^2}{b^2} - \frac{20de^{-a-bx}(a+bx)^3}{b^2} - \frac{4(bc-ad)e^{-a-bx}(a+bx)^4}{b^2} \\
 &= -\frac{24(bc-ad)e^{-a-bx}(a+bx)}{b^2} - \frac{60de^{-a-bx}(a+bx)^2}{b^2} - \frac{12(bc-ad)e^{-a-bx}(a+bx)^4}{b^2} \\
 &= -\frac{24(bc-ad)e^{-a-bx}}{b^2} - \frac{120de^{-a-bx}(a+bx)}{b^2} - \frac{24(bc-ad)e^{-a-bx}(a+bx)}{b^2} \\
 &= -\frac{120de^{-a-bx}}{b^2} - \frac{24(bc-ad)e^{-a-bx}}{b^2} - \frac{120de^{-a-bx}(a+bx)}{b^2} - \frac{24(bc-ad)e^{-a-bx}}{b^2}
 \end{aligned}$$

**Mathematica [A]**

time = 0.65, size = 191, normalized size = 0.70

$$\frac{e^{-bx}(-((120+96a+36a^2+8a^3+a^4)d) - b^2x^4(c+dx) - b^2x^3(4(1+a)c + (5+4a)dx) - 2b^2x^2(3(2+2a+a^2)c + (10+8a+3a^2)dx) - 2b^2x(2(6+6a+3a^2+a^3)c + (30+24a+9a^2+2a^3)dx) - b(24+24a+12a^2+4a^3+a^4)c + (120+96a+36a^2+8a^3+a^4)dx)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(-a - b\*x)\*(a + b\*x)^4\*(c + d\*x), x]

[Out] (E^(-a - b\*x)\*(-(120 + 96\*a + 36\*a^2 + 8\*a^3 + a^4)\*d) - b^5\*x^4\*(c + d\*x) - b^4\*x^3\*(4\*(1 + a)\*c + (5 + 4\*a)\*d\*x) - 2\*b^3\*x^2\*(3\*(2 + 2\*a + a^2)\*c + (10 + 8\*a + 3\*a^2)\*d\*x) - 2\*b^2\*x\*(2\*(6 + 6\*a + 3\*a^2 + a^3)\*c + (30 + 24\*a + 9\*a^2 + 2\*a^3)\*d\*x) - b\*((24 + 24\*a + 12\*a^2 + 4\*a^3 + a^4)\*c + (120 + 96\*a + 36\*a^2 + 8\*a^3 + a^4)\*d\*x))/b^2

**Maple [A]**

time = 0.07, size = 322, normalized size = 1.19

method	result
norman	$(-4ab^2d - cb^3 - 5b^2d)x^4e^{-bx-a} + (-6a^2bd - 4ab^2c - 16abd - 4b^2c - 20bd)x^3e^{-bx-a} +$



[Out]  $-(b^5*d*x^5 + (b^5*c + (4*a + 5)*b^4*d)*x^4 + 2*(2*(a + 1)*b^4*c + (3*a^2 + 8*a + 10)*b^3*d)*x^3 + (a^4 + 4*a^3 + 12*a^2 + 24*a + 24)*b*c + 2*(3*(a^2 + 2*a + 2)*b^3*c + (2*a^3 + 9*a^2 + 24*a + 30)*b^2*d)*x^2 + (a^4 + 8*a^3 + 36*a^2 + 96*a + 120)*d + (4*(a^3 + 3*a^2 + 6*a + 6)*b^2*c + (a^4 + 8*a^3 + 36*a^2 + 96*a + 120)*b*d)*x)*e^{(-b*x - a)/b^2}$

**Sympy** [A]

time = 0.15, size = 447, normalized size = 1.65

$$\begin{cases} \frac{(-a^6b - a^4bd - a^4d - 4a^3bc - 4a^2b^2c - 4a^2bd - 4a^2c^2 - 8a^2bd - 8a^2d - 6a^2b^2c^2 - 6a^2b^2d^2 - 12a^2bc - 18a^2bd^2 - 12a^2c^2 - 36a^2bd - 36a^2d - 4a^3c^2 - 4a^3d^2 - 12a^3c^2 - 16a^3d^2 - 24a^3bc - 48a^3bd^2 - 24a^3c^2 - 96abd - 96ad - b^5c^4 - b^5d^4 - 4a^4c^2 - 5a^4d^2 - 4a^4c^2 - 20a^4d^2 - 24a^4bc - 60a^4bd - 24a^4c^2 - 120bd - 120d)}{b^6} & \text{for } b^2 \neq 0 \\ a^4cx + \frac{b^4d^2}{6} + x^3 \cdot \left( \frac{4ab^2d}{5} + \frac{b^4c}{5} \right) + x^4 \cdot \left( \frac{3a^2b^2d}{2} + ab^2c \right) + x^5 \cdot \left( \frac{4a^2bd}{3} + 2a^2b^2c \right) + x^2 \left( \frac{a^4d}{2} + 2a^3bc \right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)\*\*4\*(d\*x+c), x)

[Out] Piecewise((((-a\*\*4\*b\*c - a\*\*4\*b\*d\*x - a\*\*4\*d - 4\*a\*\*3\*b\*\*2\*c\*x - 4\*a\*\*3\*b\*\*2\*d\*x\*\*2 - 4\*a\*\*3\*b\*c - 8\*a\*\*3\*b\*d\*x - 8\*a\*\*3\*d - 6\*a\*\*2\*b\*\*3\*c\*x\*\*2 - 6\*a\*\*2\*b\*\*3\*d\*x\*\*3 - 12\*a\*\*2\*b\*\*2\*c\*x - 18\*a\*\*2\*b\*\*2\*d\*x\*\*2 - 12\*a\*\*2\*b\*c - 36\*a\*\*2\*b\*d\*x - 36\*a\*\*2\*d - 4\*a\*b\*\*4\*c\*x\*\*3 - 4\*a\*b\*\*4\*d\*x\*\*4 - 12\*a\*b\*\*3\*c\*x\*\*2 - 16\*a\*b\*\*3\*d\*x\*\*3 - 24\*a\*b\*\*2\*c\*x - 48\*a\*b\*\*2\*d\*x\*\*2 - 24\*a\*b\*c - 96\*a\*b\*d\*x - 96\*a\*d - b\*\*5\*c\*x\*\*4 - b\*\*5\*d\*x\*\*5 - 4\*b\*\*4\*c\*x\*\*3 - 5\*b\*\*4\*d\*x\*\*4 - 12\*b\*\*3\*c\*x\*\*2 - 20\*b\*\*3\*d\*x\*\*3 - 24\*b\*\*2\*c\*x - 60\*b\*\*2\*d\*x\*\*2 - 24\*b\*c - 120\*b\*d\*x - 120\*d)\*exp(-a - b\*x)/b\*\*2, Ne(b\*\*2, 0)), (a\*\*4\*c\*x + b\*\*4\*d\*x\*\*6/6 + x\*\*5\*(4\*a\*b\*\*3\*d/5 + b\*\*4\*c/5) + x\*\*4\*(3\*a\*\*2\*b\*\*2\*d/2 + a\*b\*\*3\*c) + x\*\*3\*(4\*a\*\*3\*b\*d/3 + 2\*a\*\*2\*b\*\*2\*c) + x\*\*2\*(a\*\*4\*d/2 + 2\*a\*\*3\*b\*c), True))

**Giac** [A]

time = 1.95, size = 331, normalized size = 1.22

$$\frac{(b^6d^2 + b^6c^2 + 4ab^5d^2 + 4ab^5c^2 + 6a^2b^4d^2 + 5a^2b^4c^2 + 6a^2b^3d^2 + 4a^2b^3c^2 + 4a^2b^2d^2 + 4a^2b^2c^2 + 16ab^4d^2 + 4a^2b^3c^2 + a^2b^4d^2 + 12ab^4d^2 + 18a^2b^3d^2 + 20b^4d^2 + a^2b^3c^2 + 12a^2b^2c^2 + 8a^2b^2d^2 + 12b^4c^2 + 48ab^4d^2 + 4a^2b^3c^2 + a^2b^4d^2 + 24ab^4c^2 + 36a^2b^3d^2 + 60b^4c^2 + 12a^2b^2c^2 + 8a^2b^2d^2 + 24b^4c^2 + 96ab^4d^2 + 24ab^4c^2 + 36a^2b^3d^2 + 120b^4c^2 + 96ab^4d^2 + 120b^4c^2)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4\*(d\*x+c), x, algorithm="giac")

[Out]  $-(b^9*d*x^5 + b^9*c*x^4 + 4*a*b^8*d*x^4 + 4*a*b^8*c*x^3 + 6*a^2*b^7*d*x^3 + 5*b^8*d*x^4 + 6*a^2*b^7*c*x^2 + 4*a^3*b^6*d*x^2 + 4*b^8*c*x^3 + 16*a*b^7*d*x^3 + 4*a^3*b^6*c*x + a^4*b^5*d*x + 12*a*b^7*c*x^2 + 18*a^2*b^6*d*x^2 + 20*b^7*d*x^3 + a^4*b^5*c + 12*a^2*b^6*c*x + 8*a^3*b^5*d*x + 12*b^7*c*x^2 + 48*a*b^6*d*x^2 + 4*a^3*b^5*c + a^4*b^4*d + 24*a*b^6*c*x + 36*a^2*b^5*d*x + 60*b^6*d*x^2 + 12*a^2*b^5*c + 8*a^3*b^4*d + 24*b^6*c*x + 96*a*b^5*d*x + 24*a*b^5*c + 36*a^2*b^4*d + 120*b^5*d*x + 24*b^5*c + 96*a*b^4*d + 120*b^4*d)*e^{(-b*x - a)/b^6}$

**Mupad** [B]

time = 0.18, size = 264, normalized size = 0.97

$$\frac{e^{-bx-a} (120d + 96ad + 24bc + 36a^2d + 8a^2d + a^4d + 24abc + 12a^3bc + 4a^3bc + a^4bd)}{b^6} - x^{5-b^{-1}} (60d + 48ad + 12bc + 18a^2d + 4a^2d + 12abc + 6a^2bc) - x^{4-b^{-1}} (24c + 24ac + 12a^2c + 4a^3c + \frac{d^2 + 8d^2 + 36d^2 + 96da + 120d}{b}) - b^4d^2e^{-bx-a} - b^4c^2e^{-bx-a} (5d + 4ad + bc) - 2b^3c^{-bx-a} (10d + 8ad + 2bc + 3a^2d + 2ab)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\exp(-a - b*x)*(a + b*x)^4*(c + d*x), x)$

[Out]  $-\frac{\exp(-a - b*x)(120*d + 96*a*d + 24*b*c + 36*a^2*d + 8*a^3*d + a^4*d + 24*a*b*c + 12*a^2*b*c + 4*a^3*b*c + a^4*b*c)}{b^2} - x^2 \exp(-a - b*x)(60*d + 48*a*d + 12*b*c + 18*a^2*d + 4*a^3*d + 12*a*b*c + 6*a^2*b*c) - x \exp(-a - b*x)(24*c + 24*a*c + 12*a^2*c + 4*a^3*c + (120*d + 96*a*d + 36*a^2*d + 8*a^3*d + a^4*d)/b) - b^3*d*x^5 \exp(-a - b*x) - b^2*x^4 \exp(-a - b*x)(5*d + 4*a*d + b*c) - 2*b*x^3 \exp(-a - b*x)(10*d + 8*a*d + 2*b*c + 3*a^2*d + 2*a*b*c)$

### 3.77 $\int e^{-a-bx}(a+bx)^4 dx$

**Optimal.** Leaf size=102

$$\frac{24e^{-a-bx}}{b} - \frac{24e^{-a-bx}(a+bx)}{b} - \frac{12e^{-a-bx}(a+bx)^2}{b} - \frac{4e^{-a-bx}(a+bx)^3}{b} - \frac{e^{-a-bx}(a+bx)^4}{b}$$

[Out]  $-24*\exp(-b*x-a)/b-24*\exp(-b*x-a)*(b*x+a)/b-12*\exp(-b*x-a)*(b*x+a)^2/b-4*\exp(-b*x-a)*(b*x+a)^3/b-\exp(-b*x-a)*(b*x+a)^4/b$

**Rubi [A]**

time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2207, 2225}

$$-\frac{e^{-a-bx}(a+bx)^4}{b} - \frac{4e^{-a-bx}(a+bx)^3}{b} - \frac{12e^{-a-bx}(a+bx)^2}{b} - \frac{24e^{-a-bx}(a+bx)}{b} - \frac{24e^{-a-bx}}{b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[E^{(-a - b*x)}*(a + b*x)^4, x]$

[Out]  $(-24*E^{(-a - b*x)})/b - (24*E^{(-a - b*x)}*(a + b*x))/b - (12*E^{(-a - b*x)}*(a + b*x)^2)/b - (4*E^{(-a - b*x)}*(a + b*x)^3)/b - (E^{(-a - b*x)}*(a + b*x)^4)/b$

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x]
;/ FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol]
:> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{-a-bx}(a+bx)^4 dx &= -\frac{e^{-a-bx}(a+bx)^4}{b} + 4 \int e^{-a-bx}(a+bx)^3 dx \\
&= -\frac{4e^{-a-bx}(a+bx)^3}{b} - \frac{e^{-a-bx}(a+bx)^4}{b} + 12 \int e^{-a-bx}(a+bx)^2 dx \\
&= -\frac{12e^{-a-bx}(a+bx)^2}{b} - \frac{4e^{-a-bx}(a+bx)^3}{b} - \frac{e^{-a-bx}(a+bx)^4}{b} + 24 \int e^{-a-bx}(a+bx) dx \\
&= -\frac{24e^{-a-bx}(a+bx)}{b} - \frac{12e^{-a-bx}(a+bx)^2}{b} - \frac{4e^{-a-bx}(a+bx)^3}{b} - \frac{e^{-a-bx}(a+bx)^4}{b} + \dots \\
&= -\frac{24e^{-a-bx}}{b} - \frac{24e^{-a-bx}(a+bx)}{b} - \frac{12e^{-a-bx}(a+bx)^2}{b} - \frac{4e^{-a-bx}(a+bx)^3}{b} - \frac{e^{-a-bx}(a+bx)^4}{b} + \dots
\end{aligned}$$

**Mathematica [A]**

time = 0.23, size = 50, normalized size = 0.49

$$\frac{e^{-a-bx}(-24 - 24(a+bx) - 12(a+bx)^2 - 4(a+bx)^3 - (a+bx)^4)}{b}$$

Antiderivative was successfully verified.

`[In] Integrate[E^(-a - b*x)*(a + b*x)^4,x]``[Out] (E^(-a - b*x)*(-24 - 24*(a + b*x) - 12*(a + b*x)^2 - 4*(a + b*x)^3 - (a + b*x)^4))/b`**Maple [A]**

time = 0.06, size = 99, normalized size = 0.97

method	result
derivativedivides	$-\frac{(-bx-a)^4 e^{-bx-a} - 4e^{-bx-a}(-bx-a)^3 + 12(-bx-a)^2 e^{-bx-a} - 24(-bx-a)e^{-bx-a} + 24e^{-bx-a}}{b}$
default	$-\frac{(-bx-a)^4 e^{-bx-a} - 4e^{-bx-a}(-bx-a)^3 + 12(-bx-a)^2 e^{-bx-a} - 24(-bx-a)e^{-bx-a} + 24e^{-bx-a}}{b}$
gospers	$-\frac{(b^4 x^4 + 4b^3 x^3 a + 6a^2 b^2 x^2 + 4b^3 x^3 + 4a^3 b x + 12b^2 x^2 a + a^4 + 12a^2 b x + 12b^2 x^2 + 4a^3 + 24abx + 12a^2 + 24bx + 24a + 24)e^{-bx-a}}{b}$
risch	$-\frac{(b^4 x^4 + 4b^3 x^3 a + 6a^2 b^2 x^2 + 4b^3 x^3 + 4a^3 b x + 12b^2 x^2 a + a^4 + 12a^2 b x + 12b^2 x^2 + 4a^3 + 24abx + 12a^2 + 24bx + 24a + 24)e^{-bx-a}}{b}$
norman	$(-4b^2 a - 4b^2) x^3 e^{-bx-a} + (-4a^3 - 12a^2 - 24a - 24) x e^{-bx-a} - b^3 x^4 e^{-bx-a} - \frac{(a^4 + 4a^3 + 12a^2 + 24a + 24)e^{-bx-a}}{b}$
meijerg	$\frac{e^{-a} \left( 24 - \frac{(5b^4 x^4 + 20b^3 x^3 a + 60b^2 x^2 + 120bx + 120)e^{-bx}}{5} \right)}{b} + \frac{4e^{-a} \left( 6 - \frac{(4b^3 x^3 + 12b^2 x^2 + 24bx + 24)e^{-bx}}{4} \right)}{b} + \frac{6e^{-a} a^2 \left( 2 - \frac{(a^4 + 4a^3 + 12a^2 + 24a + 24)e^{-bx-a}}{b} \right)}{b}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(exp(-b*x-a)*(b*x+a)^4,x,method=_RETURNVERBOSE)`

[Out]  $-1/b*((-b*x-a)^4*\exp(-b*x-a)-4*\exp(-b*x-a)*(-b*x-a)^3+12*(-b*x-a)^2*\exp(-b*x-a)-24*(-b*x-a)*\exp(-b*x-a)+24*\exp(-b*x-a))$

**Maxima [A]**

time = 0.30, size = 149, normalized size = 1.46

$$\frac{4(bx+1)a^3e^{-bx-a}}{b} - \frac{a^4e^{-bx-a}}{b} - \frac{6(b^2x^2+2bx+2)a^2e^{-bx-a}}{b} - \frac{4(b^3x^3+3b^2x^2+6bx+6)ae^{-bx-a}}{b} - \frac{(b^4x^4+4b^3x^3+12b^2x^2+24bx+24)e^{-bx-a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^4,x, algorithm="maxima")`

[Out]  $-4*(b*x+1)*a^3*e^{(-b*x-a)}/b - a^4*e^{(-b*x-a)}/b - 6*(b^2*x^2+2*b*x+2)*a^2*e^{(-b*x-a)}/b - 4*(b^3*x^3+3*b^2*x^2+6*b*x+6)*a*e^{(-b*x-a)}/b - (b^4*x^4+4*b^3*x^3+12*b^2*x^2+24*b*x+24)*e^{(-b*x-a)}/b$

**Fricas [A]**

time = 0.39, size = 83, normalized size = 0.81

$$\frac{(b^4x^4+4(a+1)b^3x^3+6(a^2+2a+2)b^2x^2+a^4+4a^3+4(a^3+3a^2+6a+6)bx+12a^2+24a+24)e^{-bx-a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^4,x, algorithm="fricas")`

[Out]  $-(b^4*x^4+4*(a+1)*b^3*x^3+6*(a^2+2*a+2)*b^2*x^2+a^4+4*a^3+4*(a^3+3*a^2+6*a+6)*b*x+12*a^2+24*a+24)*e^{(-b*x-a)}/b$

**Sympy [A]**

time = 0.08, size = 158, normalized size = 1.55

$$\begin{cases} \frac{(-a^4-4a^3bx-4a^3-6a^2b^2x^2-12a^2bx-12a^2-4ab^3x^3-12ab^2x^2-24abx-24a-b^4x^4-4b^3x^3-12b^2x^2-24bx-24)e^{-a-bx}}{b} & \text{for } b \neq 0 \\ a^4x+2a^3bx^2+2a^2b^2x^3+ab^3x^4+\frac{b^4x^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)**4,x)`

[Out] `Piecewise((( -a**4 - 4*a**3*b*x - 4*a**3 - 6*a**2*b**2*x**2 - 12*a**2*b*x - 12*a**2 - 4*a*b**3*x**3 - 12*a*b**2*x**2 - 24*a*b*x - 24*a - b**4*x**4 - 4*b**3*x**3 - 12*b**2*x**2 - 24*b*x - 24)*exp(-a - b*x)/b, Ne(b, 0)), (a**4*x + 2*a**3*b*x**2 + 2*a**2*b**2*x**3 + a*b**3*x**4 + b**4*x**5/5, True))`

**Giac [A]**

time = 3.31, size = 132, normalized size = 1.29

$$\frac{(b^8x^4+4ab^7x^3+6a^2b^6x^2+4b^7x^3+4a^3b^5x+12ab^6x^2+a^4b^4+12a^2b^5x+12b^6x^2+4a^3b^4+24ab^5x+12a^2b^4+24b^5x+24ab^4+24b^4)e^{-bx-a}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4,x, algorithm="giac")

[Out]  $-(b^8x^4 + 4ab^7x^3 + 6a^2b^6x^2 + 4b^7x^3 + 4a^3b^5x + 12ab^6x^2 + a^4b^4 + 12a^2b^5x + 12b^6x^2 + 4a^3b^4 + 24ab^5x + 12a^2b^4 + 24b^5x + 24ab^4 + 24b^4)e^{-(b*x - a)}/b^5$

Mupad [B]

time = 0.11, size = 120, normalized size = 1.18

$$-b^3 x^4 e^{-a-bx} - x e^{-a-bx} (4a^3 + 12a^2 + 24a + 24) - \frac{e^{-a-bx} (a^4 + 4a^3 + 12a^2 + 24a + 24)}{b} - 6bx^2 e^{-a-bx} (a^2 + 2a + 2) - 4b^2 x^3 e^{-a-bx} (a + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(- a - b\*x)\*(a + b\*x)^4,x)

[Out]  $-b^3x^4\exp(-a-bx) - x\exp(-a-bx)*(24a + 12a^2 + 4a^3 + 24) - (\exp(-a-bx)*(24a + 12a^2 + 4a^3 + a^4 + 24))/b - 6b^2x^2\exp(-a-bx)*(2a + a^2 + 2) - 4b^2x^3\exp(-a-bx)*(a + 1)$

$$3.78 \quad \int \frac{e^{-a-bx}(a+bx)^4}{c+dx} dx$$

**Optimal.** Leaf size=277

$$-\frac{6e^{-a-bx}}{d} + \frac{2(bc-ad)e^{-a-bx}}{d^2} - \frac{(bc-ad)^2e^{-a-bx}}{d^3} + \frac{(bc-ad)^3e^{-a-bx}}{d^4} - \frac{6e^{-a-bx}(a+bx)}{d} + \frac{2(bc-ad)e^{-a-bx}(a+bx)}{d^2}$$

[Out]  $-6*\exp(-b*x-a)/d+2*(-a*d+b*c)*\exp(-b*x-a)/d^2-(-a*d+b*c)^2*\exp(-b*x-a)/d^3+(-a*d+b*c)^3*\exp(-b*x-a)/d^4-6*\exp(-b*x-a)*(b*x+a)/d+2*(-a*d+b*c)*\exp(-b*x-a)*(b*x+a)/d^2-(-a*d+b*c)^2*\exp(-b*x-a)*(b*x+a)/d^3-3*\exp(-b*x-a)*(b*x+a)^2/d+(-a*d+b*c)*\exp(-b*x-a)*(b*x+a)^2/d^2-\exp(-b*x-a)*(b*x+a)^3/d+(-a*d+b*c)^4*\exp(-a+b*c/d)*\text{Ei}(-b*(d*x+c)/d)/d^5$

**Rubi [A]**

time = 0.24, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {2230, 2225, 2207, 2209}

$$\frac{e^{\frac{b}{d}(-a-bx)}(bc-ad)^4\text{Ei}\left(\frac{-b(c+dx)}{d}\right)}{d^5} + \frac{e^{-a-bx}(bc-ad)^3}{d^4} - \frac{e^{-a-bx}(bc-ad)^2}{d^3} - \frac{e^{-a-bx}(a+bx)(bc-ad)^2}{d^3} + \frac{2e^{-a-bx}(bc-ad)}{d^2} + \frac{e^{-a-bx}(a+bx)^2(bc-ad)}{d^2} + \frac{2e^{-a-bx}(a+bx)(bc-ad)}{d^2} - \frac{6e^{-a-bx}}{d} - \frac{e^{-a-bx}(a+bx)^3}{d} - \frac{3e^{-a-bx}(a+bx)^2}{d} - \frac{6e^{-a-bx}(a+bx)}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(E^{-a-b*x})*(a+b*x)^4]/(c+d*x), x]$

[Out]  $(-6*E^{-a-b*x})/d + (2*(b*c-a*d)*E^{-a-b*x})/d^2 - ((b*c-a*d)^2*E^{-a-b*x})/d^3 + ((b*c-a*d)^3*E^{-a-b*x})/d^4 - (6*E^{-a-b*x}*(a+b*x))/d + (2*(b*c-a*d)*E^{-a-b*x}*(a+b*x))/d^2 - ((b*c-a*d)^2*E^{-a-b*x}*(a+b*x))/d^3 - (3*E^{-a-b*x}*(a+b*x)^2)/d + ((b*c-a*d)*E^{-a-b*x}*(a+b*x)^2)/d^2 - (E^{-a-b*x}*(a+b*x)^3)/d + ((b*c-a*d)^4*E^{-a-b*x}*(a+b*x)/d)*\text{ExpIntegralEi}[-((b*(c+d*x))/d)]/d^5$

Rule 2207

$\text{Int}[(b_.)*(F_)^((g_.)*((e_.)+(f_.)*(x_.)))^{(n_.)*((c_.)+(d_.)*(x_.))^{(m_.)}, x\_Symbol] :> \text{Simp}[(c+d*x)^m*((b*F^(g*(e+f*x)))^n/(f*g*n*\text{Log}[F])), x] - \text{Dist}[d*(m/(f*g*n*\text{Log}[F])), \text{Int}[(c+d*x)^{(m-1)}*(b*F^(g*(e+f*x)))^n, x], x] /; \text{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0] \&\& \text{IntegerQ}[2*m] \&\& !\text{TrueQ}[\$UseGamma]$

Rule 2209

$\text{Int}[(F_)^((g_.)*((e_.)+(f_.)*(x_.)))/((c_.)+(d_.)*(x_.)), x\_Symbol] :> \text{Simp}[(F^(g*(e-c*(f/d)))/d)*\text{ExpIntegralEi}[f*g*(c+d*x)*(\text{Log}[F]/d)], x] /; \text{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\text{TrueQ}[\$UseGamma]$

Rule 2225

```
Int[((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.), x_Symbol] := Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

### Rule 2230

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] := Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F,
c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] && !TrueQ[$UseGamma]
```

### Rubi steps

$$\begin{aligned}
\int \frac{e^{-a-bx}(a+bx)^4}{c+dx} dx &= \int \left( -\frac{b(bc-ad)^3 e^{-a-bx}}{d^4} + \frac{b(bc-ad)^2 e^{-a-bx}(a+bx)}{d^3} - \frac{b(bc-ad)e^{-a-bx}(a+bx)}{d^2} \right. \\
&= \frac{b \int e^{-a-bx}(a+bx)^3 dx}{d} - \frac{(b(bc-ad)) \int e^{-a-bx}(a+bx)^2 dx}{d^2} + \frac{(b(bc-ad)^2) \int e^{-a-bx}(a+bx) dx}{d^3} \\
&= \frac{(bc-ad)^3 e^{-a-bx}}{d^4} - \frac{(bc-ad)^2 e^{-a-bx}(a+bx)}{d^3} + \frac{(bc-ad)e^{-a-bx}(a+bx)^2}{d^2} - \frac{e^{-a-bx}(a+bx)^3}{d} \\
&= -\frac{(bc-ad)^2 e^{-a-bx}}{d^3} + \frac{(bc-ad)^3 e^{-a-bx}}{d^4} + \frac{2(bc-ad)e^{-a-bx}(a+bx)}{d^2} - \frac{(bc-ad)^2 e^{-a-bx}(a+bx)^2}{d} \\
&= \frac{2(bc-ad)e^{-a-bx}}{d^2} - \frac{(bc-ad)^2 e^{-a-bx}}{d^3} + \frac{(bc-ad)^3 e^{-a-bx}}{d^4} - \frac{6e^{-a-bx}(a+bx)}{d} + \frac{2e^{-a-bx}(a+bx)^2}{d} \\
&= -\frac{6e^{-a-bx}}{d} + \frac{2(bc-ad)e^{-a-bx}}{d^2} - \frac{(bc-ad)^2 e^{-a-bx}}{d^3} + \frac{(bc-ad)^3 e^{-a-bx}}{d^4} - \frac{6e^{-a-bx}(a+bx)}{d} + \frac{2e^{-a-bx}(a+bx)^2}{d}
\end{aligned}$$

### Mathematica [A]

time = 0.70, size = 175, normalized size = 0.63

$$\frac{e^{-a-bx} \left( -d(2(3+4a+3a^2+2a^3)d^3+2bd^2((1+2a+3a^2)c+(3+4a+3a^2)dx)+b^2d((1+4a)c^2-2(1+2a)cdx+(3+4a)d^2x^2)+b^3(-c^3+c^2dx-cd^2x^2+d^3x^3))+(bc-ad)^4e^{b(\frac{5}{3}+x)}\text{Ei}\left(-\frac{b(c+dx)}{d}\right) \right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(-a - b*x)*(a + b*x)^4)/(c + d*x), x]
```

```
[Out] (E^(-a - b*x)*(-(d*(2*(3 + 4*a + 3*a^2 + 2*a^3)*d^3 + 2*b*d^2*(-((1 + 2*a +
3*a^2)*c) + (3 + 4*a + 3*a^2)*d*x) + b^2*d*((1 + 4*a)*c^2 - 2*(1 + 2*a)*c*
d*x + (3 + 4*a)*d^2*x^2) + b^3*(-c^3 + c^2*d*x - c*d^2*x^2 + d^3*x^3))) + (
b*c - a*d)^4*E^(b*(c/d + x))*ExpIntegralEi[-((b*(c + d*x))/d)])/d^5
```

### Maple [A]

time = 0.06, size = 489, normalized size = 1.77

method	result
derivativedivides	$-\frac{b a^3 e^{-bx-a}}{d} - \frac{3b^2 a^2 c e^{-bx-a}}{d^2} - \frac{b a^2 ((-bx-a)e^{-bx-a} - e^{-bx-a})}{d} + \frac{3b^3 a c^2 e^{-bx-a}}{d^3} + \frac{2b^2 a c ((-bx-a)e^{-bx-a} - e^{-bx-a})}{d^2} + \frac{b a ((-bx-a)e^{-bx-a} - e^{-bx-a})}{d}$
default	$-\frac{b a^3 e^{-bx-a}}{d} - \frac{3b^2 a^2 c e^{-bx-a}}{d^2} - \frac{b a^2 ((-bx-a)e^{-bx-a} - e^{-bx-a})}{d} + \frac{3b^3 a c^2 e^{-bx-a}}{d^3} + \frac{2b^2 a c ((-bx-a)e^{-bx-a} - e^{-bx-a})}{d^2} + \frac{b a ((-bx-a)e^{-bx-a} - e^{-bx-a})}{d}$
risch	$-\frac{b^4 e^{-\frac{ad-cb}{d}} \expIntegral\left(1, bx+a-\frac{ad-cb}{d}\right) c^4}{d^5} - \frac{6e^{-bx-a}}{d} + \frac{2bc e^{-bx-a}}{d^2} - \frac{b^3 e^{-bx-a} x^3}{d} - \frac{3b^2 e^{-bx-a} x^2}{d} - \frac{6b e^{-bx-a} x}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(-b*x-a)*(b*x+a)^4/(d*x+c),x,method=_RETURNVERBOSE)`

[Out] 
$$-1/b*(b/d*a^3*\exp(-b*x-a)-3*b^2/d^2*a^2*c*\exp(-b*x-a)-b/d*a^2*((-b*x-a)*\exp(-b*x-a)-\exp(-b*x-a))+3*b^3/d^3*a*c^2*\exp(-b*x-a)+2*b^2/d^2*a*c*((-b*x-a)*\exp(-b*x-a)-\exp(-b*x-a))+b/d*a*((-b*x-a)^2*\exp(-b*x-a)-2*(-b*x-a)*\exp(-b*x-a)+2*\exp(-b*x-a))-b^4/d^4*c^3*\exp(-b*x-a)-b^3/d^3*c^2*((-b*x-a)*\exp(-b*x-a)-\exp(-b*x-a))-b^2/d^2*c*((-b*x-a)^2*\exp(-b*x-a)-2*(-b*x-a)*\exp(-b*x-a)+2*\exp(-b*x-a))-1/d*b*(\exp(-b*x-a)*(-b*x-a)^3-3*(-b*x-a)^2*\exp(-b*x-a)+6*(-b*x-a)*\exp(-b*x-a)-6*\exp(-b*x-a))+a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*b/d^5*\exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)$$

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c),x, algorithm="maxima")`

[Out] 
$$-a^4*e^{(-a+b*c/d)}*\exp\_integral\_e(1,(d*x+c)*b/d)/d - (b^3*d^2*x^4 + (4*a*b^2*d^2 + 3*b^2*d^2)*x^3 + (6*a^2*b*d^2 + b^2*c*d + 8*a*b*d^2 + 6*b*d^2)*x^2 + (4*a^3*d^2 - b^2*c^2 + 6*a^2*d^2 + 4*b*c*d + 4*(b*c*d + 2*d^2)*a + 6*d^2)*x)*e^{(-b*x)}/(d^3*x*e^a + c*d^2*e^a) + integrate((4*a^3*c*d^2 - b^2*c^3 + 6*a^2*c*d^2 + 4*b*c^2*d + 6*c*d^2 + 4*(b*c^2*d + 2*c*d^2)*a + (b^3*c^3 + 6*a^2*b*c*d^2 - 2*b^2*c^2*d + 6*b*c*d^2 - 4*(b^2*c^2*d - 2*b*c*d^2)*a)*x)*e^{(-b*x)}/(d^4*x^2*e^a + 2*c*d^3*x*e^a + c^2*d^2*e^a), x)$$

**Fricas** [A]

time = 0.35, size = 235, normalized size = 0.85

$$\frac{(b^4 c^4 - 4 a b^3 c^3 d + 6 a^2 b^2 c^2 d^2 - 4 a^3 b c d^3 + a^4 d^4) \operatorname{Ei}\left(-\frac{b x+a}{d}\right) e^{\frac{b x+a}{d}} - (b^4 d^4 x^3 - b^3 c^2 d + (4 a+1) b^2 c^2 d^2 - 2(3 a^2+2 a+1) b c d^3 + 2(2 a^3+3 a^2+4 a+3) d^4 - (b^3 c d^3 - (4 a+3) b^2 d^4) x^2 + (b^3 c^2 d^2 - 2(2 a+1) b^2 c d^3 + 2(3 a^2+4 a+3) b d^4) x) e^{(-b x-a)}}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c),x, algorithm="fricas")`

[Out]  $((b^4c^4 - 4a^3b^3c^3d + 6a^2b^2c^2d^2 - 4a^3b^3c^3d + a^4d^4) \cdot \text{Ei}(-b^3d^4x^3 - b^3c^3d + (4a + 1)b^2c^2d^2 - 2(3a^2 + 2a + 1)b^3cd^3 + 2(2a^3 + 3a^2 + 4a + 3)d^4 - (b^3cd^3 - (4a + 3)b^2d^4)x^2 + (b^3c^2d^2 - 2(2a + 1)b^2cd^3 + 2(3a^2 + 4a + 3)b^3d^4)x) \cdot e^{-bx - a}) / d^5$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\left( \int \frac{a^4}{ce^{bx} + dx e^{bx}} dx + \int \frac{b^4 x^4}{ce^{bx} + dx e^{bx}} dx + \int \frac{4ab^3 x^3}{ce^{bx} + dx e^{bx}} dx + \int \frac{6a^2 b^2 x^2}{ce^{bx} + dx e^{bx}} dx + \int \frac{4a^3 bx}{ce^{bx} + dx e^{bx}} dx \right) e^{-a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)**4/(d*x+c), x)`

[Out]  $(\text{Integral}(a^{**4}/(c \cdot \exp(b \cdot x) + d \cdot x \cdot \exp(b \cdot x)), x) + \text{Integral}(b^{**4} \cdot x^{**4}/(c \cdot \exp(b \cdot x) + d \cdot x \cdot \exp(b \cdot x)), x) + \text{Integral}(4 \cdot a \cdot b^{**3} \cdot x^{**3}/(c \cdot \exp(b \cdot x) + d \cdot x \cdot \exp(b \cdot x)), x) + \text{Integral}(6 \cdot a^{**2} \cdot b^{**2} \cdot x^{**2}/(c \cdot \exp(b \cdot x) + d \cdot x \cdot \exp(b \cdot x)), x) + \text{Integral}(4 \cdot a^{**3} \cdot b \cdot x/(c \cdot \exp(b \cdot x) + d \cdot x \cdot \exp(b \cdot x)), x)) \cdot \exp(-a)$

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 546 vs.  $2(266) = 532$ .

time = 2.65, size = 546, normalized size = 1.97

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c), x, algorithm="giac")`

[Out]  $-(b^3d^4x^3e^{-bx - a} - b^3c^3d^3x^2e^{-bx - a} + 4a^3b^2d^4x^2e^{-bx - a} - b^4c^4 \cdot \text{Ei}(-b^3d^4x^3 - b^3c^3d + (4a + 1)b^2c^2d^2 - 2(3a^2 + 2a + 1)b^3cd^3 + 2(2a^3 + 3a^2 + 4a + 3)d^4 - (b^3cd^3 - (4a + 3)b^2d^4)x^2 + (b^3c^2d^2 - 2(2a + 1)b^2cd^3 + 2(3a^2 + 4a + 3)b^3d^4)x) \cdot e^{-bx - a} - 6a^2b^2c^2d^2 \cdot \text{Ei}(-b^3d^4x^3 - b^3c^3d + (4a + 1)b^2c^2d^2 - 2(3a^2 + 2a + 1)b^3cd^3 + 2(2a^3 + 3a^2 + 4a + 3)d^4 - (b^3cd^3 - (4a + 3)b^2d^4)x^2 + (b^3c^2d^2 - 2(2a + 1)b^2cd^3 + 2(3a^2 + 4a + 3)b^3d^4)x) \cdot e^{-bx - a} + 4a^3b^2c^3d^3 \cdot \text{Ei}(-b^3d^4x^3 - b^3c^3d + (4a + 1)b^2c^2d^2 - 2(3a^2 + 2a + 1)b^3cd^3 + 2(2a^3 + 3a^2 + 4a + 3)d^4 - (b^3cd^3 - (4a + 3)b^2d^4)x^2 + (b^3c^2d^2 - 2(2a + 1)b^2cd^3 + 2(3a^2 + 4a + 3)b^3d^4)x) \cdot e^{-bx - a} - a^4d^4 \cdot \text{Ei}(-b^3d^4x^3 - b^3c^3d + (4a + 1)b^2c^2d^2 - 2(3a^2 + 2a + 1)b^3cd^3 + 2(2a^3 + 3a^2 + 4a + 3)d^4 - (b^3cd^3 - (4a + 3)b^2d^4)x^2 + (b^3c^2d^2 - 2(2a + 1)b^2cd^3 + 2(3a^2 + 4a + 3)b^3d^4)x) \cdot e^{-bx - a} + b^3c^2d^2x^2e^{-bx - a} - 4a^3b^2c^3d^3x^2e^{-bx - a} + 6a^2b^2c^2d^2x^2e^{-bx - a} + 3b^2d^4x^2e^{-bx - a} - b^3c^3d^3x^2e^{-bx - a} + 4a^3b^2c^2d^2x^2e^{-bx - a} - 6a^2b^2c^3d^3x^2e^{-bx - a} + 4a^3d^4x^2e^{-bx - a} - 2b^2c^3d^3x^2e^{-bx - a} + 8a^3b^2d^4x^2e^{-bx - a} + b^2c^2d^2x^2e^{-bx - a} - 4a^3b^2c^3d^3x^2e^{-bx - a} + 6a^2d^4x^2e^{-bx - a} + 6b^2d^4x^2e^{-bx - a} - 2b^2c^3d^3x^2e^{-bx - a} + 8a^3d^4x^2e^{-bx - a} + 6d^4x^2e^{-bx - a}) / d^5$

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{-a-bx} (a+bx)^4}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(- a - b*x)*(a + b*x)^4)/(c + d*x),x)
```

```
[Out] int((exp(- a - b*x)*(a + b*x)^4)/(c + d*x), x)
```

$$3.79 \quad \int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^2} dx$$

**Optimal.** Leaf size=258

$$-\frac{2be^{-a-bx}}{d^2} + \frac{4b(bc-ad)e^{-a-bx}}{d^3} - \frac{6b(bc-ad)^2e^{-a-bx}}{d^4} - \frac{(bc-ad)^4e^{-a-bx}}{d^5(c+dx)} - \frac{2b^2e^{-a-bx}(c+dx)}{d^3} + \frac{4b^2(bc-ad)e^{-a-bx}}{d^2}$$

[Out]  $-2*b*\exp(-b*x-a)/d^2+4*b*(-a*d+b*c)*\exp(-b*x-a)/d^3-6*b*(-a*d+b*c)^2*\exp(-b*x-a)/d^4-(-a*d+b*c)^4*\exp(-b*x-a)/d^5/(d*x+c)-2*b^2*\exp(-b*x-a)*(d*x+c)/d^3+4*b^2*(-a*d+b*c)*\exp(-b*x-a)*(d*x+c)/d^4-b^3*\exp(-b*x-a)*(d*x+c)^2/d^4-4*b*(-a*d+b*c)^3*\exp(-a+b*c/d)*\text{Ei}(-b*(d*x+c)/d)/d^5-b*(-a*d+b*c)^4*\exp(-a+b*c/d)*\text{Ei}(-b*(d*x+c)/d)/d^6$

**Rubi [A]**

time = 0.27, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2230, 2225, 2208, 2209, 2207}

$$-\frac{b^3e^{-a-bx}(c+dx)^2}{d^4} + \frac{4b^2e^{-a-bx}(c+dx)(bc-ad)}{d^4} - \frac{2b^2e^{-a-bx}(c+dx)}{d^3} - \frac{bc^{\frac{bc}{d}}e^{-a}(bc-ad)^4\text{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^6} - \frac{4be^{\frac{bc}{d}-a}(bc-ad)^3\text{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^6} - \frac{e^{-a-bx}(bc-ad)^4}{d^5(c+dx)} - \frac{6be^{-a-bx}(bc-ad)^2}{d^4} + \frac{4be^{-a-bx}(bc-ad)}{d^3} - \frac{2be^{-a-bx}}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(E^(-a - b\*x)\*(a + b\*x)^4)/(c + d\*x)^2,x]

[Out]  $(-2*b*E^(-a - b*x))/d^2 + (4*b*(b*c - a*d)*E^(-a - b*x))/d^3 - (6*b*(b*c - a*d)^2*E^(-a - b*x))/d^4 - ((b*c - a*d)^4*E^(-a - b*x))/(d^5*(c + d*x)) - (2*b^2*E^(-a - b*x)*(c + d*x))/d^3 + (4*b^2*(b*c - a*d)*E^(-a - b*x)*(c + d*x))/d^4 - (b^3*E^(-a - b*x)*(c + d*x)^2)/d^4 - (4*b*(b*c - a*d)^3*E^(-a + (b*c)/d)*\text{ExpIntegralEi}[-((b*(c + d*x))/d)])/d^5 - (b*(b*c - a*d)^4*E^(-a + (b*c)/d)*\text{ExpIntegralEi}[-((b*(c + d*x))/d)])/d^6$

Rule 2207

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(c + d\*x)^m\*((b\*F^(g\*(e + f\*x)))^n/(f\*g\*n\*Log[F])), x] - Dist[d\*(m/(f\*g\*n\*Log[F])), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

Rule 2208

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)\*((b\*F^(g\*(e + f\*x)))^n/(d\*(m + 1))), x] - Dist[f\*g\*n\*(Log[F]/(d\*(m + 1))), Int[(c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

Rule 2209

```
Int[(F_)^((g_)*((e_)+(f_)*(x_)))/((c_)+(d_)*(x_)), x_Symbol] := Simp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_)*((a_)+(b_)*(x_))))^(n_), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2230

```
Int[(F_)^((c_)*(v_))*(u_)^(m_)*(w_), x_Symbol] := Int[ExpandIntegrand[F^(c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && IntegerQ[m] && !TrueQ[$UseGamma]
```

Rubi steps

$$\begin{aligned} \int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^2} dx &= \int \left( \frac{6b^2(bc-ad)^2 e^{-a-bx}}{d^4} + \frac{(-bc+ad)^4 e^{-a-bx}}{d^4(c+dx)^2} - \frac{4b(bc-ad)^3 e^{-a-bx}}{d^4(c+dx)} - \frac{4b^3(bc-ad)^2 e^{-a-bx}}{d^4} \right) dx \\ &= \frac{b^4 \int e^{-a-bx}(c+dx)^2 dx}{d^4} - \frac{(4b^3(bc-ad)) \int e^{-a-bx}(c+dx) dx}{d^4} + \frac{(6b^2(bc-ad)^2) \int e^{-a-bx} dx}{d^4} \\ &= -\frac{6b(bc-ad)^2 e^{-a-bx}}{d^4} - \frac{(bc-ad)^4 e^{-a-bx}}{d^5(c+dx)} + \frac{4b^2(bc-ad) e^{-a-bx}(c+dx)}{d^4} - \frac{b^3 e^{-a-bx}}{d^4} \\ &= \frac{4b(bc-ad) e^{-a-bx}}{d^3} - \frac{6b(bc-ad)^2 e^{-a-bx}}{d^4} - \frac{(bc-ad)^4 e^{-a-bx}}{d^5(c+dx)} - \frac{2b^2 e^{-a-bx}(c+dx)}{d^3} \\ &= -\frac{2be^{-a-bx}}{d^2} + \frac{4b(bc-ad) e^{-a-bx}}{d^3} - \frac{6b(bc-ad)^2 e^{-a-bx}}{d^4} - \frac{(bc-ad)^4 e^{-a-bx}}{d^5(c+dx)} - \frac{2b^2 e^{-a-bx}}{d^3} \end{aligned}$$

Mathematica [A]

time = 1.09, size = 163, normalized size = 0.63

$$\frac{e^{-a} \left( -\frac{de^{-bx}((bc-ad)^4 + bd(3b^2c^2 - 2(1+4a)bcd + 2(1+2a+3a^2)d^2)(c+dx) - 2b^2d^2(bc-(1+2a)d)x(c+dx) + b^3d^3x^2(c+dx))}{c+dx} - b(bc - (-4+a)d)(bc-ad)^3 e^{\frac{bx}{d}} \text{Ei}\left(-\frac{b(c+dx)}{d}\right) \right)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(-a - b*x))*(a + b*x)^4)/(c + d*x)^2, x]
```



```
[Out] (-((d*((b*c - a*d)^4 + b*d*(3*b^2*c^2 - 2*(1 + 4*a)*b*c*d + 2*(1 + 2*a + 3*
a^2)*d^2)*(c + d*x) - 2*b^2*d^2*(b*c - (1 + 2*a)*d)*x*(c + d*x) + b^3*d^3*x
^2*(c + d*x)))/(E^(b*x)*(c + d*x))) - b*(b*c - (-4 + a)*d)*(b*c - a*d)^3*E^
((b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)]/(d^6*E^a)
```

**Maple [A]**

time = 0.07, size = 406, normalized size = 1.57

method	result
derivativedivides	$-\frac{3b^2a^2e^{-bx-a}}{d^2} - \frac{6b^3ace^{-bx-a}}{d^3} - \frac{2b^2a((-bx-a)e^{-bx-a} - e^{-bx-a})}{d^2} + \frac{3b^4e^2e^{-bx-a}}{d^4} + \frac{2b^3c((-bx-a)e^{-bx-a} - e^{-bx-a})}{d^3} + \frac{b^2}{d^2}$
default	$-\frac{3b^2a^2e^{-bx-a}}{d^2} - \frac{6b^3ace^{-bx-a}}{d^3} - \frac{2b^2a((-bx-a)e^{-bx-a} - e^{-bx-a})}{d^2} + \frac{3b^4e^2e^{-bx-a}}{d^4} + \frac{2b^3c((-bx-a)e^{-bx-a} - e^{-bx-a})}{d^3} + \frac{b^2}{d^2}$
risch	$-\frac{6ba^2e^{-bx-a}}{d^2} - \frac{3b^3c^2e^{-bx-a}}{d^4} + \frac{12b^2e^{-\frac{ad-cb}{d}} \expIntegral\left(1, bx+a-\frac{ad-cb}{d}\right) a^2 c}{d^3} - \frac{12b^3e^{-\frac{ad-cb}{d}} \expIntegral\left(1, bx+a-\frac{ad-cb}{d}\right)}{d^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(3*b^2/d^2*a^2*exp(-b*x-a)-6*b^3/d^3*a*c*exp(-b*x-a)-2*b^2/d^2*a*((-b*
x-a)*exp(-b*x-a)-exp(-b*x-a))+3*b^4/d^4*c^2*exp(-b*x-a)+2*b^3/d^3*c*((-b*x-
a)*exp(-b*x-a)-exp(-b*x-a))+1/d^2*b^2*((-b*x-a)^2*exp(-b*x-a)-2*(-b*x-a)*ex
p(-b*x-a)+2*exp(-b*x-a))+4/d^5*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3
)*b^2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)+(a^4*d^4-4*a^3*b*c*d^3+6*a^
2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*b^2/d^6*(-exp(-b*x-a)/(-b*x-a+(a*d-b*c
)/d)-exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -a^4*e^(-a + b*c/d)*exp_integral_e(2, (d*x + c)*b/d)/((d*x + c)*d) - (b^3*d
^2*x^4 + 2*(2*a*b^2*d^2 + b^2*d^2)*x^3 + 2*(3*a^2*b*d^2 + b^2*c*d + 2*a*b*d
^2 + b*d^2)*x^2 + 2*(2*a^3*d^2 - b^2*c^2 + 4*a*b*c*d + 2*b*c*d)*x)*e^(-b*x)
/(d^4*x^2*e^a + 2*c*d^3*x*e^a + c^2*d^2*e^a) - integrate(-2*(2*a^3*c*d^2 -
b^2*c^3 + 4*a*b*c^2*d + 2*b*c^2*d + (b^3*c^3 - 4*a*b^2*c^2*d + 6*a^2*b*c*d^
2 - 2*a^3*d^3 + b^2*c^2*d)*x)*e^(-b*x)/(d^5*x^3*e^a + 3*c*d^4*x^2*e^a + 3*c
^2*d^3*x*e^a + c^3*d^2*e^a), x)
```

**Fricas [A]**

time = 0.38, size = 353, normalized size = 1.37

$$\frac{(b^5c^5 - 4(a-1)b^4c^4d + 6(a^2 - 2a)b^3c^3d^2 - 4(a^3 - 3a^2)b^2c^2d^3 + (a^4 - 4a^3)b^2c^2d^4 + (b^5c^4d - 4(a-1)b^4c^3d^2 - 4(a^2 - 2a)b^3c^2d^3 - 4(a^3 - 3a^2)b^2c^2d^4 + (a^4 - 4a^3)b^2c^2d^5)Ei(-\frac{b^2c^2d^2}{d^2x + c^2d^2}) + (b^5c^4d - 4(a-1)b^4c^3d^2 - 4(a^2 - 2a)b^3c^2d^3 - 4(a^3 - 3a^2)b^2c^2d^4 - 4(a^4 - 4a^3)b^2c^2d^5)Ei(-\frac{b^2c^2d^2}{d^2x + c^2d^2})}{d^2x + c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c)^2,x, algorithm="fricas")

**[Out]**  $-(b^5c^5 - 4(a-1)b^4c^4d + 6(a^2 - 2a)b^3c^3d^2 - 4(a^3 - 3a^2)b^2c^2d^3 + (a^4 - 4a^3)b^2c^2d^4 + (b^5c^4d - 4(a-1)b^4c^3d^2 + 6(a^2 - 2a)b^3c^2d^3 - 4(a^3 - 3a^2)b^2c^2d^4 + (a^4 - 4a^3)b^2c^2d^5)*x)*Ei(-\frac{b^2c^2d^2}{d^2x + c^2d^2}) + (b^3d^5x^3 + b^4c^4d - (4a - 3)b^3c^3d^2 + a^4d^5 + 2*(3a^2 - 4a - 1)b^2c^2d^3 - 2*(2a^3 - 3a^2 - 2a - 1)b^2c^2d^4 - (b^3c^2d^4 - 2*(2a + 1)b^2c^2d^5)*x^2 + (b^3c^2d^3 - 4a*b^2c^2d^4 + 2*(3a^2 + 2a + 1)b^2c^2d^5)*x)*Ei(-\frac{b^2c^2d^2}{d^2x + c^2d^2})$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\left(\int \frac{a^4}{c^2e^{bx} + 2cdxe^{bx} + d^2x^2e^{bx}} dx + \int \frac{b^4x^4}{c^2e^{bx} + 2cdxe^{bx} + d^2x^2e^{bx}} dx + \int \frac{4ab^3x^3}{c^2e^{bx} + 2cdxe^{bx} + d^2x^2e^{bx}} dx + \int \frac{6a^2b^2x^2}{c^2e^{bx} + 2cdxe^{bx} + d^2x^2e^{bx}} dx + \int \frac{4a^3bx}{c^2e^{bx} + 2cdxe^{bx} + d^2x^2e^{bx}} dx\right)e^{-a}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(exp(-b\*x-a)\*(b\*x+a)\*\*4/(d\*x+c)\*\*2,x)

**[Out]**  $(\text{Integral}(a**4/(c**2*\exp(b*x) + 2*c*d*x*\exp(b*x) + d**2*x**2*\exp(b*x)), x) + \text{Integral}(b**4*x**4/(c**2*\exp(b*x) + 2*c*d*x*\exp(b*x) + d**2*x**2*\exp(b*x)), x) + \text{Integral}(4*a*b**3*x**3/(c**2*\exp(b*x) + 2*c*d*x*\exp(b*x) + d**2*x**2*\exp(b*x)), x) + \text{Integral}(6*a**2*b**2*x**2/(c**2*\exp(b*x) + 2*c*d*x*\exp(b*x) + d**2*x**2*\exp(b*x)), x) + \text{Integral}(4*a**3*b*x/(c**2*\exp(b*x) + 2*c*d*x*\exp(b*x) + d**2*x**2*\exp(b*x)), x))*\exp(-a)$

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 2861 vs. 2(249) = 498.

time = 3.18, size = 2861, normalized size = 11.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c)^2,x, algorithm="giac")

**[Out]**  $-\left(\frac{b^6c^4}{(d^2x + c)^2}Ei\left(-\frac{(d^2x + c)(b - b^2c/(d^2x + c) + a^2d/(d^2x + c))}{d}\right) + b^7c^5Ei\left(-\frac{(d^2x + c)(b - b^2c/(d^2x + c) + a^2d/(d^2x + c))}{d}\right) + b^7c^5Ei\left(-\frac{(b^2c - a^2d)}{d}\right) - 4(d^2x + c)a(b - b^2c/(d^2x + c) + a^2d/(d^2x + c))b^5c^3dEi\left(-\frac{(b^2c - a^2d)}{d}\right)\right)$



$d/(d*x + c))/d) - 3*a^3*b^2*d^5*e^{-(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) + 2*(d*x + c)^2*(b - b*c/(d*x + c) + a*d/(d*x + c))^2*b^2*d^3*e^{-(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) - 2*b^4*c^2*d^3*e^{-(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) + 4*a*b^3*c*d^4*e^{-(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) - 2*a^2*b^2*d^5*e^{-(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) + 2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*d^4*e^{-(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) + 2*b^3*c*d^4*e^{-(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) - 2*a*b^2*d^5*e^{-(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d))*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*d^8 + b*c*d^8 - a*d^9)*b)$

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{-a-bx} (a+bx)^4}{(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(- a - b\*x)\*(a + b\*x)^4)/(c + d\*x)^2,x)

[Out] int((exp(- a - b\*x)\*(a + b\*x)^4)/(c + d\*x)^2, x)

$$3.80 \quad \int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^3} dx$$

**Optimal.** Leaf size=294

$$-\frac{b^2 e^{-a-bx}}{d^3} + \frac{b^2(3bc-4ad)e^{-a-bx}}{d^4} - \frac{b^3 e^{-a-bx} x}{d^3} - \frac{(bc-ad)^4 e^{-a-bx}}{2d^5(c+dx)^2} + \frac{4b(bc-ad)^3 e^{-a-bx}}{d^5(c+dx)} + \frac{b(bc-ad)^4 e^{-a-bx}}{2d^6(c+dx)}$$

[Out]  $-b^2 \exp(-bx-a)/d^3 + b^2(-4ad+3bc) \exp(-bx-a)/d^4 - b^3 \exp(-bx-a)x/d^3 - 1/2(-ad+bc)^4 \exp(-bx-a)/d^5 + (d^2x+c)^2 + 4b(-ad+bc)^3 \exp(-bx-a)/d^5 + (d^2x+c) + 1/2b(-ad+bc)^4 \exp(-bx-a)/d^6 + (d^2x+c) + 6b^2(-ad+bc)^2 \exp(-a+bc/d) \text{Ei}(-b(d^2x+c)/d)/d^5 + 4b^2(-ad+bc)^3 \exp(-a+bc/d) \text{Ei}(-b(d^2x+c)/d)/d^6 + 1/2b^2(-ad+bc)^4 \exp(-a+bc/d) \text{Ei}(-b(d^2x+c)/d)/d^7$

**Rubi [A]**

time = 0.29, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2230, 2225, 2207, 2208, 2209}

$$-\frac{b^3 x e^{-a-bx}}{d^3} + \frac{b^2 e^{\frac{a}{d}-bx}(bc-ad)^4 \text{Ei}\left(-\frac{b(c+dx)}{d}\right)}{2d^6} + \frac{4b^2 e^{\frac{a}{d}-bx}(bc-ad)^3 \text{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^6} + \frac{6b^2 e^{\frac{a}{d}-bx}(bc-ad)^2 \text{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^6} + \frac{b^2 e^{-a-bx}(3bc-4ad)}{d^4} - \frac{b^2 e^{-a-bx}}{d^3} + \frac{be^{-a-bx}(bc-ad)^4}{2d^6(c+dx)} - \frac{e^{-a-bx}(bc-ad)^4}{2d^6(c+dx)^2} + \frac{4be^{-a-bx}(bc-ad)^3}{d^6(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(E^(-a - b\*x)\*(a + b\*x)^4)/(c + d\*x)^3, x]

[Out]  $-((b^2 E^{-a-bx})/d^3) + (b^2(3bc-4ad)E^{-a-bx})/d^4 - (b^3 E^{-a-bx}x)/d^3 - ((b^2c-ad)^4 E^{-a-bx})/(2d^5(c+dx)^2) + (4b^2(b^2c-ad)^3 E^{-a-bx})/(d^5(c+dx)) + (b(b^2c-ad)^4 E^{-a-bx})/(2d^6(c+dx)) + (6b^2(b^2c-ad)^2 E^{-a-bx}/d) \text{ExpIntegralEi}[-((b(c+dx))/d)]/d^5 + (4b^2(b^2c-ad)^3 E^{-a-bx}/d) \text{ExpIntegralEi}[-((b(c+dx))/d)]/d^6 + (b^2(b^2c-ad)^4 E^{-a-bx}/d) \text{ExpIntegralEi}[-((b(c+dx))/d)]/(2d^7)$

**Rule 2207**

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(c + d\*x)^m\*((b\*F^(g\*(e + f\*x)))^n/(f\*g\*n\*Log[F])), x] - Dist[d\*(m/(f\*g\*n\*Log[F])), Int[(c + d\*x)^(m-1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

**Rule 2208**

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(c + d\*x)^(m+1)\*((b\*F^(g\*(e + f\*x)))^n/(d\*(m+1))), x] - Dist[f\*g\*n\*(Log[F]/(d\*(m+1))), Int[(c + d\*x)^(m+1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int

egerQ[2\*m] && !TrueQ[\$UseGamma]

### Rule 2209

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - c\*(f/d)))/d)\*ExpIntegralEi[f\*g\*(c + d\*x)\*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

### Rule 2225

Int[((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(n\_), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

### Rule 2230

Int[(F\_)^((c\_)\*(v\_))\*(u\_)^(m\_)\*(w\_), x\_Symbol] := Int[ExpandIntegrand[F^(c\*ExpandToSum[v, x]), w\*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && IntegerQ[m] && !TrueQ[\$UseGamma]

### Rubi steps

$$\begin{aligned}
 \int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^3} dx &= \int \left( -\frac{b^3(3bc-4ad)e^{-a-bx}}{d^4} + \frac{b^4e^{-a-bx}x}{d^3} + \frac{(-bc+ad)^4e^{-a-bx}}{d^4(c+dx)^3} - \frac{4b(bc-ad)^3e^{-a-bx}}{d^4(c+dx)^2} \right) dx \\
 &= \frac{b^4 \int e^{-a-bx} x dx}{d^3} - \frac{(b^3(3bc-4ad)) \int e^{-a-bx} dx}{d^4} + \frac{(6b^2(bc-ad)^2) \int \frac{e^{-a-bx}}{c+dx} dx}{d^4} - \frac{4b(bc-ad)^3 \int e^{-a-bx} dx}{d^4(c+dx)} \\
 &= \frac{b^2(3bc-4ad)e^{-a-bx}}{d^4} - \frac{b^3e^{-a-bx}x}{d^3} - \frac{(bc-ad)^4e^{-a-bx}}{2d^5(c+dx)^2} + \frac{4b(bc-ad)^3e^{-a-bx}}{d^5(c+dx)} + \frac{6b^2(bc-ad)^2 \int e^{-a-bx} dx}{d^5(c+dx)} \\
 &= -\frac{b^2e^{-a-bx}}{d^3} + \frac{b^2(3bc-4ad)e^{-a-bx}}{d^4} - \frac{b^3e^{-a-bx}x}{d^3} - \frac{(bc-ad)^4e^{-a-bx}}{2d^5(c+dx)^2} + \frac{4b(bc-ad)^3e^{-a-bx}}{d^5(c+dx)} + \frac{6b^2(bc-ad)^2 \int e^{-a-bx} dx}{d^5(c+dx)} \\
 &= -\frac{b^2e^{-a-bx}}{d^3} + \frac{b^2(3bc-4ad)e^{-a-bx}}{d^4} - \frac{b^3e^{-a-bx}x}{d^3} - \frac{(bc-ad)^4e^{-a-bx}}{2d^5(c+dx)^2} + \frac{4b(bc-ad)^3e^{-a-bx}}{d^5(c+dx)} + \frac{6b^2(bc-ad)^2 \int e^{-a-bx} dx}{d^5(c+dx)}
 \end{aligned}$$

### Mathematica [A]

time = 1.81, size = 267, normalized size = 0.91

$$\frac{e^{-a} \left( \frac{dx^{-bx}(-a^4d^2+b^4c^2(c+dx)+a^3bd^2((-4+a)c+(-8+a)dx)+b^4c^2d((7-4a)c-4(-2+a)dx)-2b^2d^2((1+4a-9a^2+2a^3)c^2+2(1+4a-6a^2+a^3)cdx+(1+4a)d^2x^2)+2b^2d^2((3-10a+3a^2)c^2+(5-12a+3a^2)c^2dx+cd^2x^2-d^2x^2)}{(c+dx)^3} \right) + b^2(bc-ad)^2(b^2c^2-2(-4+a)bcd+(12-8a+a^2)d^2)e^{\frac{5}{2}\text{Ei}\left(-\frac{5(c+dx)}{d}\right)}}{2d^7}$$

Antiderivative was successfully verified.

[In] Integrate[(E^(-a - b\*x)\*(a + b\*x)^4)/(c + d\*x)^3,x]

[Out] 
$$\frac{(d*(-a^4*d^5) + b^5*c^4*(c + d*x) + a^3*b*d^4*((-4 + a)*c + (-8 + a)*d*x) + b^4*c^3*d*((7 - 4*a)*c - 4*(-2 + a)*d*x) - 2*b^2*d^3*((1 + 4*a - 9*a^2 + 2*a^3)*c^2 + 2*(1 + 4*a - 6*a^2 + a^3)*c*d*x + (1 + 4*a)*d^2*x^2) + 2*b^3*d^2*((3 - 10*a + 3*a^2)*c^3 + (5 - 12*a + 3*a^2)*c^2*d*x + c*d^2*x^2 - d^3*x^3))/(E^(b*x)*(c + d*x)^2) + b^2*(b*c - a*d)^2*(b^2*c^2 - 2*(-4 + a)*b*c*d + (12 - 8*a + a^2)*d^2)*E^((b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)]/(2*d^7*E^a)$$

**Maple [A]**

time = 0.08, size = 418, normalized size = 1.42 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c)^3,x,method=\_RETURNVERBOSE)

[Out] 
$$\begin{aligned} & -1/b*(3*b^3/d^3*a*\exp(-b*x-a)-3*b^4/d^4*c*\exp(-b*x-a)-1/d^3*b^3*((-b*x-a)*\exp(-b*x-a)-\exp(-b*x-a))-(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*b^3/d^7*(-1/2*\exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/2*\exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-1/2*\exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))+6/d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)*b^3*\exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)+4/d^6*(a^3*d^3-3*a^2*b*c*d+3*a*b^2*c^2*d-b^3*c^3)*b^3*(-\exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-\exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)) \end{aligned}$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c)^3,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -a^4*e^{(-a + b*c/d)*\exp\_integral\_e(3, (d*x + c)*b/d)/((d*x + c)^2*d) - (b^3*d^2*x^4 + (4*a*b^2*d^2 + b^2*d^2)*x^3 + 3*(2*a^2*b*d^2 + b^2*c*d)*x^2 + (4*a^3*d^2 - 3*b^2*c^2 + 12*a*b*c*d - 6*a^2*d^2)*x)*e^{(-b*x)/(d^5*x^3*e^a + 3*c*d^4*x^2*e^a + 3*c^2*d^3*x*e^a + c^3*d^2*e^a) - \integrate(-(4*a^3*c*d^2 - 3*b^2*c^3 + 12*a*b*c^2*d - 6*a^2*c*d^2 + (3*b^3*c^3 - 8*a^3*d^3 + 12*b^2*c^2*d + 6*(3*b*c*d^2 + 2*d^3)*a^2 - 12*(b^2*c^2*d + 2*b*c*d^2)*a)*x)*e^{(-b*x)/(d^6*x^4*e^a + 4*c*d^5*x^3*e^a + 6*c^2*d^4*x^2*e^a + 4*c^3*d^3*x*e^a + c^4*d^2*e^a), x) \end{aligned}$$

**Fricas [A]**

time = 0.40, size = 550, normalized size = 1.87

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c)^3,x, algorithm="fricas")

[Out]  $\frac{1}{2} * ((b^6 * c^6 - 4 * (a - 2) * b^5 * c^5 * d + 6 * (a^2 - 4 * a + 2) * b^4 * c^4 * d^2 - 4 * (a^3 - 6 * a^2 + 6 * a) * b^3 * c^3 * d^3 + (a^4 - 8 * a^3 + 12 * a^2) * b^2 * c^2 * d^4 + (b^6 * c^4 * d^2 - 4 * (a - 2) * b^5 * c^3 * d^3 + 6 * (a^2 - 4 * a + 2) * b^4 * c^2 * d^4 - 4 * (a^3 - 6 * a^2 + 6 * a) * b^3 * c * d^5 + (a^4 - 8 * a^3 + 12 * a^2) * b^2 * d^6) * x^2 + 2 * (b^6 * c^5 * d - 4 * (a - 2) * b^5 * c^4 * d^2 + 6 * (a^2 - 4 * a + 2) * b^4 * c^3 * d^3 - 4 * (a^3 - 6 * a^2 + 6 * a) * b^3 * c^2 * d^4 + (a^4 - 8 * a^3 + 12 * a^2) * b^2 * c * d^5) * x) * Ei(-(b * d * x + b * c) / d) * e^{((b * c - a * d) / d)} - (2 * b^3 * d^6 * x^3 - b^5 * c^5 * d + (4 * a - 7) * b^4 * c^4 * d^2 - 2 * (3 * a^2 - 10 * a + 3) * b^3 * c^3 * d^3 + a^4 * d^6 + 2 * (2 * a^3 - 9 * a^2 + 4 * a + 1) * b^2 * c^2 * d^4 - (a^4 - 4 * a^3) * b * c * d^5 - 2 * (b^3 * c * d^5 - (4 * a + 1) * b^2 * d^6) * x^2 - (b^5 * c^4 * d^2 - 4 * (a - 2) * b^4 * c^3 * d^3 + 2 * (3 * a^2 - 12 * a + 5) * b^3 * c^2 * d^4 - 4 * (a^3 - 6 * a^2 + 4 * a + 1) * b^2 * c * d^5 + (a^4 - 8 * a^3) * b * d^6) * x) * e^{-b * x - a}) / (d^9 * x^2 + 2 * c * d^8 * x + c^2 * d^7)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\left( \int \frac{a^4}{c^4 e^{bx} + 3c^2 d x e^{bx} + 3cd^2 x^2 e^{bx} + d^3 x^3 e^{bx}} dx + \int \frac{b^4 x^4}{c^4 e^{bx} + 3c^2 d x e^{bx} + 3cd^2 x^2 e^{bx} + d^3 x^3 e^{bx}} dx + \int \frac{4ab^3 x^3}{c^4 e^{bx} + 3c^2 d x e^{bx} + 3cd^2 x^2 e^{bx} + d^3 x^3 e^{bx}} dx + \int \frac{6a^2 b^2 x^2}{c^4 e^{bx} + 3c^2 d x e^{bx} + 3cd^2 x^2 e^{bx} + d^3 x^3 e^{bx}} dx + \int \frac{4a^3 b x}{c^4 e^{bx} + 3c^2 d x e^{bx} + 3cd^2 x^2 e^{bx} + d^3 x^3 e^{bx}} dx \right) e^{-a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)\*\*4/(d\*x+c)\*\*3,x)

[Out] (Integral(a\*\*4/(c\*\*3\*exp(b\*x) + 3\*c\*\*2\*d\*x\*exp(b\*x) + 3\*c\*d\*\*2\*x\*\*2\*exp(b\*x) + d\*\*3\*x\*\*3\*exp(b\*x)), x) + Integral(b\*\*4\*x\*\*4/(c\*\*3\*exp(b\*x) + 3\*c\*\*2\*d\*x\*exp(b\*x) + 3\*c\*d\*\*2\*x\*\*2\*exp(b\*x) + d\*\*3\*x\*\*3\*exp(b\*x)), x) + Integral(4\*a\*b\*\*3\*x\*\*3/(c\*\*3\*exp(b\*x) + 3\*c\*\*2\*d\*x\*exp(b\*x) + 3\*c\*d\*\*2\*x\*\*2\*exp(b\*x) + d\*\*3\*x\*\*3\*exp(b\*x)), x) + Integral(6\*a\*\*2\*b\*\*2\*x\*\*2/(c\*\*3\*exp(b\*x) + 3\*c\*\*2\*d\*x\*exp(b\*x) + 3\*c\*d\*\*2\*x\*\*2\*exp(b\*x) + d\*\*3\*x\*\*3\*exp(b\*x)), x) + Integral(4\*a\*\*3\*b\*x/(c\*\*3\*exp(b\*x) + 3\*c\*\*2\*d\*x\*exp(b\*x) + 3\*c\*d\*\*2\*x\*\*2\*exp(b\*x) + d\*\*3\*x\*\*3\*exp(b\*x)), x))\*exp(-a)

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 1995 vs. 2(279) = 558.

time = 2.77, size = 1995, normalized size = 6.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c)^3,x, algorithm="giac")

[Out]  $\frac{1}{2} * (b^6 * c^4 * d^2 * x^2 * Ei(-(b * d * x + b * c) / d) * e^{-a + b * c / d} - 4 * a * b^5 * c^3 * d^3 * x^2 * Ei(-(b * d * x + b * c) / d) * e^{-a + b * c / d} + 6 * a^2 * b^4 * c^2 * d^4 * x^2 * Ei(-(b * d * x + b * c) / d) * e^{-a + b * c / d} - 4 * a^3 * b^3 * c * d^5 * x^2 * Ei(-(b * d * x + b * c) / d) * e^{-a + b * c / d} + a^4 * b^2 * d^6 * x^2 * Ei(-(b * d * x + b * c) / d) * e^{-a + b * c / d} + 2 * b^6 * c^5 * d * x * Ei(-(b * d * x + b * c) / d) * e^{-a + b * c / d} - 8 * a * b^5 * c^4 * d^2 * x * Ei(-(b * d * x + b * c) / d) * e^{-a + b * c / d} + 4 * a^2 * b^4 * c^3 * d^3 * x * Ei(-(b * d * x + b * c) / d) * e^{-a + b * c / d} - 4 * a^3 * b^3 * c^2 * d^4 * x * Ei(-(b * d * x + b * c) / d) * e^{-a + b * c / d} + 2 * a^4 * b^2 * c * d^5 * x * Ei(-(b * d * x + b * c) / d) * e^{-a + b * c / d} + 2 * b^6 * c^5 * d * Ei(-(b * d * x + b * c) / d) * e^{-a + b * c / d} - 4 * a * b^5 * c^4 * d^2 * Ei(-(b * d * x + b * c) / d) * e^{-a + b * c / d} + 6 * a^2 * b^4 * c^3 * d^3 * Ei(-(b * d * x + b * c) / d) * e^{-a + b * c / d} - 4 * a^3 * b^3 * c^2 * d^4 * Ei(-(b * d * x + b * c) / d) * e^{-a + b * c / d} + a^4 * b^2 * c * d^5 * Ei(-(b * d * x + b * c) / d) * e^{-a + b * c / d}) * e^{-a}$



$$\begin{aligned} & )/d) * e^{(-a + b*c/d)} + 12*a^2*b^4*c^3*d^3*x*Ei(-(b*d*x + b*c)/d) * e^{(-a + b*c/d)} \\ & - 8*a^3*b^3*c^2*d^4*x*Ei(-(b*d*x + b*c)/d) * e^{(-a + b*c/d)} + 2*a^4*b^2*c \\ & *d^5*x*Ei(-(b*d*x + b*c)/d) * e^{(-a + b*c/d)} + 8*b^5*c^3*d^3*x^2*Ei(-(b*d*x + \\ & b*c)/d) * e^{(-a + b*c/d)} - 24*a*b^4*c^2*d^4*x^2*Ei(-(b*d*x + b*c)/d) * e^{(-a + \\ & b*c/d)} + 24*a^2*b^3*c*d^5*x^2*Ei(-(b*d*x + b*c)/d) * e^{(-a + b*c/d)} - 8*a^3* \\ & b^2*d^6*x^2*Ei(-(b*d*x + b*c)/d) * e^{(-a + b*c/d)} + b^6*c^6*Ei(-(b*d*x + b*c) \\ & /d) * e^{(-a + b*c/d)} - 4*a*b^5*c^5*d*Ei(-(b*d*x + b*c)/d) * e^{(-a + b*c/d)} + 6* \\ & a^2*b^4*c^4*d^2*Ei(-(b*d*x + b*c)/d) * e^{(-a + b*c/d)} - 4*a^3*b^3*c^3*d^3*Ei( \\ & -(b*d*x + b*c)/d) * e^{(-a + b*c/d)} + a^4*b^2*c^2*d^4*Ei(-(b*d*x + b*c)/d) * e^{( \\ & -a + b*c/d)} + 16*b^5*c^4*d^2*x*Ei(-(b*d*x + b*c)/d) * e^{(-a + b*c/d)} - 48*a*b \\ & ^4*c^3*d^3*x*Ei(-(b*d*x + b*c)/d) * e^{(-a + b*c/d)} + 48*a^2*b^3*c^2*d^4*x*Ei( \\ & -(b*d*x + b*c)/d) * e^{(-a + b*c/d)} - 16*a^3*b^2*c*d^5*x*Ei(-(b*d*x + b*c)/d) * \\ & e^{(-a + b*c/d)} + 12*b^4*c^2*d^4*x^2*Ei(-(b*d*x + b*c)/d) * e^{(-a + b*c/d)} - 2 \\ & 4*a*b^3*c*d^5*x^2*Ei(-(b*d*x + b*c)/d) * e^{(-a + b*c/d)} + 12*a^2*b^2*d^6*x^2* \\ & Ei(-(b*d*x + b*c)/d) * e^{(-a + b*c/d)} + b^5*c^4*d^2*x*e^{(-b*x - a)} - 4*a*b^4*c \\ & ^3*d^3*x*e^{(-b*x - a)} + 6*a^2*b^3*c^2*d^4*x*e^{(-b*x - a)} - 4*a^3*b^2*c*d^5 \\ & *x*e^{(-b*x - a)} + a^4*b*d^6*x*e^{(-b*x - a)} - 2*b^3*d^6*x^3*e^{(-b*x - a)} + 8 \\ & *b^5*c^5*d*Ei(-(b*d*x + b*c)/d) * e^{(-a + b*c/d)} - 24*a*b^4*c^4*d^2*Ei(-(b*d* \\ & x + b*c)/d) * e^{(-a + b*c/d)} + 24*a^2*b^3*c^3*d^3*Ei(-(b*d*x + b*c)/d) * e^{(-a \\ & + b*c/d)} - 8*a^3*b^2*c^2*d^4*Ei(-(b*d*x + b*c)/d) * e^{(-a + b*c/d)} + 24*b^4*c \\ & ^3*d^3*x*Ei(-(b*d*x + b*c)/d) * e^{(-a + b*c/d)} - 48*a*b^3*c^2*d^4*x*Ei(-(b*d* \\ & x + b*c)/d) * e^{(-a + b*c/d)} + 24*a^2*b^2*c*d^5*x*Ei(-(b*d*x + b*c)/d) * e^{(-a \\ & + b*c/d)} + b^5*c^5*d*e^{(-b*x - a)} - 4*a*b^4*c^4*d^2*e^{(-b*x - a)} + 6*a^2*b^ \\ & 3*c^3*d^3*e^{(-b*x - a)} - 4*a^3*b^2*c^2*d^4*e^{(-b*x - a)} + a^4*b*c*d^5*e^{(-b \\ & *x - a)} + 8*b^4*c^3*d^3*x*e^{(-b*x - a)} - 24*a*b^3*c^2*d^4*x*e^{(-b*x - a)} + \\ & 24*a^2*b^2*c*d^5*x*e^{(-b*x - a)} - 8*a^3*b*d^6*x*e^{(-b*x - a)} + 2*b^3*c*d^5* \\ & x^2*e^{(-b*x - a)} - 8*a*b^2*d^6*x^2*e^{(-b*x - a)} + 12*b^4*c^4*d^2*Ei(-(b*d*x \\ & + b*c)/d) * e^{(-a + b*c/d)} - 24*a*b^3*c^3*d^3*Ei(-(b*d*x + b*c)/d) * e^{(-a + b \\ & *c/d)} + 12*a^2*b^2*c^2*d^4*Ei(-(b*d*x + b*c)/d) * e^{(-a + b*c/d)} + 7*b^4*c^4* \\ & d^2*e^{(-b*x - a)} - 20*a*b^3*c^3*d^3*e^{(-b*x - a)} + 18*a^2*b^2*c^2*d^4*e^{(-b \\ & *x - a)} - 4*a^3*b*c*d^5*e^{(-b*x - a)} - a^4*d^6*e^{(-b*x - a)} + 10*b^3*c^2*d^ \\ & 4*x*e^{(-b*x - a)} - 16*a*b^2*c*d^5*x*e^{(-b*x - a)} - 2*b^2*d^6*x^2*e^{(-b*x - \\ & a)} + 6*b^3*c^3*d^3*e^{(-b*x - a)} - 8*a*b^2*c^2*d^4*e^{(-b*x - a)} - 4*b^2*c*d^ \\ & 5*x*e^{(-b*x - a)} - 2*b^2*c^2*d^4*e^{(-b*x - a)})/(d^9*x^2 + 2*c*d^8*x + c^2*d \\ & ^7) \end{aligned}$$

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{-a-bx} (a+bx)^4}{(c+dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(- a - b\*x)\*(a + b\*x)^4)/(c + d\*x)^3,x)

[Out] int((exp(- a - b\*x)\*(a + b\*x)^4)/(c + d\*x)^3, x)

$$3.81 \quad \int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^4} dx$$

**Optimal.** Leaf size=396

$$-\frac{b^3 e^{-a-bx}}{d^4} - \frac{(bc-ad)^4 e^{-a-bx}}{3d^5(c+dx)^3} + \frac{2b(bc-ad)^3 e^{-a-bx}}{d^5(c+dx)^2} + \frac{b(bc-ad)^4 e^{-a-bx}}{6d^6(c+dx)^2} - \frac{6b^2(bc-ad)^2 e^{-a-bx}}{d^5(c+dx)} - \frac{2b^2(bc-ad)^3}{d^6(c+dx)}$$

[Out]  $-b^3 \exp(-bx-a)/d^4 - 1/3 * (-a+bx)^4 \exp(-bx-a)/d^5 / (dx+c)^3 + 2b * (-a+bx)^3 \exp(-bx-a)/d^5 / (dx+c)^2 + 1/6 * b * (-a+bx)^4 \exp(-bx-a)/d^6 / (dx+c)^2 - 6 * b^2 * (-a+bx)^2 \exp(-bx-a)/d^5 / (dx+c) - 2 * b^2 * (-a+bx)^3 \exp(-bx-a)/d^6 / (dx+c) - 1/6 * b^2 * (-a+bx)^4 \exp(-bx-a)/d^7 / (dx+c) - 4 * b^3 * (-a+bx) * \exp(-a+bc/d) * \text{Ei}(-b * (dx+c)/d) / d^5 - 6 * b^3 * (-a+bx)^2 \exp(-a+bc/d) * \text{Ei}(-b * (dx+c)/d) / d^6 - 2 * b^3 * (-a+bx)^3 \exp(-a+bc/d) * \text{Ei}(-b * (dx+c)/d) / d^7 - 1/6 * b^3 * (-a+bx)^4 \exp(-a+bc/d) * \text{Ei}(-b * (dx+c)/d) / d^8$

**Rubi [A]**

time = 0.36, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {2230, 2225, 2208, 2209}

$$\frac{b^3 e^{-a-bx} \text{Ei}\left(-\frac{b(c+dx)}{d}\right)}{6d^6} - \frac{2b^2 e^{-a-bx} (bc-ad)^2 \text{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^6} - \frac{6b^2 e^{-a-bx} (bc-ad)^2 \text{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^6} - \frac{4b^2 e^{-a-bx} (bc-ad) \text{Ei}\left(-\frac{b(c+dx)}{d}\right)}{d^6} - \frac{b^2 e^{-a-bx}}{d^6} - \frac{b^2 e^{-a-bx} (bc-ad)^4}{6d^6(c+dx)} - \frac{2b^2 e^{-a-bx} (bc-ad)^3}{d^6(c+dx)} - \frac{6b^2 e^{-a-bx} (bc-ad)^2}{d^6(c+dx)} - \frac{bc^{-a-bx} (bc-ad)^4}{6d^6(c+dx)^2} - \frac{e^{-a-bx} (bc-ad)^4}{3d^6(c+dx)^3} + \frac{2bc^{-a-bx} (bc-ad)^3}{d^6(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Int[(E^(-a - b\*x)\*(a + b\*x)^4)/(c + d\*x)^4, x]

[Out]  $-\left(\frac{b^3 E^{-a-bx}}{d^4}\right) - \left(\frac{(bc-a*d)^4 E^{-a-bx}}{3*d^5*(c+d*x)^3}\right) + \left(\frac{2*b*(b*c-a*d)^3 E^{-a-bx}}{d^5*(c+d*x)^2}\right) + \left(\frac{b*(b*c-a*d)^4 E^{-a-bx}}{(6*d^6*(c+d*x)^2)} - \frac{(6*b^2*(b*c-a*d)^2 E^{-a-bx}}{d^5*(c+d*x)}\right) - \left(\frac{2*b^2*(b*c-a*d)^3 E^{-a-bx}}{d^6*(c+d*x)} - \frac{b^2*(b*c-a*d)^4 E^{-a-bx}}{(6*d^7*(c+d*x))}\right) - \left(\frac{4*b^3*(b*c-a*d)*E^{-a+(b*c)/d}}{d}\right) * \text{ExpIntegralEi}\left[-\left(\frac{b*(c+d*x)}{d}\right)\right] / d^5 - \left(\frac{6*b^3*(b*c-a*d)^2 E^{-a+(b*c)/d}}{d}\right) * \text{ExpIntegralEi}\left[-\left(\frac{b*(c+d*x)}{d}\right)\right] / d^6 - \left(\frac{2*b^3*(b*c-a*d)^3 E^{-a+(b*c)/d}}{d}\right) * \text{ExpIntegralEi}\left[-\left(\frac{b*(c+d*x)}{d}\right)\right] / d^7 - \left(\frac{b^3*(b*c-a*d)^4 E^{-a+(b*c)/d}}{(6*d^8)}\right) * \text{ExpIntegralEi}\left[-\left(\frac{b*(c+d*x)}{d}\right)\right] / (6*d^8)$

**Rule 2208**

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.)+(f\_.)\*(x\_))))^(n\_.)\*((c\_.)+(d\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(c + d\*x)^(m + 1)\*((b\*F^(g\*(e + f\*x)))^n/(d\*(m + 1))), x] - Dist[f\*g\*n\*(Log[F]/(d\*(m + 1))), Int[(c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

**Rule 2209**

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))/((c_) + (d_)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

### Rule 2225

```
Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] := Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

### Rule 2230

```
Int[(F_)^((c_)*(v_))*(u_)^(m_)*(w_), x_Symbol] := Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F,
c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] && !TrueQ[$UseGamma]
```

### Rubi steps

$$\begin{aligned}
\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^4} dx &= \int \left( \frac{b^4 e^{-a-bx}}{d^4} + \frac{(-bc+ad)^4 e^{-a-bx}}{d^4(c+dx)^4} - \frac{4b(bc-ad)^3 e^{-a-bx}}{d^4(c+dx)^3} + \frac{6b^2(bc-ad)^2 e^{-a-bx}}{d^4(c+dx)^2} - \frac{4b^3(bc-ad) e^{-a-bx}}{d^4(c+dx)} + \frac{b^4 e^{-a-bx}}{d^4} \right) dx \\
&= \frac{b^4 \int e^{-a-bx} dx}{d^4} - \frac{(4b^3(bc-ad)) \int \frac{e^{-a-bx}}{c+dx} dx}{d^4} + \frac{(6b^2(bc-ad)^2) \int \frac{e^{-a-bx}}{(c+dx)^2} dx}{d^4} - \frac{(4b^3(bc-ad)) \int \frac{e^{-a-bx}}{c+dx} dx}{d^4} + \frac{b^4 \int e^{-a-bx} dx}{d^4} \\
&= -\frac{b^3 e^{-a-bx}}{d^4} - \frac{(bc-ad)^4 e^{-a-bx}}{3d^5(c+dx)^3} + \frac{2b(bc-ad)^3 e^{-a-bx}}{d^5(c+dx)^2} - \frac{6b^2(bc-ad)^2 e^{-a-bx}}{d^5(c+dx)} - \frac{4b^3(bc-ad) e^{-a-bx}}{d^5(c+dx)} + \frac{b^4 e^{-a-bx}}{d^4} \\
&= -\frac{b^3 e^{-a-bx}}{d^4} - \frac{(bc-ad)^4 e^{-a-bx}}{3d^5(c+dx)^3} + \frac{2b(bc-ad)^3 e^{-a-bx}}{d^5(c+dx)^2} + \frac{b(bc-ad)^4 e^{-a-bx}}{6d^6(c+dx)^2} - \frac{6b^2(bc-ad)^2 e^{-a-bx}}{6d^6(c+dx)^2} - \frac{4b^3(bc-ad) e^{-a-bx}}{6d^6(c+dx)^2} + \frac{b^4 e^{-a-bx}}{d^4} \\
&= -\frac{b^3 e^{-a-bx}}{d^4} - \frac{(bc-ad)^4 e^{-a-bx}}{3d^5(c+dx)^3} + \frac{2b(bc-ad)^3 e^{-a-bx}}{d^5(c+dx)^2} + \frac{b(bc-ad)^4 e^{-a-bx}}{6d^6(c+dx)^2} - \frac{6b^2(bc-ad)^2 e^{-a-bx}}{6d^6(c+dx)^2} - \frac{4b^3(bc-ad) e^{-a-bx}}{6d^6(c+dx)^2} + \frac{b^4 e^{-a-bx}}{d^4} \\
&= -\frac{b^3 e^{-a-bx}}{d^4} - \frac{(bc-ad)^4 e^{-a-bx}}{3d^5(c+dx)^3} + \frac{2b(bc-ad)^3 e^{-a-bx}}{d^5(c+dx)^2} + \frac{b(bc-ad)^4 e^{-a-bx}}{6d^6(c+dx)^2} - \frac{6b^2(bc-ad)^2 e^{-a-bx}}{6d^6(c+dx)^2} - \frac{4b^3(bc-ad) e^{-a-bx}}{6d^6(c+dx)^2} + \frac{b^4 e^{-a-bx}}{d^4}
\end{aligned}$$

### Mathematica [A]

time = 2.23, size = 389, normalized size = 0.98

$$\frac{b^4 e^{-a-bx} \operatorname{Ei}\left(-\frac{b(c+dx)}{d}\right) - (bc-ad)^4 e^{-a-bx} \operatorname{Ei}\left(-\frac{c+dx}{d}\right) + 2b(bc-ad)^3 e^{-a-bx} \operatorname{Ei}\left(-\frac{2(c+dx)}{d}\right) - 6b^2(bc-ad)^2 e^{-a-bx} \operatorname{Ei}\left(-\frac{3(c+dx)}{d}\right) - 4b^3(bc-ad) e^{-a-bx} \operatorname{Ei}\left(-\frac{4(c+dx)}{d}\right) + b^4 e^{-a-bx} \operatorname{Ei}\left(-\frac{5(c+dx)}{d}\right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(-a - b*x)*(a + b*x)^4)/(c + d*x)^4, x]
```

```
[Out] (-(d*(2*a^4*d^6 + b^6*c^4*(c + d*x)^2 - a^3*b*d^5*((-4 + a)*c + (-12 + a)*d*x) - b^5*c^3*d*(c + d*x)*((-11 + 4*a)*c + 4*(-3 + a)*d*x) + a^2*b^2*d^4*(12 - 8*a + a^2)*c^2 + 2*(18 - 10*a + a^2)*c*d*x + (-6 + a)^2*d^2*x^2) + 2*b^4*c^2*d^2*((13 - 16*a + 3*a^2)*c^2 + 2*(15 - 17*a + 3*a^2)*c*d*x + 3*(6 - 6*a + a^2)*d^2*x^2) + 2*b^3*d^3*((3 - 22*a + 15*a^2 - 2*a^3)*c^3 + (9 - 54*a + 33*a^2 - 4*a^3)*c^2*d*x + (9 - 36*a + 18*a^2 - 2*a^3)*c*d^2*x^2 + 3*d^3*x^3)))/(E^(b*x)*(c + d*x)^3) - b^3*(b^4*c^4 - 4*(-3 + a)*b^3*c^3*d + 6*(6 - 6*a + a^2)*b^2*c^2*d^2 - 4*(-6 + 18*a - 9*a^2 + a^3)*b*c*d^3 + a*(-24 + 36*a - 12*a^2 + a^3)*d^4)*E^((b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)]/(6*d^8*E^a)
```

**Maple [A]**

time = 0.08, size = 511, normalized size = 1.29

method	result
derivativedivides	$-\frac{b^4 e^{-bx-a}}{d^4} + \frac{(a^4 d^4 - 4a^3 bc d^3 + 6a^2 b^2 c^2 d^2 - 4a b^3 c^3 d + b^4 c^4) b^4}{d^8} \left( -\frac{e^{-bx-a}}{3(-bx-a + \frac{ad-cb}{d})^3} - \frac{e^{-bx-a}}{6(-bx-a + \frac{ad-cb}{d})^2} - \frac{e^{-bx-a}}{6(-bx-a + \frac{ad-cb}{d})} \right)$
default	$-\frac{b^4 e^{-bx-a}}{d^4} + \frac{(a^4 d^4 - 4a^3 bc d^3 + 6a^2 b^2 c^2 d^2 - 4a b^3 c^3 d + b^4 c^4) b^4}{d^8} \left( -\frac{e^{-bx-a}}{3(-bx-a + \frac{ad-cb}{d})^3} - \frac{e^{-bx-a}}{6(-bx-a + \frac{ad-cb}{d})^2} - \frac{e^{-bx-a}}{6(-bx-a + \frac{ad-cb}{d})} \right)$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(b^4/d^4*exp(-b*x-a)+(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*b^4/d^8*(-1/3*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^3-1/6*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/6*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-1/6*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))-4/d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*b^4*(-1/2*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/2*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-1/2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))+4/d^5*(a*d-b*c)*b^4*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)+6/d^6*(a^2*d^2-2*a*b*c*d+b^2*c^2)*b^4*(-exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -a^4*e^(-a + b*c/d)*exp_integral_e(4, (d*x + c)*b/d)/((d*x + c)^3*d) - (b^3
*d^2*x^4 + 4*a*b^2*d^2*x^3 + 2*(3*a^2*b*d^2 + 2*b^2*c*d - 2*a*b*d^2)*x^2 +
4*(a^3*d^2 - b^2*c^2 - 3*a^2*d^2 - 2*b*c*d + 2*(2*b*c*d + d^2)*a)*x)*e^(-b*
x)/(d^6*x^4*e^a + 4*c*d^5*x^3*e^a + 6*c^2*d^4*x^2*e^a + 4*c^3*d^3*x*e^a + c
^4*d^2*e^a) - integrate(-4*(a^3*c*d^2 - b^2*c^3 - 3*a^2*c*d^2 - 2*b*c^2*d +
2*(2*b*c^2*d + c*d^2)*a + (b^3*c^3 - 3*a^3*d^3 + 7*b^2*c^2*d + 6*b*c*d^2 +
3*(2*b*c*d^2 + 3*d^3)*a^2 - 2*(2*b^2*c^2*d + 8*b*c*d^2 + 3*d^3)*a)*x)*e^(-
b*x)/(d^7*x^5*e^a + 5*c*d^6*x^4*e^a + 10*c^2*d^5*x^3*e^a + 10*c^3*d^4*x^2*e
^a + 5*c^4*d^3*x*e^a + c^5*d^2*e^a), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 793 vs. 2(377) = 754.  
time = 0.39, size = 793, normalized size = 2.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/6*((b^7*c^7 - 4*(a - 3)*b^6*c^6*d + 6*(a^2 - 6*a + 6)*b^5*c^5*d^2 - 4*(a
^3 - 9*a^2 + 18*a - 6)*b^4*c^4*d^3 + (a^4 - 12*a^3 + 36*a^2 - 24*a)*b^3*c^3
*d^4 + (b^7*c^4*d^3 - 4*(a - 3)*b^6*c^3*d^4 + 6*(a^2 - 6*a + 6)*b^5*c^2*d^5
- 4*(a^3 - 9*a^2 + 18*a - 6)*b^4*c*d^6 + (a^4 - 12*a^3 + 36*a^2 - 24*a)*b
^3*d^7)*x^3 + 3*(b^7*c^5*d^2 - 4*(a - 3)*b^6*c^4*d^3 + 6*(a^2 - 6*a + 6)*b^5
*c^3*d^4 - 4*(a^3 - 9*a^2 + 18*a - 6)*b^4*c^2*d^5 + (a^4 - 12*a^3 + 36*a^2
- 24*a)*b^3*c*d^6)*x^2 + 3*(b^7*c^6*d - 4*(a - 3)*b^6*c^5*d^2 + 6*(a^2 - 6*
a + 6)*b^5*c^4*d^3 - 4*(a^3 - 9*a^2 + 18*a - 6)*b^4*c^3*d^4 + (a^4 - 12*a^3
+ 36*a^2 - 24*a)*b^3*c^2*d^5)*x)*Ei(-(b*d*x + b*c)/d)*e^((b*c - a*d)/d) +
(b^6*c^6*d - (4*a - 11)*b^5*c^5*d^2 + 6*b^3*d^7*x^3 + 2*(3*a^2 - 16*a + 13)
*b^4*c^4*d^3 - 2*(2*a^3 - 15*a^2 + 22*a - 3)*b^3*c^3*d^4 + 2*a^4*d^7 + (a^4
- 8*a^3 + 12*a^2)*b^2*c^2*d^5 - (a^4 - 4*a^3)*b*c*d^6 + (b^6*c^4*d^3 - 4*(
a - 3)*b^5*c^3*d^4 + 6*(a^2 - 6*a + 6)*b^4*c^2*d^5 - 2*(2*a^3 - 18*a^2 + 36
*a - 9)*b^3*c*d^6 + (a^4 - 12*a^3 + 36*a^2)*b^2*d^7)*x^2 + (2*b^6*c^5*d^2 -
(8*a - 23)*b^5*c^4*d^3 + 4*(3*a^2 - 17*a + 15)*b^4*c^3*d^4 - 2*(4*a^3 - 33
*a^2 + 54*a - 9)*b^3*c^2*d^5 + 2*(a^4 - 10*a^3 + 18*a^2)*b^2*c*d^6 - (a^4 -
12*a^3)*b*d^7)*x)*e^(-b*x - a))/(d^11*x^3 + 3*c*d^10*x^2 + 3*c^2*d^9*x + c
^3*d^8)
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\left( \int \frac{a^4}{c^4 d^4 + 4c^3 d^3 e^{bx} + 6c^2 d^2 e^{2bx} + 4cd e^{3bx} + d^4 e^{4bx}} dx + \int \frac{b^4 x^4}{c^4 d^4 + 4c^3 d^3 e^{bx} + 6c^2 d^2 e^{2bx} + 4cd e^{3bx} + d^4 e^{4bx}} dx + \int \frac{4ab^3 x^3}{c^4 d^4 + 4c^3 d^3 e^{bx} + 6c^2 d^2 e^{2bx} + 4cd e^{3bx} + d^4 e^{4bx}} dx + \int \frac{6a^2 b^2 x^2}{c^4 d^4 + 4c^3 d^3 e^{bx} + 6c^2 d^2 e^{2bx} + 4cd e^{3bx} + d^4 e^{4bx}} dx + \int \frac{4a^3 b x}{c^4 d^4 + 4c^3 d^3 e^{bx} + 6c^2 d^2 e^{2bx} + 4cd e^{3bx} + d^4 e^{4bx}} dx \right) e^{-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b*x-a)*(b*x+a)**4/(d*x+c)**4,x)
```

```
[Out] (Integral(a**4/(c**4*exp(b*x) + 4*c**3*d*x*exp(b*x) + 6*c**2*d**2*x**2*exp(b*x) + 4*c*d**3*x**3*exp(b*x) + d**4*x**4*exp(b*x)), x) + Integral(b**4*x**4/(c**4*exp(b*x) + 4*c**3*d*x*exp(b*x) + 6*c**2*d**2*x**2*exp(b*x) + 4*c*d**3*x**3*exp(b*x) + d**4*x**4*exp(b*x)), x) + Integral(4*a*b**3*x**3/(c**4*exp(b*x) + 4*c**3*d*x*exp(b*x) + 6*c**2*d**2*x**2*exp(b*x) + 4*c*d**3*x**3*exp(b*x) + d**4*x**4*exp(b*x)), x) + Integral(6*a**2*b**2*x**2/(c**4*exp(b*x) + 4*c**3*d*x*exp(b*x) + 6*c**2*d**2*x**2*exp(b*x) + 4*c*d**3*x**3*exp(b*x) + d**4*x**4*exp(b*x)), x) + Integral(4*a**3*b*x/(c**4*exp(b*x) + 4*c**3*d*x*exp(b*x) + 6*c**2*d**2*x**2*exp(b*x) + 4*c*d**3*x**3*exp(b*x) + d**4*x**4*exp(b*x)), x))*exp(-a)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 3178 vs. 2(377) = 754.

time = 1.77, size = 3178, normalized size = 8.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/6*(b^7*c^4*d^3*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 4*a*b^6*c^3*d^4*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 6*a^2*b^5*c^2*d^5*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 4*a^3*b^4*c*d^6*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + a^4*b^3*d^7*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 3*b^7*c^5*d^2*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 12*a*b^6*c^4*d^3*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 18*a^2*b^5*c^3*d^4*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 12*a^3*b^4*c^2*d^5*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 3*a^4*b^3*c*d^6*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 12*b^6*c^3*d^4*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 36*a*b^5*c^2*d^5*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 36*a^2*b^4*c*d^6*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 12*a^3*b^3*d^7*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 3*b^7*c^6*d*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 12*a*b^6*c^5*d^2*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 18*a^2*b^5*c^4*d^3*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 12*a^3*b^4*c^3*d^4*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 3*a^4*b^3*c^2*d^5*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 36*b^6*c^4*d^3*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 108*a*b^5*c^3*d^4*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 108*a^2*b^4*c^2*d^5*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 36*a^3*b^3*c*d^6*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 36*b^5*c^2*d^5*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 72*a*b^4*c*d^6*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 36*a^2*b^3*d^7*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + b^6*c^4*d^3*x^2*e^(-b*x - a) - 4*a*b^5*c^3*d^4*x^2*e^(-b*x - a) + 6*a^2*b^4*c^2*d^5*x^2*e^(-b*x - a) - 4*a^3*b^3*c*d^6*x^2*e^(-b*x - a) + a^4*b^2*d^7*x^2*e^(-b*x - a) + b^7*c^7*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 4*a*b^6*c^6*d*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 6*a^2*b^5*c^5*d^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 4*a^3*b^4*c^4*d^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d)
```

$$\begin{aligned}
& (-a + b*c/d) + a^4*b^3*c^3*d^4*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 36*b^6 \\
& *c^5*d^2*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 108*a*b^5*c^4*d^3*x*Ei(-(b \\
& *d*x + b*c)/d)*e^{(-a + b*c/d)} + 108*a^2*b^4*c^3*d^4*x*Ei(-(b*d*x + b*c)/d)* \\
& e^{(-a + b*c/d)} - 36*a^3*b^3*c^2*d^5*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + \\
& 108*b^5*c^3*d^4*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 216*a*b^4*c^2*d^ \\
& 5*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 108*a^2*b^3*c*d^6*x^2*Ei(-(b*d* \\
& x + b*c)/d)*e^{(-a + b*c/d)} + 24*b^4*c*d^6*x^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + \\
& b*c/d)} - 24*a*b^3*d^7*x^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 2*b^6*c^5*d \\
& ^2*x*e^{(-b*x - a)} - 8*a*b^5*c^4*d^3*x*e^{(-b*x - a)} + 12*a^2*b^4*c^3*d^4*x*e \\
& ^(-b*x - a) - 8*a^3*b^3*c^2*d^5*x*e^{(-b*x - a)} + 2*a^4*b^2*c*d^6*x*e^{(-b*x \\
& - a)} + 12*b^5*c^3*d^4*x^2*e^{(-b*x - a)} - 36*a*b^4*c^2*d^5*x^2*e^{(-b*x - a)} \\
& + 36*a^2*b^3*c*d^6*x^2*e^{(-b*x - a)} - 12*a^3*b^2*d^7*x^2*e^{(-b*x - a)} + 12* \\
& b^6*c^6*d*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 36*a*b^5*c^5*d^2*Ei(-(b*d*x \\
& + b*c)/d)*e^{(-a + b*c/d)} + 36*a^2*b^4*c^4*d^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + \\
& b*c/d)} - 12*a^3*b^3*c^3*d^4*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 108*b^5* \\
& c^4*d^3*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 216*a*b^4*c^3*d^4*x*Ei(-(b* \\
& d*x + b*c)/d)*e^{(-a + b*c/d)} + 108*a^2*b^3*c^2*d^5*x*Ei(-(b*d*x + b*c)/d)*e \\
& ^(-a + b*c/d) + 72*b^4*c^2*d^5*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 72 \\
& *a*b^3*c*d^6*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + b^6*c^6*d*e^{(-b*x - \\
& a)} - 4*a*b^5*c^5*d^2*e^{(-b*x - a)} + 6*a^2*b^4*c^4*d^3*e^{(-b*x - a)} - 4*a^3* \\
& b^3*c^3*d^4*e^{(-b*x - a)} + a^4*b^2*c^2*d^5*e^{(-b*x - a)} + 23*b^5*c^4*d^3*x* \\
& e^{(-b*x - a)} - 68*a*b^4*c^3*d^4*x*e^{(-b*x - a)} + 66*a^2*b^3*c^2*d^5*x*e^{(-b \\
& *x - a)} - 20*a^3*b^2*c*d^6*x*e^{(-b*x - a)} - a^4*b*d^7*x*e^{(-b*x - a)} + 36*b \\
& ^4*c^2*d^5*x^2*e^{(-b*x - a)} - 72*a*b^3*c*d^6*x^2*e^{(-b*x - a)} + 36*a^2*b^2* \\
& d^7*x^2*e^{(-b*x - a)} + 6*b^3*d^7*x^3*e^{(-b*x - a)} + 36*b^5*c^5*d^2*Ei(-(b*d \\
& *x + b*c)/d)*e^{(-a + b*c/d)} - 72*a*b^4*c^4*d^3*Ei(-(b*d*x + b*c)/d)*e^{(-a + \\
& b*c/d)} + 36*a^2*b^3*c^3*d^4*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 72*b^4*c \\
& ^3*d^4*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - 72*a*b^3*c^2*d^5*x*Ei(-(b*d* \\
& x + b*c)/d)*e^{(-a + b*c/d)} + 11*b^5*c^5*d^2*e^{(-b*x - a)} - 32*a*b^4*c^4*d^3 \\
& *e^{(-b*x - a)} + 30*a^2*b^3*c^3*d^4*e^{(-b*x - a)} - 8*a^3*b^2*c^2*d^5*e^{(-b*x \\
& - a)} - a^4*b*c*d^6*e^{(-b*x - a)} + 60*b^4*c^3*d^4*x*e^{(-b*x - a)} - 108*a*b^ \\
& 3*c^2*d^5*x*e^{(-b*x - a)} + 36*a^2*b^2*c*d^6*x*e^{(-b*x - a)} + 12*a^3*b*d^7*x \\
& *e^{(-b*x - a)} + 18*b^3*c*d^6*x^2*e^{(-b*x - a)} + 24*b^4*c^4*d^3*Ei(-(b*d*x + \\
& b*c)/d)*e^{(-a + b*c/d)} - 24*a*b^3*c^3*d^4*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c \\
& /d)} + 26*b^4*c^4*d^3*e^{(-b*x - a)} - 44*a*b^3*c^3*d^4*e^{(-b*x - a)} + 12*a^2* \\
& b^2*c^2*d^5*e^{(-b*x - a)} + 4*a^3*b*c*d^6*e^{(-b*x - a)} + 2*a^4*d^7*e^{(-b*x - \\
& a)} + 18*b^3*c^2*d^5*x*e^{(-b*x - a)} + 6*b^3*c^3*d^4*e^{(-b*x - a)})/(d^11*x^3 \\
& + 3*c*d^10*x^2 + 3*c^2*d^9*x + c^3*d^8)
\end{aligned}$$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{-a-bx} (a+bx)^4}{(c+dx)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(- a - b*x)*(a + b*x)^4)/(c + d*x)^4,x)
```

```
[Out] int((exp(- a - b*x)*(a + b*x)^4)/(c + d*x)^4, x)
```



$$3.82 \quad \int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^5} dx$$

**Optimal.** Leaf size=557

$$-\frac{(bc-ad)^4 e^{-a-bx}}{4d^5(c+dx)^4} + \frac{4b(bc-ad)^3 e^{-a-bx}}{3d^5(c+dx)^3} + \frac{b(bc-ad)^4 e^{-a-bx}}{12d^6(c+dx)^3} - \frac{3b^2(bc-ad)^2 e^{-a-bx}}{d^5(c+dx)^2} - \frac{2b^2(bc-ad)^3 e^{-a-bx}}{3d^6(c+dx)^2} - \frac{b^3 e^{-a-bx}}{d^5(c+dx)}$$

[Out]  $-1/4*(-a*d+b*c)^4*\exp(-b*x-a)/d^5/(d*x+c)^4+4/3*b*(-a*d+b*c)^3*\exp(-b*x-a)/d^5/(d*x+c)^3+1/12*b*(-a*d+b*c)^4*\exp(-b*x-a)/d^6/(d*x+c)^3-3*b^2*(-a*d+b*c)^2*\exp(-b*x-a)/d^5/(d*x+c)^2-2/3*b^2*(-a*d+b*c)^3*\exp(-b*x-a)/d^6/(d*x+c)^2-1/24*b^2*(-a*d+b*c)^4*\exp(-b*x-a)/d^7/(d*x+c)^2+4*b^3*(-a*d+b*c)*\exp(-b*x-a)/d^5/(d*x+c)+3*b^3*(-a*d+b*c)^2*\exp(-b*x-a)/d^6/(d*x+c)+2/3*b^3*(-a*d+b*c)^3*\exp(-b*x-a)/d^7/(d*x+c)+1/24*b^3*(-a*d+b*c)^4*\exp(-b*x-a)/d^8/(d*x+c)+b^4*\exp(-a+b*c/d)*\text{Ei}(-b*(d*x+c)/d)/d^5+4*b^4*(-a*d+b*c)*\exp(-a+b*c/d)*\text{Ei}(-b*(d*x+c)/d)/d^6+3*b^4*(-a*d+b*c)^2*\exp(-a+b*c/d)*\text{Ei}(-b*(d*x+c)/d)/d^7+2/3*b^4*(-a*d+b*c)^3*\exp(-a+b*c/d)*\text{Ei}(-b*(d*x+c)/d)/d^8+1/24*b^4*(-a*d+b*c)^4*\exp(-a+b*c/d)*\text{Ei}(-b*(d*x+c)/d)/d^9$

**Rubi [A]**

time = 0.48, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$ , Rules used = {2230, 2208, 2209}

$$\frac{b^4(bc-ad)^4 e^{-a-bx}}{24d^9} + \frac{2b^3(bc-ad)^3 e^{-a-bx}}{3d^8} + \frac{b^2(bc-ad)^2 e^{-a-bx}}{d^7} + \frac{b(bc-ad) e^{-a-bx}}{d^6} + \frac{e^{-a-bx}}{d^5} + \frac{b^4 e^{-a-bx}(bc-ad)^4}{24d^9(c+dx)} + \frac{2b^3 e^{-a-bx}(bc-ad)^3}{3d^8(c+dx)} + \frac{b^2 e^{-a-bx}(bc-ad)^2}{d^7(c+dx)} + \frac{b e^{-a-bx}(bc-ad)}{d^6(c+dx)} + \frac{e^{-a-bx}(bc-ad)}{d^5(c+dx)} + \frac{2b^2 e^{-a-bx}(bc-ad)^2}{24d^8(c+dx)^2} + \frac{b e^{-a-bx}(bc-ad)}{3d^7(c+dx)^2} + \frac{e^{-a-bx}(bc-ad)}{d^6(c+dx)^2} + \frac{2b e^{-a-bx}(bc-ad)}{12d^5(c+dx)^2} + \frac{e^{-a-bx}(bc-ad)}{3d^4(c+dx)^2} + \frac{e^{-a-bx}(bc-ad)}{4d^3(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Int[(E^(-a - b\*x)\*(a + b\*x)^4)/(c + d\*x)^5, x]

[Out]  $-1/4*((b*c - a*d)^4*E^{-a - b*x})/(d^5*(c + d*x)^4) + (4*b*(b*c - a*d)^3*E^{-a - b*x})/(3*d^5*(c + d*x)^3) + (b*(b*c - a*d)^4*E^{-a - b*x})/(12*d^6*(c + d*x)^3) - (3*b^2*(b*c - a*d)^2*E^{-a - b*x})/(d^5*(c + d*x)^2) - (2*b^2*(b*c - a*d)^3*E^{-a - b*x})/(3*d^6*(c + d*x)^2) - (b^2*(b*c - a*d)^4*E^{-a - b*x})/(24*d^7*(c + d*x)^2) + (4*b^3*(b*c - a*d)*E^{-a - b*x})/(d^5*(c + d*x)) + (3*b^3*(b*c - a*d)^2*E^{-a - b*x})/(d^6*(c + d*x)) + (2*b^3*(b*c - a*d)^3*E^{-a - b*x})/(3*d^7*(c + d*x)) + (b^3*(b*c - a*d)^4*E^{-a - b*x})/(24*d^8*(c + d*x)) + (b^4*E^{-a + (b*c)/d}*\text{ExpIntegralEi}[-((b*(c + d*x))/d)])/d^5 + (4*b^4*(b*c - a*d)*E^{-a + (b*c)/d}*\text{ExpIntegralEi}[-((b*(c + d*x))/d)])/d^6 + (3*b^4*(b*c - a*d)^2*E^{-a + (b*c)/d}*\text{ExpIntegralEi}[-((b*(c + d*x))/d)])/d^7 + (2*b^4*(b*c - a*d)^3*E^{-a + (b*c)/d}*\text{ExpIntegralEi}[-((b*(c + d*x))/d)])/d^8 + (b^4*(b*c - a*d)^4*E^{-a + (b*c)/d}*\text{ExpIntegralEi}[-((b*(c + d*x))/d)])/d^9$

**Rule 2208**

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(c + d\*x)^(m + 1)\*((b\*F^(g\*(e + f\*x)))^n/(d\*(m + 1)))

```
, x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

### Rule 2209

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))/((c_) + (d_)*(x_)), x_Symbol] := Simp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

### Rule 2230

```
Int[(F_)^((c_)*(v_))*(u_)^(m_)*(w_), x_Symbol] := Int[ExpandIntegrand[F^(c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && IntegerQ[m] && !TrueQ[$UseGamma]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^5} dx &= \int \left( \frac{(-bc+ad)^4 e^{-a-bx}}{d^4(c+dx)^5} - \frac{4b(bc-ad)^3 e^{-a-bx}}{d^4(c+dx)^4} + \frac{6b^2(bc-ad)^2 e^{-a-bx}}{d^4(c+dx)^3} - \frac{4b^3(bc-ad)}{d^4(c+dx)^2} + \frac{b^4}{d^4} \right) dx \\
 &= \frac{b^4 \int \frac{e^{-a-bx}}{c+dx} dx}{d^4} - \frac{(4b^3(bc-ad)) \int \frac{e^{-a-bx}}{(c+dx)^2} dx}{d^4} + \frac{(6b^2(bc-ad)^2) \int \frac{e^{-a-bx}}{(c+dx)^3} dx}{d^4} - \frac{(4b^3(bc-ad)) \int \frac{e^{-a-bx}}{(c+dx)^4} dx}{d^4} + \frac{b^4 \int \frac{e^{-a-bx}}{c+dx} dx}{d^4} \\
 &= -\frac{(bc-ad)^4 e^{-a-bx}}{4d^5(c+dx)^4} + \frac{4b(bc-ad)^3 e^{-a-bx}}{3d^5(c+dx)^3} - \frac{3b^2(bc-ad)^2 e^{-a-bx}}{d^5(c+dx)^2} + \frac{4b^3(bc-ad) e^{-a-bx}}{d^5(c+dx)} - \frac{b^4 \int \frac{e^{-a-bx}}{c+dx} dx}{d^4} \\
 &= -\frac{(bc-ad)^4 e^{-a-bx}}{4d^5(c+dx)^4} + \frac{4b(bc-ad)^3 e^{-a-bx}}{3d^5(c+dx)^3} + \frac{b(bc-ad)^4 e^{-a-bx}}{12d^6(c+dx)^3} - \frac{3b^2(bc-ad)^2 e^{-a-bx}}{d^5(c+dx)^2} - \frac{b^4 \int \frac{e^{-a-bx}}{c+dx} dx}{d^4} \\
 &= -\frac{(bc-ad)^4 e^{-a-bx}}{4d^5(c+dx)^4} + \frac{4b(bc-ad)^3 e^{-a-bx}}{3d^5(c+dx)^3} + \frac{b(bc-ad)^4 e^{-a-bx}}{12d^6(c+dx)^3} - \frac{3b^2(bc-ad)^2 e^{-a-bx}}{d^5(c+dx)^2} - \frac{b^4 \int \frac{e^{-a-bx}}{c+dx} dx}{d^4} \\
 &= -\frac{(bc-ad)^4 e^{-a-bx}}{4d^5(c+dx)^4} + \frac{4b(bc-ad)^3 e^{-a-bx}}{3d^5(c+dx)^3} + \frac{b(bc-ad)^4 e^{-a-bx}}{12d^6(c+dx)^3} - \frac{3b^2(bc-ad)^2 e^{-a-bx}}{d^5(c+dx)^2} - \frac{b^4 \int \frac{e^{-a-bx}}{c+dx} dx}{d^4} \\
 &= -\frac{(bc-ad)^4 e^{-a-bx}}{4d^5(c+dx)^4} + \frac{4b(bc-ad)^3 e^{-a-bx}}{3d^5(c+dx)^3} + \frac{b(bc-ad)^4 e^{-a-bx}}{12d^6(c+dx)^3} - \frac{3b^2(bc-ad)^2 e^{-a-bx}}{d^5(c+dx)^2} - \frac{b^4 \int \frac{e^{-a-bx}}{c+dx} dx}{d^4}
 \end{aligned}$$

### Mathematica [A]

time = 2.28, size = 669, normalized size = 1.20

---



$$(-b*x-a+(a*d-b*c)/d)^4-1/12*\exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^3-1/24*\exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/24*\exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-1/24*\exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c)^5,x, algorithm="maxima")

[Out]  $-(b^3*d^2*x^4 + (4*a*b^2*d^2 - b^2*d^2)*x^3 + (6*a^2*b*d^2 + 5*b^2*c*d - 8*a*b*d^2 + 2*b*d^2)*x^2 + (4*a^3*d^2 - 5*b^2*c^2 - 18*a^2*d^2 - 20*b*c*d + 4*(5*b*c*d + 6*d^2)*a - 6*d^2)*x)*e^{-b*x}/(d^7*x^5*e^a + 5*c*d^6*x^4*e^a + 10*c^2*d^5*x^3*e^a + 10*c^3*d^4*x^2*e^a + 5*c^4*d^3*x*e^a + c^5*d^2*e^a) - a^4*e^{-a+b*c/d}*\exp\_integral\_e(5, (d*x+c)*b/d)/((d*x+c)^4*d) - \integrate(-(4*a^3*c*d^2 - 5*b^2*c^3 - 18*a^2*c*d^2 - 20*b*c^2*d - 6*c*d^2 + 4*(5*b*c^2*d + 6*c*d^2)*a + (5*b^3*c^3 - 16*a^3*d^3 + 50*b^2*c^2*d + 90*b*c*d^2 + 6*(5*b*c*d^2 + 12*d^3)*a^2 + 24*d^3 - 4*(5*b^2*c^2*d + 30*b*c*d^2 + 24*d^3)*a)*x)*e^{-b*x}/(d^8*x^6*e^a + 6*c*d^7*x^5*e^a + 15*c^2*d^6*x^4*e^a + 20*c^3*d^5*x^3*e^a + 15*c^4*d^4*x^2*e^a + 6*c^5*d^3*x*e^a + c^6*d^2*e^a), x)$

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 1084 vs.  $2(524) = 1048$ .

time = 0.38, size = 1084, normalized size = 1.95

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c)^5,x, algorithm="fricas")

[Out]  $1/24*((b^8*c^8 - 4*(a-4)*b^7*c^7*d + 6*(a^2 - 8*a + 12)*b^6*c^6*d^2 - 4*(a^3 - 12*a^2 + 36*a - 24)*b^5*c^5*d^3 + (a^4 - 16*a^3 + 72*a^2 - 96*a + 24)*b^4*c^4*d^4 + (b^8*c^4*d^4 - 4*(a-4)*b^7*c^3*d^5 + 6*(a^2 - 8*a + 12)*b^6*c^2*d^6 - 4*(a^3 - 12*a^2 + 36*a - 24)*b^5*c*d^7 + (a^4 - 16*a^3 + 72*a^2 - 96*a + 24)*b^4*d^8)*x^4 + 4*(b^8*c^5*d^3 - 4*(a-4)*b^7*c^4*d^4 + 6*(a^2 - 8*a + 12)*b^6*c^3*d^5 - 4*(a^3 - 12*a^2 + 36*a - 24)*b^5*c^2*d^6 + (a^4 - 16*a^3 + 72*a^2 - 96*a + 24)*b^4*c*d^7)*x^3 + 6*(b^8*c^6*d^2 - 4*(a-4)*b^7*c^5*d^3 + 6*(a^2 - 8*a + 12)*b^6*c^4*d^4 - 4*(a^3 - 12*a^2 + 36*a - 24)*b^5*c^3*d^5 + (a^4 - 16*a^3 + 72*a^2 - 96*a + 24)*b^4*c^2*d^6)*x^2 + 4*(b^8*c^7*d - 4*(a-4)*b^7*c^6*d^2 + 6*(a^2 - 8*a + 12)*b^6*c^5*d^3 - 4*(a^3 - 12*a^2 + 36*a - 24)*b^5*c^4*d^4 + (a^4 - 16*a^3 + 72*a^2 - 96*a + 24)*b^4*c^3*d^5)*x)*Ei(-(b*d*x+b*c)/d)*e^{(b*c-a*d)/d} + (b^7*c^7*d - (4*a-15)*b^6*c^6*d^2 + 2*(3*a^2-22*a+29)*b^5*c^5*d^3 - 2*(2*a^3-21*a^2+52*a-25)*b^4*c^4*d^4 + (a^4-12*a^3+36*a^2-24*a)*b^3*c^3*d^5 - 6*a^4*d$

$$\begin{aligned} &^8 - (a^4 - 8a^3 + 12a^2)*b^2*c^2*d^6 + 2*(a^4 - 4a^3)*b*c*d^7 + (b^7*c^4*d^4 - 4*(a - 4)*b^6*c^3*d^5 + 6*(a^2 - 8a + 12)*b^5*c^2*d^6 - 4*(a^3 - 12a^2 + 36a - 24)*b^4*c*d^7 + (a^4 - 16a^3 + 72a^2 - 96a)*b^3*d^8)*x^3 \\ &+ (3*b^7*c^5*d^3 - (12*a - 47)*b^6*c^4*d^4 + 2*(9*a^2 - 70*a + 100)*b^5*c^3*d^5 - 6*(2*a^3 - 23*a^2 + 64*a - 36)*b^4*c^2*d^6 + (3*a^4 - 44*a^3 + 168*a^2 - 144*a)*b^3*c*d^7 - (a^4 - 16*a^3 + 72*a^2)*b^2*d^8)*x^2 + (3*b^7*c^6*d^2 - 2*(6*a - 23)*b^6*c^5*d^3 + 2*(9*a^2 - 68*a + 93)*b^5*c^4*d^4 - 4*(3*a^3 - 33*a^2 + 86*a - 44)*b^4*c^3*d^5 + (3*a^4 - 40*a^3 + 132*a^2 - 96*a)*b^3*c^2*d^6 - 2*(a^4 - 12*a^3 + 24*a^2)*b^2*c*d^7 + 2*(a^4 - 16*a^3)*b*d^8)*x \\ &*e^{(-b*x - a)}/(d^{13}*x^4 + 4*c*d^{12}*x^3 + 6*c^2*d^{11}*x^2 + 4*c^3*d^{10}*x + c^4*d^9) \end{aligned}$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\left( \frac{\int \frac{a^4}{2c^4 + 5cd^4 + 10c^2d^4 + 10c^3d^4 + 5cd^4 + d^4} dx + \int \frac{4bd^4}{2c^4 + 5cd^4 + 10c^2d^4 + 10c^3d^4 + 5cd^4 + d^4} dx + \int \frac{6bd^4}{2c^4 + 5cd^4 + 10c^2d^4 + 10c^3d^4 + 5cd^4 + d^4} dx + \int \frac{4bd^4}{2c^4 + 5cd^4 + 10c^2d^4 + 10c^3d^4 + 5cd^4 + d^4} dx \right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)\*\*4/(d\*x+c)\*\*5,x)

[Out] (Integral(a\*\*4/(c\*\*5\*exp(b\*x) + 5\*c\*\*4\*d\*x\*exp(b\*x) + 10\*c\*\*3\*d\*\*2\*x\*\*2\*exp(b\*x) + 10\*c\*\*2\*d\*\*3\*x\*\*3\*exp(b\*x) + 5\*c\*d\*\*4\*x\*\*4\*exp(b\*x) + d\*\*5\*x\*\*5\*exp(b\*x)), x) + Integral(b\*\*4\*x\*\*4/(c\*\*5\*exp(b\*x) + 5\*c\*\*4\*d\*x\*exp(b\*x) + 10\*c\*\*3\*d\*\*2\*x\*\*2\*exp(b\*x) + 10\*c\*\*2\*d\*\*3\*x\*\*3\*exp(b\*x) + 5\*c\*d\*\*4\*x\*\*4\*exp(b\*x) + d\*\*5\*x\*\*5\*exp(b\*x)), x) + Integral(4\*a\*b\*\*3\*x\*\*3/(c\*\*5\*exp(b\*x) + 5\*c\*\*4\*d\*x\*exp(b\*x) + 10\*c\*\*3\*d\*\*2\*x\*\*2\*exp(b\*x) + 10\*c\*\*2\*d\*\*3\*x\*\*3\*exp(b\*x) + 5\*c\*d\*\*4\*x\*\*4\*exp(b\*x) + d\*\*5\*x\*\*5\*exp(b\*x)), x) + Integral(6\*a\*\*2\*b\*\*2\*x\*\*2/(c\*\*5\*exp(b\*x) + 5\*c\*\*4\*d\*x\*exp(b\*x) + 10\*c\*\*3\*d\*\*2\*x\*\*2\*exp(b\*x) + 10\*c\*\*2\*d\*\*3\*x\*\*3\*exp(b\*x) + 5\*c\*d\*\*4\*x\*\*4\*exp(b\*x) + d\*\*5\*x\*\*5\*exp(b\*x)), x) + Integral(4\*a\*\*3\*b\*x/(c\*\*5\*exp(b\*x) + 5\*c\*\*4\*d\*x\*exp(b\*x) + 10\*c\*\*3\*d\*\*2\*x\*\*2\*exp(b\*x) + 10\*c\*\*2\*d\*\*3\*x\*\*3\*exp(b\*x) + 5\*c\*d\*\*4\*x\*\*4\*exp(b\*x) + d\*\*5\*x\*\*5\*exp(b\*x)), x))\*exp(-a)

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 16988 vs.  $2(524) = 1048$ .

time = 2.85, size = 16988, normalized size = 30.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-b\*x-a)\*(b\*x+a)^4/(d\*x+c)^5,x, algorithm="giac")

[Out]  $1/24*((d*x + c)^4*(b - b*c/(d*x + c) + a*d/(d*x + c))^4*b^9*c^4*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{(b*c - a*d)/d} + 4*(d*x + c)^3*(b - b*c/(d*x + c) + a*d/(d*x + c))^3*b^10*c^5*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{(b*c - a*d)/d} + 6$

$$\begin{aligned}
&*(d*x + c)^2*(b - b*c/(d*x + c) + a*d/(d*x + c))^2*b^{11}*c^6*Ei(-((d*x + c)* \\
&(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d)} + 4*( \\
&d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^{12}*c^7*Ei(-((d*x + c)*(b - b \\
&*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d)} + b^{13}*c^8* \\
&Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c \\
&- a*d)/d)} - 4*(d*x + c)^4*a*(b - b*c/(d*x + c) + a*d/(d*x + c))^4*b^8*c^3*d \\
&*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c \\
&- a*d)/d)} - 20*(d*x + c)^3*a*(b - b*c/(d*x + c) + a*d/(d*x + c))^3*b^9*c^4 \\
&*d*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b \\
&*c - a*d)/d)} - 36*(d*x + c)^2*a*(b - b*c/(d*x + c) + a*d/(d*x + c))^2*b^{10}* \\
&c^5*d*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{ \\
&((b*c - a*d)/d)} - 28*(d*x + c)*a*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^{11}*c \\
&^6*d*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{( \\
&(b*c - a*d)/d)} - 8*a*b^{12}*c^7*d*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d* \\
&x + c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d)} + 6*(d*x + c)^4*a^2*(b - b*c/(d*x \\
&+ c) + a*d/(d*x + c))^4*b^7*c^2*d^2*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a* \\
&d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d)} + 40*(d*x + c)^3*a^2*(b - b* \\
&c/(d*x + c) + a*d/(d*x + c))^3*b^8*c^3*d^2*Ei(-((d*x + c)*(b - b*c/(d*x + c \\
&+ a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d)} + 90*(d*x + c)^2*a^2*( \\
&b - b*c/(d*x + c) + a*d/(d*x + c))^2*b^9*c^4*d^2*Ei(-((d*x + c)*(b - b*c/(d \\
&>*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d)} + 84*(d*x + c)*a \\
&^2*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^{10}*c^5*d^2*Ei(-((d*x + c)*(b - b*c \\
&/d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d)} + 28*a^2*b^{11} \\
&*c^6*d^2*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) \\
&*e^{((b*c - a*d)/d)} - 4*(d*x + c)^4*a^3*(b - b*c/(d*x + c) + a*d/(d*x + c))^ \\
&4*b^6*c*d^3*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d) \\
&/d)*e^{((b*c - a*d)/d)} - 40*(d*x + c)^3*a^3*(b - b*c/(d*x + c) + a*d/(d*x + \\
&c))^3*b^7*c^2*d^3*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c \\
&- a*d)/d)*e^{((b*c - a*d)/d)} - 120*(d*x + c)^2*a^3*(b - b*c/(d*x + c) + a*d/ \\
&(d*x + c))^2*b^8*c^3*d^3*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) \\
&+ b*c - a*d)/d)*e^{((b*c - a*d)/d)} - 140*(d*x + c)*a^3*(b - b*c/(d*x + c) + \\
&a*d/(d*x + c))*b^9*c^4*d^3*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + \\
&c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d)} - 56*a^3*b^{10}*c^5*d^3*Ei(-((d*x + c)* \\
&(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d)} + (d* \\
&x + c)^4*a^4*(b - b*c/(d*x + c) + a*d/(d*x + c))^4*b^5*d^4*Ei(-((d*x + c)*( \\
&b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d)} + 20*( \\
&d*x + c)^3*a^4*(b - b*c/(d*x + c) + a*d/(d*x + c))^3*b^6*c*d^4*Ei(-((d*x + \\
&c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d)} + \\
&90*(d*x + c)^2*a^4*(b - b*c/(d*x + c) + a*d/(d*x + c))^2*b^7*c^2*d^4*Ei(-(( \\
&d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a*d) \\
&/d)} + 140*(d*x + c)*a^4*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^8*c^3*d^4*Ei( \\
&-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a \\
&*d)/d)} + 70*a^4*b^9*c^4*d^4*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + \\
&c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d)} - 4*(d*x + c)^3*a^5*(b - b*c/(d*x + c \\
&+ a*d/(d*x + c))^3*b^5*d^5*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x +
\end{aligned}$$

$c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d) - 36*(d*x + c)^2*a^5*(b - b*c/(d*x + c) + a*d/(d*x + c))^2*b^6*c*d^5*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d) - 84*(d*x + c)*a^5*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^7*c^2*d^5*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d) - 56*a^5*b^8*c^3*d^5*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + 6*(d*x + c)^2*a^6*(b - b*c/(d*x + c) + a*d/(d*x + c))^2*b^5*d^6*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d) + 28*(d*x + c)*a^6*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^6*c*d^6*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d) + 28*a^6*b^7*c^2*d^6*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d) - 4*(d*x + c)*a^7*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^5*d^7*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d) - 8*a^7*b^6*c*d^7*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d) + a^8*b^5*d^8*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d) + 16*(d*x + c)^4*(b - b*c/(d*x + c) + a*d/(d*x + c))^4*b^8*c^3*d*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)))...}$

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{-a-bx} (a+bx)^4}{(c+dx)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(- a - b\*x)\*(a + b\*x)^4)/(c + d\*x)^5, x)

[Out] int((exp(- a - b\*x)\*(a + b\*x)^4)/(c + d\*x)^5, x)

### 3.83 $\int F^{c(a+bx)} x^m \log^n(dx) (e + en + e(1 + m + bcx \log(F))) \log(dx)$

Optimal. Leaf size=24

$$eF^{c(a+bx)} x^{1+m} \log^{1+n}(dx)$$

[Out] e\*F^(c\*(b\*x+a))\*x^(1+m)\*ln(d\*x)^(1+n)

Rubi [A]

time = 0.10, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ ,

Rules used = {2233}

$$ex^{m+1} \log^{n+1}(dx) F^{c(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))\*x^m\*Log[d\*x]^n\*(e + e\*n + e\*(1 + m + b\*c\*x\*Log[F])\*Log[d\*x]), x]

[Out] e\*F^(c\*(a + b\*x))\*x^(1 + m)\*Log[d\*x]^(1 + n)

Rule 2233

Int[Log[(d\_.)\*(x\_)]^(n\_.)\*(F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_)))\*(x\_)^(m\_.)\*((e\_ + Log[(d\_.)\*(x\_)]\*(h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := Simp[e\*x^(m + 1)\*F^(c\*(a + b\*x))\*(Log[d\*x]^(n + 1)/(n + 1)), x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e\*(m + 1) - f\*h\*(n + 1), 0] && EqQ[g\*h\*(n + 1) - b\*c\*e\*Log[F], 0] && NeQ[n, -1]

Rubi steps

$$\int F^{c(a+bx)} x^m \log^n(dx) (e + en + e(1 + m + bcx \log(F))) \log(dx) dx = eF^{c(a+bx)} x^{1+m} \log^{1+n}(dx)$$

Mathematica [A]

time = 0.19, size = 24, normalized size = 1.00

$$eF^{c(a+bx)} x^{1+m} \log^{1+n}(dx)$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*x^m\*Log[d\*x]^n\*(e + e\*n + e\*(1 + m + b\*c\*x\*Log[F])\*Log[d\*x]), x]

[Out] e\*F^(c\*(a + b\*x))\*x^(1 + m)\*Log[d\*x]^(1 + n)



**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.  
time = 0.20, size = 192, normalized size = 8.00

method	result
risch	$\frac{(2ex F^{c(bx+a)} \ln(x) + ix F^{c(bx+a)} e\pi \operatorname{csgn}(id) \operatorname{csgn}(idx)^2 + ix F^{c(bx+a)} e\pi \operatorname{csgn}(ix) \operatorname{csgn}(idx)^2 - ix F^{c(bx+a)} e\pi \operatorname{csgn}(id) \operatorname{csgn}(ix) \operatorname{csgn}(idx)^2)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*x^m*ln(d*x)^n*(e+e*n+e*(1+m+b*c*x*ln(F))*ln(d*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*e*x*F^(c*(b*x+a))*ln(x)+I*x*F^(c*(b*x+a))*e*Pi*csgn(I*d)*csgn(I*d*x)^2+I*x*F^(c*(b*x+a))*e*Pi*csgn(I*x)*csgn(I*d*x)^2-I*x*F^(c*(b*x+a))*e*Pi*csgn(I*d)*csgn(I*x)*csgn(I*d*x)-I*x*F^(c*(b*x+a))*e*Pi*csgn(I*d*x)^3+2*x*F^(c*(b*x+a))*e*ln(d))*x^m*(ln(d)+ln(x)-1/2*I*Pi*csgn(I*d*x)*(-csgn(I*d*x)+csgn(I*d))*(-csgn(I*d*x)+csgn(I*x)))^n
```

**Maxima [A]**

time = 0.35, size = 44, normalized size = 1.83

$$(F^{ac} x e \log(d) + F^{ac} x e \log(x)) e^{(bcx \log(F) + m \log(x) + n \log(\log(d) + \log(x)))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*x^m*log(d*x)^n*(e+e*n+e*(1+m+b*c*x*log(F))*log(d*x)),x,algorithm="maxima")
```

```
[Out] (F^(a*c)*x*e*log(d) + F^(a*c)*x*e*log(x))*e^(b*c*x*log(F) + m*log(x) + n*log(log(d) + log(x)))
```

**Fricas [A]**

time = 0.41, size = 34, normalized size = 1.42

$$(x e \log(d) + x e \log(x)) F^{bcx+ac} x^m (\log(d) + \log(x))^n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*x^m*log(d*x)^n*(e+e*n+e*(1+m+b*c*x*log(F))*log(d*x)),x,algorithm="fricas")
```

```
[Out] (x*e*log(d) + x*e*log(x))*F^(b*c*x + a*c)*x^m*(log(d) + log(x))^n
```

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*x**m*ln(d*x)**n*(e+e*n+e*(1+m+b*c*x*ln(F))*ln(d*x)
),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

**Giac [F]**

```
time = 0.00, size = 0, normalized size = 0.00
```

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*x^m*log(d*x)^n*(e+e*n+e*(1+m+b*c*x*log(F))*log(d*x)
),x, algorithm="giac")
```

```
[Out] integrate(((b*c*x*log(F) + m + 1)*e*log(d*x) + n*e + e)*F^((b*x + a)*c)*x^m
*log(d*x)^n, x)
```

**Mupad [B]**

```
time = 3.59, size = 25, normalized size = 1.04
```

$$F^{ac+bcx} e^{x^{m+1}} \ln(dx)^{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*x^m*log(d*x)^n*(e + e*n + e*log(d*x)*(m + b*c*x*log(F)
+ 1)),x)
```

```
[Out] F^(a*c + b*c*x)*e*x^(m + 1)*log(d*x)^(n + 1)
```

### 3.84 $\int F^{c(a+bx)} x^2 \log^n(dx) (e + en + e(3 + bcx \log(F))) \log(dx)$

Optimal. Leaf size=22

$$eF^{c(a+bx)} x^3 \log^{1+n}(dx)$$

[Out]  $eF^{c(bx+a)} x^3 \ln(dx)^{(1+n)}$

Rubi [A]

time = 0.09, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ ,

Rules used = {2233}

$$ex^3 \log^{n+1}(dx) F^{c(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))\*x^2\*Log[d\*x]^n\*(e + e\*n + e\*(3 + b\*c\*x\*Log[F])\*Log[d\*x]),x]

[Out]  $eF^{c(a + bx)} x^3 \text{Log}[d*x]^{(1 + n)}$

Rule 2233

Int[Log[(d\_.)\*(x\_)]^(n\_.)\*(F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_)))\*(x\_)^(m\_.)\*((e\_ + Log[(d\_.)\*(x\_)]\*(h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] :> Simp[e\*x^(m + 1)\*F^(c\*(a + b\*x))\*(Log[d\*x]^(n + 1)/(n + 1)), x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e\*(m + 1) - f\*h\*(n + 1), 0] && EqQ[g\*h\*(n + 1) - b\*c\*e\*Log[F], 0] && NeQ[n, -1]

Rubi steps

$$\int F^{c(a+bx)} x^2 \log^n(dx) (e + en + e(3 + bcx \log(F))) \log(dx) dx = eF^{c(a+bx)} x^3 \log^{1+n}(dx)$$

Mathematica [A]

time = 0.11, size = 23, normalized size = 1.05

$$eF^{ac+bcx} x^3 \log^{1+n}(dx)$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*x^2\*Log[d\*x]^n\*(e + e\*n + e\*(3 + b\*c\*x\*Log[F])\*Log[d\*x]),x]

[Out]  $eF^{(a*c + b*c*x)} x^3 \text{Log}[d*x]^{(1 + n)}$

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.  
time = 0.12, size = 198, normalized size = 9.00

method	result
risch	$\left( -\frac{i\pi e x^3 \operatorname{csgn}(id) \operatorname{csgn}(ix) \operatorname{csgn}(idx) F^{c(bx+a)}}{2} + \frac{i\pi e x^3 \operatorname{csgn}(id) \operatorname{csgn}(idx)^2 F^{c(bx+a)}}{2} + \frac{i\pi e x^3 \operatorname{csgn}(ix) \operatorname{csgn}(idx)^2 F^{c(bx+a)}}{2} - \frac{i\pi e x^3 \operatorname{csgn}(id) \operatorname{csgn}(ix) \operatorname{csgn}(idx)^2 F^{c(bx+a)}}{2} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*x^2*ln(d*x)^n*(e+e*n+e*(3+b*c*x*ln(F))*ln(d*x)),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*I*Pi*e*x^3*csgn(I*d)*csgn(I*x)*csgn(I*d*x)*F^(c*(b*x+a))+1/2*I*Pi*e*x^3*csgn(I*d)*csgn(I*d*x)^2*F^(c*(b*x+a))+1/2*I*Pi*e*x^3*csgn(I*x)*csgn(I*d*x)^2*F^(c*(b*x+a))-1/2*I*Pi*e*x^3*csgn(I*d*x)^3*F^(c*(b*x+a))+ln(d)*e*x^3*F^(c*(b*x+a))+e*x^3*F^(c*(b*x+a))*ln(x))*(ln(d)+ln(x)-1/2*I*Pi*csgn(I*d*x)*(-csgn(I*d*x)+csgn(I*d))*(-csgn(I*d*x)+csgn(I*x)))^n
```

**Maxima [A]**

time = 0.37, size = 44, normalized size = 2.00

$$(F^{ac} x^3 e \log(d) + F^{ac} x^3 e \log(x)) e^{(bcx \log(F) + n \log(\log(d) + \log(x)))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*x^2*log(d*x)^n*(e+e*n+e*(3+b*c*x*log(F))*log(d*x)),x,algorithm="maxima")
```

```
[Out] (F^(a*c)*x^3*e*log(d) + F^(a*c)*x^3*e*log(x))*e^(b*c*x*log(F) + n*log(log(d) + log(x)))
```

**Fricas [A]**

time = 0.44, size = 26, normalized size = 1.18

$$F^{bcx+ac} x^3 \log(dx)^n e \log(dx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*x^2*log(d*x)^n*(e+e*n+e*(3+b*c*x*log(F))*log(d*x)),x,algorithm="fricas")
```

```
[Out] F^(b*c*x + a*c)*x^3*log(d*x)^n*e*log(d*x)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$e \left( \int F^{ac} F^{bcx} x^2 \log(dx)^n dx + \int F^{ac} F^{bcx} n x^2 \log(dx)^n dx + \int 3 F^{ac} F^{bcx} x^2 \log(dx) \log(dx)^n dx + \int F^{ac} F^{bcx} b c x^3 \log(F) \log(dx) \log(dx)^n dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*x**2*ln(d*x)**n*(e+e*n+e*(3+b*c*x*ln(F))*ln(d*x)),
x)
```

```
[Out] e*(Integral(F**(a*c)*F**(b*c*x)*x**2*log(d*x)**n, x) + Integral(F**(a*c)*F*
*(b*c*x)*n*x**2*log(d*x)**n, x) + Integral(3*F**(a*c)*F**(b*c*x)*x**2*log(d
*x)*log(d*x)**n, x) + Integral(F**(a*c)*F**(b*c*x)*b*c*x**3*log(F)*log(d*x)
*log(d*x)**n, x))
```

**Giac** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*x^2*log(d*x)^n*(e+e*n+e*(3+b*c*x*log(F))*log(d*x)),
x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, infinity is unsigned, perhaps you meant +infinite
Warning, infinity is unsigned, perhaps you meant +infinityUnable to divide
, perhaps
```

**Mupad** [B]

time = 3.50, size = 23, normalized size = 1.05

$$F^{ac+bcx} e x^3 \ln(dx)^{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*x^2*log(d*x)^n*(e + e*n + e*log(d*x)*(b*c*x*log(F) + 3)
),x)
```

```
[Out] F^(a*c + b*c*x)*e*x^3*log(d*x)^(n + 1)
```

### 3.85 $\int F^{c(a+bx)} x \log^n(dx) (e + en + e(2 + bcx \log(F)) \log(dx)) dx$

Optimal. Leaf size=22

$$eF^{c(a+bx)} x^2 \log^{1+n}(dx)$$

[Out] e\*F^(c\*(b\*x+a))\*x^2\*ln(d\*x)^(1+n)

Rubi [A]

time = 0.06, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$ ,

Rules used = {2233}

$$ex^2 \log^{n+1}(dx) F^{c(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))\*x\*Log[d\*x]^n\*(e + e\*n + e\*(2 + b\*c\*x\*Log[F])\*Log[d\*x]), x]

[Out] e\*F^(c\*(a + b\*x))\*x^2\*Log[d\*x]^(1 + n)

Rule 2233

Int[Log[(d\_.)\*(x\_.)]^(n\_.)\*(F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_.)))\*(x\_)^(m\_.)\*((e\_ + Log[(d\_.)\*(x\_.)]\*(h\_.)\*((f\_.) + (g\_.)\*(x\_.))), x\_Symbol] := Simp[e\*x^(m + 1)\*F^(c\*(a + b\*x))\*(Log[d\*x]^(n + 1)/(n + 1)), x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e\*(m + 1) - f\*h\*(n + 1), 0] && EqQ[g\*h\*(n + 1) - b\*c\*e\*Log[F], 0] && NeQ[n, -1]

Rubi steps

$$\int F^{c(a+bx)} x \log^n(dx) (e + en + e(2 + bcx \log(F)) \log(dx)) dx = eF^{c(a+bx)} x^2 \log^{1+n}(dx)$$

Mathematica [A]

time = 0.09, size = 23, normalized size = 1.05

$$eF^{ac+bcx} x^2 \log^{1+n}(dx)$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*x\*Log[d\*x]^n\*(e + e\*n + e\*(2 + b\*c\*x\*Log[F])\*Log[d\*x]), x]

[Out] e\*F^(a\*c + b\*c\*x)\*x^2\*Log[d\*x]^(1 + n)

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.  
time = 0.12, size = 198, normalized size = 9.00

method	result
risch	$\left( -\frac{i\pi e x^2 \operatorname{csgn}(id) \operatorname{csgn}(ix) \operatorname{csgn}(idx) F^{c(bx+a)}}{2} + \frac{i\pi e x^2 \operatorname{csgn}(id) \operatorname{csgn}(idx)^2 F^{c(bx+a)}}{2} + \frac{i\pi e x^2 \operatorname{csgn}(ix) \operatorname{csgn}(idx)^2 F^{c(bx+a)}}{2} - \frac{i\pi e x^2 \operatorname{csgn}(id) \operatorname{csgn}(ix) \operatorname{csgn}(idx)^2 F^{c(bx+a)}}{2} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a))*x*ln(d*x)^n*(e+e*n+e*(2+b*c*x*ln(F))*ln(d*x)),x,method=_RETURVERBOSE)`

[Out] 
$$\begin{aligned} & (-1/2*I*Pi*e*x^2*csgn(I*d)*csgn(I*x)*csgn(I*d*x)*F^(c*(b*x+a))+1/2*I*Pi*e*x^2*csgn(I*d)*csgn(I*d*x)^2*F^(c*(b*x+a))+1/2*I*Pi*e*x^2*csgn(I*x)*csgn(I*d*x)^2*F^(c*(b*x+a))-1/2*I*Pi*e*x^2*csgn(I*d*x)^3*F^(c*(b*x+a))+\ln(d)*e*x^2*F^(c*(b*x+a))+e*x^2*F^(c*(b*x+a))*\ln(x))*(\ln(d)+\ln(x)-1/2*I*Pi*csgn(I*d*x)*(-csgn(I*d*x)+csgn(I*d))*(-csgn(I*d*x)+csgn(I*x)))^n \end{aligned}$$

**Maxima [A]**

time = 0.35, size = 44, normalized size = 2.00

$$(F^{ac} x^2 e \log(d) + F^{ac} x^2 e \log(x)) e^{(bcx \log(F) + n \log(\log(d) + \log(x)))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*x*log(d*x)^n*(e+e*n+e*(2+b*c*x*log(F))*log(d*x)),x,algorithm="maxima")`

[Out] 
$$(F^{(a*c)} * x^2 * e * \log(d) + F^{(a*c)} * x^2 * e * \log(x)) * e^{(b*c*x*\log(F) + n*\log(\log(d) + \log(x)))}$$

**Fricas [A]**

time = 0.45, size = 26, normalized size = 1.18

$$F^{bcx+ac} x^2 \log(dx)^n e \log(dx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*x*log(d*x)^n*(e+e*n+e*(2+b*c*x*log(F))*log(d*x)),x,algorithm="fricas")`

[Out] 
$$F^{(b*c*x + a*c)} * x^2 * \log(d*x)^n * e * \log(d*x)$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$e \left( \int F^{ac} F^{bcx} x \log(dx)^n dx + \int F^{ac} F^{bcx} n x \log(dx)^n dx + \int 2 F^{ac} F^{bcx} x \log(dx) \log(dx)^n dx + \int F^{ac} F^{bcx} bcx^2 \log(F) \log(dx) \log(dx)^n dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*x*ln(d*x)**n*(e+e*n+e*(2+b*c*x*ln(F))*ln(d*x)),x)
[Out] e*(Integral(F**(a*c)*F**(b*c*x)*x*log(d*x)**n, x) + Integral(F**(a*c)*F**(b
*c*x)*n*x*log(d*x)**n, x) + Integral(2*F**(a*c)*F**(b*c*x)*x*log(d*x)*log(d
*x)**n, x) + Integral(F**(a*c)*F**(b*c*x)*b*c*x**2*log(F)*log(d*x)*log(d*x)
**n, x))
```

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*x*log(d*x)^n*(e+e*n+e*(2+b*c*x*log(F))*log(d*x)),x,
algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, infinity is unsigned, perhaps you meant +infinite
Warning, infinity is unsigned, perhaps you meant +infinityUnable to divide
, perhaps
```

**Mupad [B]**

time = 3.39, size = 23, normalized size = 1.05

$$F^{ac+bcx} e x^2 \ln(dx)^{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*x*log(d*x)^n*(e + e*n + e*log(d*x)*(b*c*x*log(F) + 2)),
x)
```

```
[Out] F^(a*c + b*c*x)*e*x^2*log(d*x)^(n + 1)
```



### 3.86 $\int F^{c(a+bx)} \log^n(dx) (e + en + e(1 + bcx \log(F))) \log(dx) dx$

Optimal. Leaf size=20

$$eF^{c(a+bx)}x \log^{1+n}(dx)$$

[Out]  $eF^{c(bx+a)}x \ln(dx)^{(1+n)}$

Rubi [A]

time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$ , Rules used = {2232}

$$ex \log^{n+1}(dx) F^{c(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[F^(c\*(a + b\*x))\*Log[d\*x]^n\*(e + e\*n + e\*(1 + b\*c\*x\*Log[F])\*Log[d\*x]),x]

[Out]  $eF^{c(a + b*x)}x \text{Log}[d*x]^{(1 + n)}$

Rule 2232

```
Int[Log[(d_.)*(x_.)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_.)))*((e_) + Log[(d_.)*(x_.)]*(h_.)*((f_.) + (g_.)*(x_.))), x_Symbol] := Simp[e*x*F^(c*(a + b*x))
*(Log[d*x]^(n + 1)/(n + 1)), x] /; FreeQ[{F, a, b, c, d, e, f, g, h, n}, x]
&& EqQ[e - f*h*(n + 1), 0] && EqQ[g*h*(n + 1) - b*c*e*Log[F], 0] && NeQ[n,
-1]
```

Rubi steps

$$\int F^{c(a+bx)} \log^n(dx) (e + en + e(1 + bcx \log(F))) \log(dx) dx = eF^{c(a+bx)}x \log^{1+n}(dx)$$

Mathematica [A]

time = 0.09, size = 21, normalized size = 1.05

$$eF^{ac+bcx}x \log^{1+n}(dx)$$

Antiderivative was successfully verified.

[In] Integrate[F^(c\*(a + b\*x))\*Log[d\*x]^n\*(e + e\*n + e\*(1 + b\*c\*x\*Log[F])\*Log[d\*x]),x]

[Out]  $eF^{(a*c + b*c*x)}x \text{Log}[d*x]^{(1 + n)}$

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.  
time = 0.13, size = 186, normalized size = 9.30

method	result
risch	$\left( -\frac{ix F^{c(bx+a)} e\pi \operatorname{csgn}(id) \operatorname{csgn}(ix) \operatorname{csgn}(idx)}{2} + \frac{ix F^{c(bx+a)} e\pi \operatorname{csgn}(id) \operatorname{csgn}(idx)^2}{2} + \frac{ix F^{c(bx+a)} e\pi \operatorname{csgn}(ix) \operatorname{csgn}(idx)^2}{2} - \frac{ix F^{c(bx+a)} e\pi \operatorname{csgn}(id) \operatorname{csgn}(idx)^3}{2} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a))*ln(d*x)^n*(e+e*n+e*(1+b*c*x*ln(F))*ln(d*x)),x,method=_RETURNVERBOSE)`

[Out] 
$$\begin{aligned} & (-1/2*I*x*F^{c*(b*x+a)}*e*Pi*csgn(I*d)*csgn(I*x)*csgn(I*d*x)+1/2*I*x*F^{c*(b*x+a)}*e*Pi*csgn(I*d)*csgn(I*d*x)^2+1/2*I*x*F^{c*(b*x+a)}*e*Pi*csgn(I*x)*csgn(I*d*x)^2-1/2*I*x*F^{c*(b*x+a)}*e*Pi*csgn(I*d*x)^3+x*F^{c*(b*x+a)}*e*ln(d)+e*x*F^{c*(b*x+a)}*ln(x))*(ln(d)+ln(x)-1/2*I*Pi*csgn(I*d*x)*(-csgn(I*d*x)+csgn(I*d)))*(-csgn(I*d*x)+csgn(I*x))^n \end{aligned}$$

**Maxima [A]**

time = 0.36, size = 40, normalized size = 2.00

$$(F^{ac} x e \log(d) + F^{ac} x e \log(x)) e^{(bcx \log(F) + n \log(\log(d) + \log(x)))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*log(d*x)^n*(e+e*n+e*(1+b*c*x*log(F))*log(d*x)),x,algorithm="maxima")`

[Out] 
$$(F^{(a*c)}*x*e*\log(d) + F^{(a*c)}*x*e*\log(x))*e^{(b*c*x*\log(F) + n*\log(\log(d) + \log(x)))}$$

**Fricas [A]**

time = 0.50, size = 24, normalized size = 1.20

$$F^{bcx+ac} x \log(dx)^n e \log(dx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*log(d*x)^n*(e+e*n+e*(1+b*c*x*log(F))*log(d*x)),x,algorithm="fricas")`

[Out] 
$$F^{(b*c*x + a*c)}*x*\log(d*x)^n*e*\log(d*x)$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$e \left( \int F^{ac} F^{bcx} \log(dx)^n dx + \int F^{ac} F^{bcx} n \log(dx)^n dx + \int F^{ac} F^{bcx} \log(dx) \log(dx)^n dx + \int F^{ac} F^{bcx} bcx \log(F) \log(dx) \log(dx)^n dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*ln(d*x)**n*(e+e*n+e*(1+b*c*x*ln(F))*ln(d*x)),x)
[Out] e*(Integral(F**(a*c)*F**(b*c*x)*log(d*x)**n, x) + Integral(F**(a*c)*F**(b*c*x)*n*log(d*x)**n, x) + Integral(F**(a*c)*F**(b*c*x)*log(d*x)*log(d*x)**n, x) + Integral(F**(a*c)*F**(b*c*x)*b*c*x*log(F)*log(d*x)*log(d*x)**n, x))
```

**Giac** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*log(d*x)^n*(e+e*n+e*(1+b*c*x*log(F))*log(d*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, infinity is unsigned, perhaps you meant +infinity
Warning, infinity is unsigned, perhaps you meant +infinityUnable to divide
, perhaps
```

**Mupad** [B]

time = 3.48, size = 21, normalized size = 1.05

$$F^{ac+bcx} e x \ln(dx)^{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*log(d*x)^n*(e + e*n + e*log(d*x)*(b*c*x*log(F) + 1)),x)
```

```
[Out] F^(a*c + b*c*x)*e*x*log(d*x)^(n + 1)
```

$$3.87 \quad \int \frac{F^{c(a+bx)} \log^n(dx) (e + en + bcex \log(F) \log(dx))}{x} dx$$

Optimal. Leaf size=19

$$eF^{c(a+bx)} \log^{1+n}(dx)$$

[Out] e\*F^(c\*(b\*x+a))\*ln(d\*x)^(1+n)

Rubi [A]

time = 0.09, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$ , Rules used = {2233}

$$e \log^{n+1}(dx) F^{c(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(F^(c\*(a + b\*x))\*Log[d\*x]^n\*(e + e\*n + b\*c\*e\*x\*Log[F]\*Log[d\*x]))/x,x]

[Out] e\*F^(c\*(a + b\*x))\*Log[d\*x]^(1 + n)

Rule 2233

Int[Log[(d\_.)\*(x\_)]^(n\_.)\*(F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_)))\*(x\_)^(m\_.)\*((e\_)+Log[(d\_.)\*(x\_)]\*(h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] :> Simp[e\*x^(m + 1)\*F^(c\*(a + b\*x))\*(Log[d\*x]^(n + 1)/(n + 1)), x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e\*(m + 1) - f\*h\*(n + 1), 0] && EqQ[g\*h\*(n + 1) - b\*c\*e\*Log[F], 0] && NeQ[n, -1]

Rubi steps

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + bcex \log(F) \log(dx))}{x} dx = eF^{c(a+bx)} \log^{1+n}(dx)$$

Mathematica [A]

time = 0.03, size = 19, normalized size = 1.00

$$eF^{c(a+bx)} \log^{1+n}(dx)$$

Antiderivative was successfully verified.

[In] Integrate[(F^(c\*(a + b\*x))\*Log[d\*x]^n\*(e + e\*n + b\*c\*e\*x\*Log[F]\*Log[d\*x]))/x,x]

[Out] e\*F^(c\*(a + b\*x))\*Log[d\*x]^(1 + n)

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.  
time = 0.12, size = 180, normalized size = 9.47

method	result
risch	$\left( -\frac{i\pi e \operatorname{csgn}(id)\operatorname{csgn}(ix)\operatorname{csgn}(idx)F^{c(bx+a)}}{2} + \frac{i\pi e \operatorname{csgn}(id)\operatorname{csgn}(idx)^2 F^{c(bx+a)}}{2} + \frac{i\pi e \operatorname{csgn}(ix)\operatorname{csgn}(idx)^2 F^{c(bx+a)}}{2} - \frac{i\pi e \operatorname{csgn}(i)}{2} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a))*ln(d*x)^n*(e+e*n+b*c*e*x*ln(F)*ln(d*x))/x,x,method=_RETURNVERBOSE)`

[Out] 
$$\begin{aligned} & (-1/2*I*Pi*e*csgn(I*d)*csgn(I*x)*csgn(I*d*x)*F^{c*(b*x+a)} + 1/2*I*Pi*e*csgn(I*d)*csgn(I*d*x)^2*F^{c*(b*x+a)} + 1/2*I*Pi*e*csgn(I*x)*csgn(I*d*x)^2*F^{c*(b*x+a)} \\ & - 1/2*I*Pi*e*csgn(I*d*x)^3*F^{c*(b*x+a)} + \ln(d)*e*F^{c*(b*x+a)} + e*F^{c*(b*x+a)}* \ln(x)) * (\ln(d) + \ln(x) - 1/2*I*Pi*csgn(I*d*x)*(-csgn(I*d*x) + csgn(I*d))*(-csgn(I*d*x) + csgn(I*x)))^n \end{aligned}$$

**Maxima [A]**

time = 0.36, size = 38, normalized size = 2.00

$$(F^{ac}e \log(d) + F^{ac}e \log(x))e^{(bcx \log(F) + n \log(\log(d) + \log(x)))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*log(d*x)^n*(e+e*n+b*c*e*x*log(F)*log(d*x))/x,x, algorithm="maxima")`

[Out] 
$$(F^{a*c}*e*\log(d) + F^{a*c}*e*\log(x))*e^{(b*c*x*\log(F) + n*\log(\log(d) + \log(x)))}$$

**Fricas [A]**

time = 0.49, size = 23, normalized size = 1.21

$$F^{bcx+ac} \log(dx)^n e \log(dx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*log(d*x)^n*(e+e*n+b*c*e*x*log(F)*log(d*x))/x,x, algorithm="fricas")`

[Out] 
$$F^{(b*c*x + a*c)*\log(d*x)^n*e*\log(d*x)}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$e \left( \int \frac{F^{ac} F^{bcx} \log(dx)^n}{x} dx + \int \frac{F^{ac} F^{bcx} n \log(dx)^n}{x} dx + \int F^{ac} F^{bcx} bc \log(F) \log(dx) \log(dx)^n dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F\*\*(c\*(b\*x+a))\*ln(d\*x)\*\*n\*(e+e\*n+b\*c\*e\*x\*ln(F)\*ln(d\*x))/x,x)

[Out] e\*(Integral(F\*\*(a\*c)\*F\*\*(b\*c\*x)\*log(d\*x)\*\*n/x, x) + Integral(F\*\*(a\*c)\*F\*\*(b\*c\*x)\*n\*log(d\*x)\*\*n/x, x) + Integral(F\*\*(a\*c)\*F\*\*(b\*c\*x)\*b\*c\*log(F)\*log(d\*x)\*log(d\*x)\*\*n, x))

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c\*(b\*x+a))\*log(d\*x)^n\*(e+e\*n+b\*c\*e\*x\*log(F)\*log(d\*x))/x,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command: INPUT:sage2OUTPUT:Warning, infinity is unsigned, perhaps you meant +infinityWarning, infinity is unsigned, perhaps you meant +infinityUnable to divide , perhaps

Mupad [B]

time = 3.52, size = 20, normalized size = 1.05

$$F^{ac+bcx} e \ln(dx)^{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((F^(c\*(a + b\*x))\*log(d\*x)^n\*(e + e\*n + b\*c\*e\*x\*log(d\*x)\*log(F)))/x,x)

[Out] F^(a\*c + b\*c\*x)\*e\*log(d\*x)^(n + 1)

$$3.88 \quad \int \frac{F^{c(a+bx)} \log^n(dx) (e + en + e(-1 + bcx \log(F)) \log(dx))}{x^2} dx$$

Optimal. Leaf size=22

$$\frac{eF^{c(a+bx)} \log^{1+n}(dx)}{x}$$

[Out] e\*F^(c\*(b\*x+a))\*ln(d\*x)^(1+n)/x

Rubi [A]

time = 0.10, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {2233}

$$\frac{e \log^{n+1}(dx) F^{c(a+bx)}}{x}$$

Antiderivative was successfully verified.

[In] Int[(F^(c\*(a + b\*x))\*Log[d\*x]^n\*(e + e\*n + e\*(-1 + b\*c\*x\*Log[F])\*Log[d\*x]))/x^2,x]

[Out] (e\*F^(c\*(a + b\*x))\*Log[d\*x]^(1 + n))/x

Rule 2233

Int[Log[(d\_.)\*(x\_)]^(n\_.)\*(F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_)))\*(x\_)^(m\_.)\*((e\_ + Log[(d\_.)\*(x\_)]\*(h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] :> Simp[e\*x^(m + 1)\*F^(c\*(a + b\*x))\*(Log[d\*x]^(n + 1)/(n + 1)), x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e\*(m + 1) - f\*h\*(n + 1), 0] && EqQ[g\*h\*(n + 1) - b\*c\*e\*Log[F], 0] && NeQ[n, -1]

Rubi steps

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + e(-1 + bcx \log(F)) \log(dx))}{x^2} dx = \frac{eF^{c(a+bx)} \log^{1+n}(dx)}{x}$$

Mathematica [A]

time = 0.11, size = 23, normalized size = 1.05

$$\frac{eF^{ac+bcx} \log^{1+n}(dx)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(F^(c\*(a + b\*x))\*Log[d\*x]^n\*(e + e\*n + e\*(-1 + b\*c\*x\*Log[F])\*Log[d\*x]))/x^2,x]

[Out]  $(e^c F^{a+c} + b^c x) \text{Log}[d^c x]^{(1+n)}/x$

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.  
time = 0.13, size = 136, normalized size = 6.18

method	result
risch	$\frac{F^{c(bx+a)} e^{(2 \ln(d)+2 \ln(x)-i\pi \operatorname{csgn}(id)\operatorname{csgn}(ix)\operatorname{csgn}(idx)+i\pi \operatorname{csgn}(id)\operatorname{csgn}(idx)^2+i\pi \operatorname{csgn}(ix)\operatorname{csgn}(idx)^2-i\pi \operatorname{csgn}(idx)^3)} (\ln(d)+\ln(x))}{2x}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a))*ln(d*x)^n*(e+e*n+e*(-1+b*c*x*ln(F))*ln(d*x))/x^2,x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{1}{2} F^{c(bx+a)} e^{(2 \ln(d)+2 \ln(x)-I \pi \operatorname{csgn}(I d) \operatorname{csgn}(I x) \operatorname{csgn}(I d x)+I \pi \operatorname{csgn}(I d) \operatorname{csgn}(I d x)^2+I \pi \operatorname{csgn}(I x) \operatorname{csgn}(I d x)^2-I \pi \operatorname{csgn}(I d x)^3)} (\ln(d)+\ln(x)-1/2 I \pi \operatorname{csgn}(I d x) (-\operatorname{csgn}(I d x)+\operatorname{csgn}(I d)) (-\operatorname{csgn}(I d x)+\operatorname{csgn}(I x)))^n$$

**Maxima [A]**

time = 0.35, size = 41, normalized size = 1.86

$$\frac{(F^{ac} e \log(d) + F^{ac} e \log(x)) e^{(bcx \log(F) + n \log(\log(d) + \log(x)))}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*log(d*x)^n*(e+e*n+e*(-1+b*c*x*log(F))*log(d*x))/x^2,x,algorithm="maxima")`

[Out]  $(F^{a+c} e \log(d) + F^{a+c} e \log(x)) e^{(b^c x \log(F) + n \log(\log(d) + \log(x)))}/x$

**Fricas [A]**

time = 0.46, size = 26, normalized size = 1.18

$$\frac{F^{bcx+ac} \log(dx)^n e \log(dx)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*log(d*x)^n*(e+e*n+e*(-1+b*c*x*log(F))*log(d*x))/x^2,x,algorithm="fricas")`

[Out]  $F^{(b^c x + a^c)} \log(d^c x)^n e \log(d^c x)/x$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$e \left( \int \frac{F^{ac} F^{bcx} \log(dx)^n}{x^2} dx + \int \frac{F^{ac} F^{bcx} n \log(dx)^n}{x^2} dx + \int \left( -\frac{F^{ac} F^{bcx} \log(dx) \log(dx)^n}{x^2} \right) dx + \int \frac{F^{ac} F^{bcx} bc \log(F) \log(dx) \log(dx)^n}{x} dx \right)$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*ln(d*x)**n*(e+e*n+e*(-1+b*c*x*ln(F))*ln(d*x))/x**2, x)
```

```
[Out] e*(Integral(F**(a*c)*F**(b*c*x)*log(d*x)**n/x**2, x) + Integral(F**(a*c)*F**(b*c*x)*n*log(d*x)**n/x**2, x) + Integral(-F**(a*c)*F**(b*c*x)*log(d*x)*log(d*x)**n/x**2, x) + Integral(F**(a*c)*F**(b*c*x)*b*c*log(F)*log(d*x)*log(d*x)**n/x, x))
```

**Giac** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*log(d*x)^n*(e+e*n+e*(-1+b*c*x*log(F))*log(d*x))/x^2, x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command: INPUT:sage2OUTPUT:Warning, infinity is unsigned, perhaps you meant +infinityWarning, infinity is unsigned, perhaps you meant +infinityUnable to divide, perhaps
```

**Mupad** [B]

time = 3.57, size = 23, normalized size = 1.05

$$\frac{F^{ac+bcx} e \ln(dx)^{n+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((F^(c*(a + b*x))*log(d*x)^n*(e + e*n + e*log(d*x)*(b*c*x*log(F) - 1)))/x^2, x)
```

```
[Out] (F^(a*c + b*c*x)*e*log(d*x)^(n + 1))/x
```

$$3.89 \quad \int \frac{F^{c(a+bx)} \log^n(dx) (e + en + e(-2 + bcx \log(F)) \log(dx))}{x^3} dx$$

Optimal. Leaf size=22

$$\frac{eF^{c(a+bx)} \log^{1+n}(dx)}{x^2}$$

[Out] e\*F^(c\*(b\*x+a))\*ln(d\*x)^(1+n)/x^2

Rubi [A]

time = 0.09, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {2233}

$$\frac{e \log^{n+1}(dx) F^{c(a+bx)}}{x^2}$$

Antiderivative was successfully verified.

[In] Int[(F^(c\*(a + b\*x))\*Log[d\*x]^n\*(e + e\*n + e\*(-2 + b\*c\*x\*Log[F])\*Log[d\*x]))/x^3,x]

[Out] (e\*F^(c\*(a + b\*x))\*Log[d\*x]^(1 + n))/x^2

Rule 2233

Int[Log[(d\_.)\*(x\_)]^(n\_.)\*(F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_)))\*(x\_)^(m\_.)\*((e\_ + Log[(d\_.)\*(x\_)]\*(h\_.)\*((f\_.) + (g\_.)\*(x\_))), x\_Symbol] :> Simp[e\*x^(m + 1)\*F^(c\*(a + b\*x))\*(Log[d\*x]^(n + 1)/(n + 1)), x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e\*(m + 1) - f\*h\*(n + 1), 0] && EqQ[g\*h\*(n + 1) - b\*c\*e\*Log[F], 0] && NeQ[n, -1]

Rubi steps

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + e(-2 + bcx \log(F)) \log(dx))}{x^3} dx = \frac{eF^{c(a+bx)} \log^{1+n}(dx)}{x^2}$$

Mathematica [A]

time = 0.11, size = 23, normalized size = 1.05

$$\frac{eF^{ac+bcx} \log^{1+n}(dx)}{x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(F^(c\*(a + b\*x))\*Log[d\*x]^n\*(e + e\*n + e\*(-2 + b\*c\*x\*Log[F])\*Log[d\*x]))/x^3,x]

[Out]  $(e \cdot F^{(a \cdot c + b \cdot c \cdot x)} \cdot \text{Log}[d \cdot x]^{(1 + n)}) / x^2$

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.  
time = 0.13, size = 136, normalized size = 6.18

method	result
risch	$\frac{F^{c(bx+a)} e^{(2 \ln(d) + 2 \ln(x) - i\pi \operatorname{csgn}(id) \operatorname{csgn}(ix) \operatorname{csgn(idx) + i\pi \operatorname{csgn}(id) \operatorname{csgn(idx)}^2 + i\pi \operatorname{csgn}(ix) \operatorname{csgn(idx)}^2 - i\pi \operatorname{csgn(idx)}^3)} (\ln(d) + \ln(x))}{2x^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F^(c*(b*x+a))*ln(d*x)^n*(e+e*n+e*(-2+b*c*x*ln(F))*ln(d*x))/x^3,x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{1}{2} F^{c(bx+a)} e^{(2 \ln(d) + 2 \ln(x) - i\pi \operatorname{csgn}(I \cdot d) \operatorname{csgn}(I \cdot x) \operatorname{csgn}(I \cdot d \cdot x) + i\pi \operatorname{csgn}(I \cdot d) \operatorname{csgn}(I \cdot d \cdot x)^2 + i\pi \operatorname{csgn}(I \cdot x) \operatorname{csgn}(I \cdot d \cdot x)^2 - i\pi \operatorname{csgn}(I \cdot d \cdot x)^3)} (\ln(d) + \ln(x) - 1/2 i\pi \operatorname{csgn}(I \cdot d \cdot x) (-\operatorname{csgn}(I \cdot d \cdot x) + \operatorname{csgn}(I \cdot d)) (-\operatorname{csgn}(I \cdot d \cdot x) + \operatorname{csgn}(I \cdot x)))^n$$

**Maxima [A]**

time = 0.36, size = 41, normalized size = 1.86

$$\frac{(F^{ac} e \log(d) + F^{ac} e \log(x)) e^{(bcx \log(F) + n \log(\log(d) + \log(x)))}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*log(d*x)^n*(e+e*n+e*(-2+b*c*x*log(F))*log(d*x))/x^3,x,algorithm="maxima")`

[Out] 
$$(F^{(a \cdot c)} \cdot e \cdot \log(d) + F^{(a \cdot c)} \cdot e \cdot \log(x)) \cdot e^{(b \cdot c \cdot x \cdot \log(F) + n \cdot \log(\log(d) + \log(x)))} / x^2$$

**Fricas [A]**

time = 0.39, size = 26, normalized size = 1.18

$$\frac{F^{bcx+ac} \log(dx)^n e \log(dx)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F^(c*(b*x+a))*log(d*x)^n*(e+e*n+e*(-2+b*c*x*log(F))*log(d*x))/x^3,x,algorithm="fricas")`

[Out] 
$$F^{(b \cdot c \cdot x + a \cdot c)} \cdot \log(d \cdot x)^n \cdot e \cdot \log(d \cdot x) / x^2$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$e \left( \int \frac{F^{ac} F^{bcx} \log(dx)^n}{x^3} dx + \int \frac{F^{ac} F^{bcx} n \log(dx)^n}{x^3} dx + \int \left( -\frac{2F^{ac} F^{bcx} \log(dx) \log(dx)^n}{x^3} \right) dx + \int \frac{F^{ac} F^{bcx} bc \log(F) \log(dx) \log(dx)^n}{x^2} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*ln(d*x)**n*(e+e*n+e*(-2+b*c*x*ln(F))*ln(d*x))/x**3, x)
```

```
[Out] e*(Integral(F**(a*c)*F**(b*c*x)*log(d*x)**n/x**3, x) + Integral(F**(a*c)*F*(b*c*x)*n*log(d*x)**n/x**3, x) + Integral(-2*F**(a*c)*F**(b*c*x)*log(d*x)*log(d*x)**n/x**3, x) + Integral(F**(a*c)*F**(b*c*x)*b*c*log(F)*log(d*x)*log(d*x)**n/x**2, x))
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*log(d*x)^n*(e+e*n+e*(-2+b*c*x*log(F))*log(d*x))/x^3, x, algorithm="giac")
```

```
[Out] integrate((((b*c*x*log(F) - 2)*e*log(d*x) + n*e + e)*F^((b*x + a)*c)*log(d*x)^n/x^3, x)
```

**Mupad [B]**

time = 3.58, size = 23, normalized size = 1.05

$$\frac{F^{ac+bcx} e \ln(dx)^{n+1}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((F^(c*(a + b*x))*log(d*x)^n*(e + e*n + e*log(d*x)*(b*c*x*log(F) - 2)))/x^3, x)
```

```
[Out] (F^(a*c + b*c*x)*e*log(d*x)^(n + 1))/x^2
```

### 3.90 $\int \sqrt{e^{a+bx}} x^4 dx$

**Optimal.** Leaf size=91

$$\frac{768\sqrt{e^{a+bx}}}{b^5} - \frac{384\sqrt{e^{a+bx}}x}{b^4} + \frac{96\sqrt{e^{a+bx}}x^2}{b^3} - \frac{16\sqrt{e^{a+bx}}x^3}{b^2} + \frac{2\sqrt{e^{a+bx}}x^4}{b}$$

[Out]  $768*\exp(b*x+a)^{(1/2)}/b^5-384*x*\exp(b*x+a)^{(1/2)}/b^4+96*x^2*\exp(b*x+a)^{(1/2)}/b^3-16*x^3*\exp(b*x+a)^{(1/2)}/b^2+2*x^4*\exp(b*x+a)^{(1/2)}/b$

**Rubi [A]**

time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2207, 2225}

$$\frac{768\sqrt{e^{a+bx}}}{b^5} - \frac{384x\sqrt{e^{a+bx}}}{b^4} + \frac{96x^2\sqrt{e^{a+bx}}}{b^3} - \frac{16x^3\sqrt{e^{a+bx}}}{b^2} + \frac{2x^4\sqrt{e^{a+bx}}}{b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[E^(a + b\*x)]\*x^4,x]

[Out]  $(768*\text{Sqrt}[E^{(a + b*x)}])/b^5 - (384*\text{Sqrt}[E^{(a + b*x)}]*x)/b^4 + (96*\text{Sqrt}[E^{(a + b*x)}]*x^2)/b^3 - (16*\text{Sqrt}[E^{(a + b*x)}]*x^3)/b^2 + (2*\text{Sqrt}[E^{(a + b*x)}]*x^4)/b$

Rule 2207

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(c + d\*x)^m\*((b\*F^(g\*(e + f\*x)))^n/(f\*g\*n\*Log[F])), x] - Dist[d\*(m/(f\*g\*n\*Log[F])), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

Rule 2225

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned}
\int \sqrt{e^{a+bx}} x^4 dx &= \frac{2\sqrt{e^{a+bx}} x^4}{b} - \frac{8 \int \sqrt{e^{a+bx}} x^3 dx}{b} \\
&= -\frac{16\sqrt{e^{a+bx}} x^3}{b^2} + \frac{2\sqrt{e^{a+bx}} x^4}{b} + \frac{48 \int \sqrt{e^{a+bx}} x^2 dx}{b^2} \\
&= \frac{96\sqrt{e^{a+bx}} x^2}{b^3} - \frac{16\sqrt{e^{a+bx}} x^3}{b^2} + \frac{2\sqrt{e^{a+bx}} x^4}{b} - \frac{192 \int \sqrt{e^{a+bx}} x dx}{b^3} \\
&= -\frac{384\sqrt{e^{a+bx}} x}{b^4} + \frac{96\sqrt{e^{a+bx}} x^2}{b^3} - \frac{16\sqrt{e^{a+bx}} x^3}{b^2} + \frac{2\sqrt{e^{a+bx}} x^4}{b} + \frac{384 \int \sqrt{e^{a+bx}} dx}{b^4} \\
&= \frac{768\sqrt{e^{a+bx}}}{b^5} - \frac{384\sqrt{e^{a+bx}} x}{b^4} + \frac{96\sqrt{e^{a+bx}} x^2}{b^3} - \frac{16\sqrt{e^{a+bx}} x^3}{b^2} + \frac{2\sqrt{e^{a+bx}} x^4}{b}
\end{aligned}$$

**Mathematica [A]**

time = 0.05, size = 45, normalized size = 0.49

$$\frac{2\sqrt{e^{a+bx}} (384 - 192bx + 48b^2x^2 - 8b^3x^3 + b^4x^4)}{b^5}$$

Antiderivative was successfully verified.

`[In] Integrate[Sqrt[E^(a + b*x)]*x^4,x]``[Out] (2*Sqrt[E^(a + b*x)]*(384 - 192*b*x + 48*b^2*x^2 - 8*b^3*x^3 + b^4*x^4))/b^5`**Maple [A]**

time = 0.04, size = 43, normalized size = 0.47

method	result	size
gospers	$\frac{2(b^4x^4 - 8b^3x^3 + 48b^2x^2 - 192bx + 384)\sqrt{e^{bx+a}}}{b^5}$	43
risch	$\frac{2(b^4x^4 - 8b^3x^3 + 48b^2x^2 - 192bx + 384)\sqrt{e^{bx+a}}}{b^5}$	43
meijerg	$-\frac{32e^{-\frac{5a}{2} - \frac{bx}{2}} \sqrt{e^{bx+a}} \left( 24 - \frac{\left( \frac{5b^4x^4 e^{2a}}{16} - \frac{5b^3x^3 e^{\frac{3a}{2}}}{2} + 15b^2x^2 e^a - 60bx e^{\frac{a}{2}} + 120 \right) e^{\frac{bx}{2}}}{5}}{b^5} \right)}{b^5}$	84

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^4*exp(b*x+a)^(1/2),x,method=_RETURNVERBOSE)``[Out] 2*(b^4*x^4-8*b^3*x^3+48*b^2*x^2-192*b*x+384)*exp(b*x+a)^(1/2)/b^5`

**Maxima [A]**

time = 0.30, size = 60, normalized size = 0.66

$$\frac{2 \left( b^4 x^4 e^{\left(\frac{1}{2} a\right)} - 8 b^3 x^3 e^{\left(\frac{1}{2} a\right)} + 48 b^2 x^2 e^{\left(\frac{1}{2} a\right)} - 192 b x e^{\left(\frac{1}{2} a\right)} + 384 e^{\left(\frac{1}{2} a\right)} \right) e^{\left(\frac{1}{2} b x\right)}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^4*exp(b*x+a)^(1/2),x, algorithm="maxima")`

```
[Out] 2*(b^4*x^4*e^(1/2*a) - 8*b^3*x^3*e^(1/2*a) + 48*b^2*x^2*e^(1/2*a) - 192*b*x
*e^(1/2*a) + 384*e^(1/2*a))*e^(1/2*b*x)/b^5
```

**Fricas [A]**

time = 0.41, size = 43, normalized size = 0.47

$$\frac{2 \left( b^4 x^4 - 8 b^3 x^3 + 48 b^2 x^2 - 192 b x + 384 \right) e^{\left(\frac{1}{2} b x + \frac{1}{2} a\right)}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^4*exp(b*x+a)^(1/2),x, algorithm="fricas")`

```
[Out] 2*(b^4*x^4 - 8*b^3*x^3 + 48*b^2*x^2 - 192*b*x + 384)*e^(1/2*b*x + 1/2*a)/b^5
```

**Sympy [A]**

time = 0.05, size = 51, normalized size = 0.56

$$\begin{cases} \frac{(2b^4x^4 - 16b^3x^3 + 96b^2x^2 - 384bx + 768)\sqrt{e^{a+bx}}}{b^5} & \text{for } b^5 \neq 0 \\ \frac{x^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x**4*exp(b*x+a)**(1/2),x)`

```
[Out] Piecewise(((2*b**4*x**4 - 16*b**3*x**3 + 96*b**2*x**2 - 384*b*x + 768)*sqrt
(exp(a + b*x))/b**5, Ne(b**5, 0)), (x**5/5, True))
```

**Giac [A]**

time = 1.76, size = 43, normalized size = 0.47

$$\frac{2 \left( b^4 x^4 - 8 b^3 x^3 + 48 b^2 x^2 - 192 b x + 384 \right) e^{\left(\frac{1}{2} b x + \frac{1}{2} a\right)}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^4*exp(b*x+a)^(1/2),x, algorithm="giac")`

[Out]  $2*(b^4*x^4 - 8*b^3*x^3 + 48*b^2*x^2 - 192*b*x + 384)*e^{(1/2*b*x + 1/2*a)}/b^5$

**Mupad [B]**

time = 0.13, size = 45, normalized size = 0.49

$$\sqrt{e^{a+bx}} \left( \frac{768}{b^5} - \frac{384x}{b^4} + \frac{2x^4}{b} - \frac{16x^3}{b^2} + \frac{96x^2}{b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*exp(a + b*x)^(1/2),x)`

[Out]  $\exp(a + b*x)^{(1/2)}*(768/b^5 - (384*x)/b^4 + (2*x^4)/b - (16*x^3)/b^2 + (96*x^2)/b^3)$



### 3.91 $\int \sqrt{e^{a+bx}} x^3 dx$

Optimal. Leaf size=72

$$-\frac{96\sqrt{e^{a+bx}}}{b^4} + \frac{48\sqrt{e^{a+bx}} x}{b^3} - \frac{12\sqrt{e^{a+bx}} x^2}{b^2} + \frac{2\sqrt{e^{a+bx}} x^3}{b}$$

[Out]  $-96*\exp(b*x+a)^{(1/2)}/b^4+48*x*\exp(b*x+a)^{(1/2)}/b^3-12*x^2*\exp(b*x+a)^{(1/2)}/b^2+2*x^3*\exp(b*x+a)^{(1/2)}/b$

**Rubi** [A]

time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2207, 2225}

$$-\frac{96\sqrt{e^{a+bx}}}{b^4} + \frac{48x\sqrt{e^{a+bx}}}{b^3} - \frac{12x^2\sqrt{e^{a+bx}}}{b^2} + \frac{2x^3\sqrt{e^{a+bx}}}{b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[E^(a + b\*x)]\*x^3,x]

[Out]  $(-96*\text{Sqrt}[E^{(a + b*x)}])/b^4 + (48*\text{Sqrt}[E^{(a + b*x)}]*x)/b^3 - (12*\text{Sqrt}[E^{(a + b*x)}]*x^2)/b^2 + (2*\text{Sqrt}[E^{(a + b*x)}]*x^3)/b$

Rule 2207

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(c + d\*x)^m\*((b\*F^(g\*(e + f\*x)))^n/(f\*g\*n\*Log[F])), x] - Dist[d\*(m/(f\*g\*n\*Log[F])), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

Rule 2225

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned}
\int \sqrt{e^{a+bx}} x^3 dx &= \frac{2\sqrt{e^{a+bx}} x^3}{b} - \frac{6 \int \sqrt{e^{a+bx}} x^2 dx}{b} \\
&= -\frac{12\sqrt{e^{a+bx}} x^2}{b^2} + \frac{2\sqrt{e^{a+bx}} x^3}{b} + \frac{24 \int \sqrt{e^{a+bx}} x dx}{b^2} \\
&= \frac{48\sqrt{e^{a+bx}} x}{b^3} - \frac{12\sqrt{e^{a+bx}} x^2}{b^2} + \frac{2\sqrt{e^{a+bx}} x^3}{b} - \frac{48 \int \sqrt{e^{a+bx}} dx}{b^3} \\
&= -\frac{96\sqrt{e^{a+bx}}}{b^4} + \frac{48\sqrt{e^{a+bx}} x}{b^3} - \frac{12\sqrt{e^{a+bx}} x^2}{b^2} + \frac{2\sqrt{e^{a+bx}} x^3}{b}
\end{aligned}$$

**Mathematica [A]**

time = 0.04, size = 37, normalized size = 0.51

$$\frac{2\sqrt{e^{a+bx}} (-48 + 24bx - 6b^2x^2 + b^3x^3)}{b^4}$$

Antiderivative was successfully verified.

`[In] Integrate[Sqrt[E^(a + b*x)]*x^3,x]``[Out] (2*Sqrt[E^(a + b*x)]*(-48 + 24*b*x - 6*b^2*x^2 + b^3*x^3))/b^4`**Maple [A]**

time = 0.01, size = 35, normalized size = 0.49

method	result	size
gospers	$\frac{2(b^3x^3 - 6b^2x^2 + 24bx - 48)\sqrt{e^{bx+a}}}{b^4}$	35
risch	$\frac{2(b^3x^3 - 6b^2x^2 + 24bx - 48)\sqrt{e^{bx+a}}}{b^4}$	35
meijerg	$16 e^{-2a - \frac{bx}{2}} \sqrt{e^{bx+a}} \left( 6 - \frac{\left( -\frac{b^3x^3 e^{\frac{3a}{2}}}{2} + 3b^2x^2 e^a - 12bx e^{\frac{a}{2}} + 24 \right) e^{\frac{bx}{2}}}{4} \right)$	72

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^3*exp(b*x+a)^(1/2),x,method=_RETURNVERBOSE)``[Out] 2*(b^3*x^3-6*b^2*x^2+24*b*x-48)*exp(b*x+a)^(1/2)/b^4`**Maxima [A]**

time = 0.28, size = 48, normalized size = 0.67

$$\frac{2 \left( b^3 x^3 e^{\left(\frac{1}{2} a\right)} - 6 b^2 x^2 e^{\left(\frac{1}{2} a\right)} + 24 b x e^{\left(\frac{1}{2} a\right)} - 48 e^{\left(\frac{1}{2} a\right)} \right) e^{\left(\frac{1}{2} b x\right)}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>3</sup>\*exp(b\*x+a)<sup>(1/2)</sup>,x, algorithm="maxima")

[Out] 2\*(b<sup>3</sup>\*x<sup>3</sup>\*e<sup>(1/2\*a)</sup> - 6\*b<sup>2</sup>\*x<sup>2</sup>\*e<sup>(1/2\*a)</sup> + 24\*b\*x\*e<sup>(1/2\*a)</sup> - 48\*e<sup>(1/2\*a)</sup>)\*e<sup>(1/2\*b\*x)</sup>/b<sup>4</sup>

**Fricas** [A]

time = 0.49, size = 35, normalized size = 0.49

$$\frac{2(b^3x^3 - 6b^2x^2 + 24bx - 48)e^{\left(\frac{1}{2}bx + \frac{1}{2}a\right)}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>3</sup>\*exp(b\*x+a)<sup>(1/2)</sup>,x, algorithm="fricas")

[Out] 2\*(b<sup>3</sup>\*x<sup>3</sup> - 6\*b<sup>2</sup>\*x<sup>2</sup> + 24\*b\*x - 48)\*e<sup>(1/2\*b\*x + 1/2\*a)</sup>/b<sup>4</sup>

**Sympy** [A]

time = 0.04, size = 42, normalized size = 0.58

$$\begin{cases} \frac{(2b^3x^3 - 12b^2x^2 + 48bx - 96)\sqrt{e^{a+bx}}}{b^4} & \text{for } b^4 \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*exp(b\*x+a)\*\*(1/2),x)

[Out] Piecewise(((2\*b\*\*3\*x\*\*3 - 12\*b\*\*2\*x\*\*2 + 48\*b\*x - 96)\*sqrt(exp(a + b\*x))/b\*\*4, Ne(b\*\*4, 0)), (x\*\*4/4, True))

**Giac** [A]

time = 1.86, size = 35, normalized size = 0.49

$$\frac{2(b^3x^3 - 6b^2x^2 + 24bx - 48)e^{\left(\frac{1}{2}bx + \frac{1}{2}a\right)}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>3</sup>\*exp(b\*x+a)<sup>(1/2)</sup>,x, algorithm="giac")

[Out] 2\*(b<sup>3</sup>\*x<sup>3</sup> - 6\*b<sup>2</sup>\*x<sup>2</sup> + 24\*b\*x - 48)\*e<sup>(1/2\*b\*x + 1/2\*a)</sup>/b<sup>4</sup>

**Mupad** [B]

time = 0.05, size = 37, normalized size = 0.51

$$\sqrt{e^{a+bx}} \left( \frac{48x}{b^3} - \frac{96}{b^4} + \frac{2x^3}{b} - \frac{12x^2}{b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>3</sup>\*exp(a + b\*x)<sup>(1/2)</sup>,x)

[Out] exp(a + b\*x)<sup>(1/2)</sup>\*((48\*x)/b<sup>3</sup> - 96/b<sup>4</sup> + (2\*x<sup>3</sup>)/b - (12\*x<sup>2</sup>)/b<sup>2</sup>)

### 3.92 $\int \sqrt{e^{a+bx}} x^2 dx$

Optimal. Leaf size=53

$$\frac{16\sqrt{e^{a+bx}}}{b^3} - \frac{8\sqrt{e^{a+bx}} x}{b^2} + \frac{2\sqrt{e^{a+bx}} x^2}{b}$$

[Out]  $16*\exp(b*x+a)^{(1/2)}/b^3-8*x*\exp(b*x+a)^{(1/2)}/b^2+2*x^2*\exp(b*x+a)^{(1/2)}/b$

Rubi [A]

time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2207, 2225}

$$\frac{16\sqrt{e^{a+bx}}}{b^3} - \frac{8x\sqrt{e^{a+bx}}}{b^2} + \frac{2x^2\sqrt{e^{a+bx}}}{b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[E^(a + b\*x)]\*x^2,x]

[Out]  $(16*\text{Sqrt}[E^{(a + b*x)}])/b^3 - (8*\text{Sqrt}[E^{(a + b*x)}]*x)/b^2 + (2*\text{Sqrt}[E^{(a + b*x)}]*x^2)/b$

Rule 2207

Int[((b\_)\*(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[(c + d\*x)^m\*((b\*F^(g\*(e + f\*x)))^n/(f\*g\*n\*Log[F])), x] - Dist[d\*(m/(f\*g\*n\*Log[F])), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

Rule 2225

Int[((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(n\_), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned} \int \sqrt{e^{a+bx}} x^2 dx &= \frac{2\sqrt{e^{a+bx}} x^2}{b} - \frac{4 \int \sqrt{e^{a+bx}} x dx}{b} \\ &= -\frac{8\sqrt{e^{a+bx}} x}{b^2} + \frac{2\sqrt{e^{a+bx}} x^2}{b} + \frac{8 \int \sqrt{e^{a+bx}} dx}{b^2} \\ &= \frac{16\sqrt{e^{a+bx}}}{b^3} - \frac{8\sqrt{e^{a+bx}} x}{b^2} + \frac{2\sqrt{e^{a+bx}} x^2}{b} \end{aligned}$$

**Mathematica [A]**

time = 0.04, size = 29, normalized size = 0.55

$$\frac{2\sqrt{e^{a+bx}}(8-4bx+b^2x^2)}{b^3}$$

Antiderivative was successfully verified.

`[In] Integrate[Sqrt[E^(a + b*x)]*x^2,x]``[Out] (2*Sqrt[E^(a + b*x)]*(8 - 4*b*x + b^2*x^2))/b^3`**Maple [A]**

time = 0.01, size = 27, normalized size = 0.51

method	result	size
gospers	$\frac{2(b^2x^2-4bx+8)\sqrt{e^{bx+a}}}{b^3}$	27
risch	$\frac{2(b^2x^2-4bx+8)\sqrt{e^{bx+a}}}{b^3}$	27
meijerg	$\frac{8e^{-\frac{3a}{2}-\frac{bx}{2}\frac{a}{2}}\sqrt{e^{bx+a}}\left(2-\frac{\left(\frac{3b^2x^2e^a}{4}-3bx\frac{a}{2}+6\right)e^{\frac{bx}{2}\frac{a}{2}}}{3}\right)}{b^3}$	60

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2*exp(b*x+a)^(1/2),x,method=_RETURNVERBOSE)``[Out] 2*(b^2*x^2-4*b*x+8)*exp(b*x+a)^(1/2)/b^3`**Maxima [A]**

time = 0.28, size = 36, normalized size = 0.68

$$\frac{2\left(b^2x^2e^{\left(\frac{1}{2}a\right)}-4bx e^{\left(\frac{1}{2}a\right)}+8e^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}bx\right)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2*exp(b*x+a)^(1/2),x, algorithm="maxima")``[Out] 2*(b^2*x^2*e^(1/2*a) - 4*b*x*e^(1/2*a) + 8*e^(1/2*a))*e^(1/2*b*x)/b^3`**Fricas [A]**

time = 0.36, size = 27, normalized size = 0.51

$$\frac{2(b^2x^2-4bx+8)e^{\left(\frac{1}{2}bx+\frac{1}{2}a\right)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*exp(b\*x+a)^(1/2),x, algorithm="fricas")

[Out] 2\*(b^2\*x^2 - 4\*b\*x + 8)\*e^(1/2\*b\*x + 1/2\*a)/b^3

**Sympy [A]**

time = 0.04, size = 34, normalized size = 0.64

$$\begin{cases} \frac{(2b^2x^2-8bx+16)\sqrt{e^{a+bx}}}{b^3} & \text{for } b^3 \neq 0 \\ \frac{x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*exp(b\*x+a)\*\*(1/2),x)

[Out] Piecewise(((2\*b\*\*2\*x\*\*2 - 8\*b\*x + 16)\*sqrt(exp(a + b\*x))/b\*\*3, Ne(b\*\*3, 0)), (x\*\*3/3, True))

**Giac [A]**

time = 1.88, size = 27, normalized size = 0.51

$$\frac{2(b^2x^2 - 4bx + 8)e^{(\frac{1}{2}bx + \frac{1}{2}a)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*exp(b\*x+a)^(1/2),x, algorithm="giac")

[Out] 2\*(b^2\*x^2 - 4\*b\*x + 8)\*e^(1/2\*b\*x + 1/2\*a)/b^3

**Mupad [B]**

time = 0.06, size = 29, normalized size = 0.55

$$\sqrt{e^{a+bx}} \left( \frac{16}{b^3} - \frac{8x}{b^2} + \frac{2x^2}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*exp(a + b\*x)^(1/2),x)

[Out] exp(a + b\*x)^(1/2)\*(16/b^3 - (8\*x)/b^2 + (2\*x^2)/b)

### 3.93 $\int \sqrt{e^{a+bx}} x dx$

**Optimal.** Leaf size=34

$$-\frac{4\sqrt{e^{a+bx}}}{b^2} + \frac{2\sqrt{e^{a+bx}} x}{b}$$

[Out]  $-4*\exp(b*x+a)^{(1/2)}/b^2+2*x*\exp(b*x+a)^{(1/2)}/b$

**Rubi [A]**

time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2207, 2225}

$$\frac{2x\sqrt{e^{a+bx}}}{b} - \frac{4\sqrt{e^{a+bx}}}{b^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[E^(a + b\*x)]\*x,x]

[Out]  $(-4*\text{Sqrt}[E^{(a + b*x)}])/b^2 + (2*\text{Sqrt}[E^{(a + b*x)}]*x)/b$

Rule 2207

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(c + d\*x)^m\*((b\*F^(g\*(e + f\*x)))^n/(f\*g\*n\*Log[F])), x] - Dist[d\*(m/(f\*g\*n\*Log[F])), Int[(c + d\*x)^(m - 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

Rule 2225

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned} \int \sqrt{e^{a+bx}} x dx &= \frac{2\sqrt{e^{a+bx}} x}{b} - \frac{2 \int \sqrt{e^{a+bx}} dx}{b} \\ &= -\frac{4\sqrt{e^{a+bx}}}{b^2} + \frac{2\sqrt{e^{a+bx}} x}{b} \end{aligned}$$

**Mathematica [A]**

time = 0.03, size = 21, normalized size = 0.62

$$\frac{2\sqrt{e^{a+bx}}(-2 + bx)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[E^(a + b\*x)]\*x,x]

[Out] (2\*Sqrt[E^(a + b\*x)]\*(-2 + b\*x))/b^2

**Maple [A]**

time = 0.01, size = 19, normalized size = 0.56

method	result	size
gosper	$\frac{2(bx-2)\sqrt{e^{bx+a}}}{b^2}$	19
risch	$\frac{2(bx-2)\sqrt{e^{bx+a}}}{b^2}$	19
meijerg	$\frac{4\sqrt{e^{bx+a}} e^{-a-\frac{bx}{2}} \left(1 - \frac{(-bx e^{\frac{a}{2}} + 2) e^{\frac{bx}{2}}}{2}\right)}{b^2}$	50

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*exp(b\*x+a)^(1/2),x,method=\_RETURNVERBOSE)

[Out] 2\*(b\*x-2)\*exp(b\*x+a)^(1/2)/b^2

**Maxima [A]**

time = 0.27, size = 24, normalized size = 0.71

$$\frac{2 \left( bxe^{\left(\frac{1}{2}a\right)} - 2e^{\left(\frac{1}{2}a\right)} \right) e^{\left(\frac{1}{2}bx\right)}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*exp(b\*x+a)^(1/2),x, algorithm="maxima")

[Out] 2\*(b\*x\*e^(1/2\*a) - 2\*e^(1/2\*a))\*e^(1/2\*b\*x)/b^2

**Fricas [A]**

time = 0.34, size = 19, normalized size = 0.56

$$\frac{2(bx-2)e^{\left(\frac{1}{2}bx+\frac{1}{2}a\right)}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*exp(b\*x+a)^(1/2),x, algorithm="fricas")

[Out] 2\*(b\*x - 2)\*e^(1/2\*b\*x + 1/2\*a)/b^2



**Sympy [A]**

time = 0.04, size = 26, normalized size = 0.76

$$\begin{cases} \frac{(2bx-4)\sqrt{e^{a+bx}}}{b^2} & \text{for } b^2 \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*exp(b*x+a)**(1/2),x)``[Out] Piecewise(((2*b*x - 4)*sqrt(exp(a + b*x))/b**2, Ne(b**2, 0)), (x**2/2, True))`**Giac [A]**

time = 1.99, size = 19, normalized size = 0.56

$$\frac{2(bx - 2)e^{(\frac{1}{2}bx + \frac{1}{2}a)}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*exp(b*x+a)^(1/2),x, algorithm="giac")``[Out] 2*(b*x - 2)*e^(1/2*b*x + 1/2*a)/b^2`**Mupad [B]**

time = 0.04, size = 18, normalized size = 0.53

$$\frac{2\sqrt{e^{a+bx}}(bx - 2)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*exp(a + b*x)^(1/2),x)``[Out] (2*exp(a + b*x)^(1/2)*(b*x - 2))/b^2`

### 3.94 $\int \sqrt{e^{a+bx}} dx$

Optimal. Leaf size=16

$$\frac{2\sqrt{e^{a+bx}}}{b}$$

[Out]  $2*\exp(b*x+a)^{(1/2)}/b$

Rubi [A]

time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2225}

$$\frac{2\sqrt{e^{a+bx}}}{b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[E^(a + b\*x)],x]

[Out] (2\*Sqrt[E^(a + b\*x)])/b

Rule 2225

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\int \sqrt{e^{a+bx}} dx = \frac{2\sqrt{e^{a+bx}}}{b}$$

Mathematica [A]

time = 0.02, size = 16, normalized size = 1.00

$$\frac{2\sqrt{e^{a+bx}}}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[E^(a + b\*x)],x]

[Out] (2\*Sqrt[E^(a + b\*x)])/b

Maple [A]

time = 0.01, size = 14, normalized size = 0.88

method	result	size
gospers	$\frac{2\sqrt{e^{bx+a}}}{b}$	14
derivativdivides	$\frac{2\sqrt{e^{bx+a}}}{b}$	14
default	$\frac{2\sqrt{e^{bx+a}}}{b}$	14
risch	$\frac{2\sqrt{e^{bx+a}}}{b}$	14
meijerg	$-\frac{2\sqrt{e^{bx+a}} e^{-\frac{a}{2} - \frac{bx}{2}} \left(1 - e^{\frac{bx}{2}}\right)}{b}$	40

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(b*x+a)^(1/2),x,method=_RETURNVERBOSE)`

[Out]  $2*\exp(b*x+a)^{(1/2)}/b$

**Maxima** [A]

time = 0.27, size = 14, normalized size = 0.88

$$\frac{2e^{(\frac{1}{2}bx + \frac{1}{2}a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(b*x+a)^(1/2),x, algorithm="maxima")`

[Out]  $2*e^{(1/2*b*x + 1/2*a)}/b$

**Fricas** [A]

time = 0.38, size = 14, normalized size = 0.88

$$\frac{2e^{(\frac{1}{2}bx + \frac{1}{2}a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(b*x+a)^(1/2),x, algorithm="fricas")`

[Out]  $2*e^{(1/2*b*x + 1/2*a)}/b$

**Sympy** [A]

time = 0.03, size = 14, normalized size = 0.88

$$\begin{cases} \frac{2\sqrt{e^{a+bx}}}{b} & \text{for } b \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(b*x+a)**(1/2),x)`

[Out] `Piecewise((2*sqrt(exp(a + b*x))/b, Ne(b, 0)), (x, True))`

**Giac** [A]

time = 1.89, size = 14, normalized size = 0.88

$$\frac{2 e^{(\frac{1}{2}bx + \frac{1}{2}a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(b*x+a)^(1/2),x, algorithm="giac")`

[Out] `2*e^(1/2*b*x + 1/2*a)/b`

**Mupad** [B]

time = 3.39, size = 13, normalized size = 0.81

$$\frac{2 \sqrt{e^{a+bx}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(a + b*x)^(1/2),x)`

[Out] `(2*exp(a + b*x)^(1/2))/b`

$$3.95 \quad \int \frac{\sqrt{e^{a+bx}}}{x} dx$$

Optimal. Leaf size=27

$$e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \operatorname{Ei}\left(\frac{bx}{2}\right)$$

[Out] Ei(1/2\*b\*x)\*exp(b\*x+a)^(1/2)/exp(1/2\*b\*x)

Rubi [A]

time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2213, 2209}

$$e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \operatorname{Ei}\left(\frac{bx}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[E^(a + b\*x)]/x,x]

[Out] (Sqrt[E^(a + b\*x)]\*ExpIntegralEi[(b\*x)/2])/E^((b\*x)/2)

Rule 2209

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - c\*(f/d)))/d)\*ExpIntegralEi[f\*g\*(c + d\*x)\*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

Rule 2213

Int[((b\_)\*(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :> Dist[(b\*F^(g\*(e + f\*x)))^n/F^(g\*n\*(e + f\*x)), Int[(c + d\*x)^m\*F^(g\*n\*(e + f\*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{e^{a+bx}}}{x} dx &= \left( e^{\frac{1}{2}(-a-bx)} \sqrt{e^{a+bx}} \right) \int \frac{e^{\frac{1}{2}(a+bx)}}{x} dx \\ &= e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \operatorname{Ei}\left(\frac{bx}{2}\right) \end{aligned}$$

Mathematica [A]

time = 0.03, size = 27, normalized size = 1.00

$$e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \operatorname{Ei}\left(\frac{bx}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[E^(a + b\*x)]/x,x]

[Out] (Sqrt[E^(a + b\*x)]\*ExpIntegralEi[(b\*x)/2])/E^((b\*x)/2)

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(21) = 42.

time = 0.02, size = 57, normalized size = 2.11

method	result	size
meijerg	$\sqrt{e^{bx+a}} e^{-\frac{bx e^{\frac{a}{2}}}{2}} \left( -\ln\left(-\frac{bx e^{\frac{a}{2}}}{2}\right) - \text{expIntegral}\left(1, -\frac{bx e^{\frac{a}{2}}}{2}\right) + \ln(x) - \ln(2) + \ln(-b e^{\frac{a}{2}})\right)$	57

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(b\*x+a)^(1/2)/x,x,method=\_RETURNVERBOSE)

[Out] exp(b\*x+a)^(1/2)\*exp(-1/2\*b\*x\*exp(1/2\*a))\*(-ln(-1/2\*b\*x\*exp(1/2\*a))-Ei(1,-1/2\*b\*x\*exp(1/2\*a))+ln(x)-ln(2)+ln(-b\*exp(1/2\*a)))

**Maxima [A]**

time = 0.32, size = 10, normalized size = 0.37

$$\text{Ei}\left(\frac{1}{2}bx\right) e^{\left(\frac{1}{2}a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)^(1/2)/x,x, algorithm="maxima")

[Out] Ei(1/2\*b\*x)\*e^(1/2\*a)

**Fricas [A]**

time = 0.38, size = 10, normalized size = 0.37

$$\text{Ei}\left(\frac{1}{2}bx\right) e^{\left(\frac{1}{2}a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)^(1/2)/x,x, algorithm="fricas")

[Out] Ei(1/2\*b\*x)\*e^(1/2\*a)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e^a e^{bx}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(b*x+a)**(1/2)/x,x)`

[Out] `Integral(sqrt(exp(a)*exp(b*x))/x, x)`

**Giac** [A]

time = 2.43, size = 10, normalized size = 0.37

$$\operatorname{Ei}\left(\frac{1}{2}bx\right)e^{\left(\frac{1}{2}a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(b*x+a)^(1/2)/x,x, algorithm="giac")`

[Out] `Ei(1/2*b*x)*e^(1/2*a)`

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e^{a+bx}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(a + b*x)^(1/2)/x,x)`

[Out] `int(exp(a + b*x)^(1/2)/x, x)`

### 3.96

$$\int \frac{\sqrt{e^{a+bx}}}{x^2} dx$$

Optimal. Leaf size=48

$$-\frac{\sqrt{e^{a+bx}}}{x} + \frac{1}{2}be^{-\frac{bx}{2}}\sqrt{e^{a+bx}}\operatorname{Ei}\left(\frac{bx}{2}\right)$$

[Out]  $-\exp(b*x+a)^{(1/2)}/x+1/2*b*Ei(1/2*b*x)*\exp(b*x+a)^{(1/2)}/\exp(1/2*b*x)$

Rubi [A]

time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2208, 2213, 2209}

$$\frac{1}{2}be^{-\frac{bx}{2}}\sqrt{e^{a+bx}}\operatorname{Ei}\left(\frac{bx}{2}\right) - \frac{\sqrt{e^{a+bx}}}{x}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[E^(a + b\*x)]/x^2,x]

[Out]  $-(\operatorname{Sqrt}[E^{(a + b*x)}]/x) + (b*\operatorname{Sqrt}[E^{(a + b*x)}]*\operatorname{ExpIntegralEi}[(b*x)/2])/(2*E^{((b*x)/2)})$

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1))), x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2213

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Dist[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)), Int[(c + d*x)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

Rubi steps



$$\begin{aligned}
\int \frac{\sqrt{e^{a+bx}}}{x^2} dx &= -\frac{\sqrt{e^{a+bx}}}{x} + \frac{1}{2}b \int \frac{\sqrt{e^{a+bx}}}{x} dx \\
&= -\frac{\sqrt{e^{a+bx}}}{x} + \frac{1}{2} \left( b e^{\frac{1}{2}(-a-bx)} \sqrt{e^{a+bx}} \right) \int \frac{e^{\frac{1}{2}(a+bx)}}{x} dx \\
&= -\frac{\sqrt{e^{a+bx}}}{x} + \frac{1}{2} b e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \operatorname{Ei}\left(\frac{bx}{2}\right)
\end{aligned}$$

**Mathematica [A]**

time = 0.04, size = 47, normalized size = 0.98

$$\frac{e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \left( -2e^{\frac{bx}{2}} + bx \operatorname{Ei}\left(\frac{bx}{2}\right) \right)}{2x}$$

Antiderivative was successfully verified.

`[In] Integrate[Sqrt[E^(a + b*x)]/x^2,x]``[Out] (Sqrt[E^(a + b*x)]*(-2*E^((b*x)/2) + b*x*ExpIntegralEi[(b*x)/2]))/(2*E^((b*x)/2)*x)`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(37) = 74.

time = 0.02, size = 116, normalized size = 2.42

method	result
meijerg	$\frac{\sqrt{e^{bx+a}} e^{\frac{a}{2} - \frac{bx}{2}} b \left( -\frac{e^{-\frac{a}{2}} (2+bx e^{\frac{a}{2}})}{bx} + 2e^{-\frac{a}{2} + \frac{bx}{2}} + \ln\left(-\frac{bx e^{\frac{a}{2}}}{2}\right) + \operatorname{expIntegral}\left(1, -\frac{bx e^{\frac{a}{2}}}{2}\right) + 1 - \ln(x) + \ln(2) - \ln(-b e^{\frac{a}{2}}) \right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(exp(b*x+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)`
`[Out] -1/2*exp(b*x+a)^(1/2)*exp(1/2*a-1/2*b*x*exp(1/2*a))*b*(-1/b/x*exp(-1/2*a))*`  
`(2+b*x*exp(1/2*a))+2/b/x*exp(-1/2*a+1/2*b*x*exp(1/2*a))+ln(-1/2*b*x*exp(1/2*`  
`a))+Ei(1,-1/2*b*x*exp(1/2*a))+1-ln(x)+ln(2)-ln(-b*exp(1/2*a))+2/x/b*exp(-1/`  
`2*a))`
**Maxima [A]**

time = 0.32, size = 13, normalized size = 0.27

$$\frac{1}{2} b e^{\left(\frac{1}{2} a\right)} \Gamma\left(-1, -\frac{1}{2} b x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)^(1/2)/x^2,x, algorithm="maxima")

[Out] 1/2\*b\*e^(1/2\*a)\*gamma(-1, -1/2\*b\*x)

**Fricas** [A]

time = 0.38, size = 29, normalized size = 0.60

$$\frac{bx\text{Ei}\left(\frac{1}{2}bx\right)e^{\left(\frac{1}{2}a\right)} - 2e^{\left(\frac{1}{2}bx + \frac{1}{2}a\right)}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)^(1/2)/x^2,x, algorithm="fricas")

[Out] 1/2\*(b\*x\*Ei(1/2\*b\*x)\*e^(1/2\*a) - 2\*e^(1/2\*b\*x + 1/2\*a))/x

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e^a e^{bx}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)\*\*(1/2)/x\*\*2,x)

[Out] Integral(sqrt(exp(a)\*exp(b\*x))/x\*\*2, x)

**Giac** [A]

time = 2.18, size = 29, normalized size = 0.60

$$\frac{bx\text{Ei}\left(\frac{1}{2}bx\right)e^{\left(\frac{1}{2}a\right)} - 2e^{\left(\frac{1}{2}bx + \frac{1}{2}a\right)}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)^(1/2)/x^2,x, algorithm="giac")

[Out] 1/2\*(b\*x\*Ei(1/2\*b\*x)\*e^(1/2\*a) - 2\*e^(1/2\*b\*x + 1/2\*a))/x

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{e^{a+bx}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(a + b\*x)^(1/2)/x^2,x)

[Out] int(exp(a + b\*x)^(1/2)/x^2, x)

$$3.97 \quad \int \frac{\sqrt{e^{a+bx}}}{x^3} dx$$

**Optimal.** Leaf size=71

$$-\frac{\sqrt{e^{a+bx}}}{2x^2} - \frac{b\sqrt{e^{a+bx}}}{4x} + \frac{1}{8}b^2e^{-\frac{bx}{2}}\sqrt{e^{a+bx}}\operatorname{Ei}\left(\frac{bx}{2}\right)$$

[Out]  $-1/2*\exp(b*x+a)^{(1/2)}/x^2-1/4*b*\exp(b*x+a)^{(1/2)}/x+1/8*b^2*Ei(1/2*b*x)*\exp(b*x+a)^{(1/2)}/\exp(1/2*b*x)$

**Rubi [A]**

time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ ,

Rules used = {2208, 2213, 2209}

$$\frac{1}{8}b^2e^{-\frac{bx}{2}}\sqrt{e^{a+bx}}\operatorname{Ei}\left(\frac{bx}{2}\right) - \frac{\sqrt{e^{a+bx}}}{2x^2} - \frac{b\sqrt{e^{a+bx}}}{4x}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[E^{(a + b*x)}]/x^3, x]$

[Out]  $-1/2*\operatorname{Sqrt}[E^{(a + b*x)}]/x^2 - (b*\operatorname{Sqrt}[E^{(a + b*x)}])/(4*x) + (b^2*\operatorname{Sqrt}[E^{(a + b*x)}]*\operatorname{ExpIntegralEi}[(b*x)/2])/(8*E^{((b*x)/2)})$

Rule 2208

$\operatorname{Int}[(b_*)*(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))^{(n_*)*((c_*) + (d_*)*(x_*))^{(m_*)}}, x\_Symbol] :> \operatorname{Simp}[(c + d*x)^{(m + 1)}*((b*F^{(g*(e + f*x)))^n/(d*(m + 1)))], x] - \operatorname{Dist}[f*g*n*(\operatorname{Log}[F]/(d*(m + 1))), \operatorname{Int}[(c + d*x)^{(m + 1)}*(b*F^{(g*(e + f*x)))^n, x], x] /; \operatorname{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{IntegerQ}[2*m] \&\& !\operatorname{TrueQ}[\$UseGamma]$

Rule 2209

$\operatorname{Int}[(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))^{(c_*) + (d_*)*(x_*)}}, x\_Symbol] :> \operatorname{Simp}[(F^{(g*(e - c*(f/d))})/d)*\operatorname{ExpIntegralEi}[f*g*(c + d*x)*(\operatorname{Log}[F]/d)], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\operatorname{TrueQ}[\$UseGamma]$

Rule 2213

$\operatorname{Int}[(b_*)*(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))^{(n_*)*((c_*) + (d_*)*(x_*))^{(m_*)}}, x\_Symbol] :> \operatorname{Dist}[(b*F^{(g*(e + f*x)))^n/F^{(g*n*(e + f*x))}, \operatorname{Int}[(c + d*x)^m*F^{(g*n*(e + f*x))}, x], x] /; \operatorname{FreeQ}\{F, b, c, d, e, f, g, m, n\}, x]$

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{e^{a+bx}}}{x^3} dx &= -\frac{\sqrt{e^{a+bx}}}{2x^2} + \frac{1}{4}b \int \frac{\sqrt{e^{a+bx}}}{x^2} dx \\
&= -\frac{\sqrt{e^{a+bx}}}{2x^2} - \frac{b\sqrt{e^{a+bx}}}{4x} + \frac{1}{8}b^2 \int \frac{\sqrt{e^{a+bx}}}{x} dx \\
&= -\frac{\sqrt{e^{a+bx}}}{2x^2} - \frac{b\sqrt{e^{a+bx}}}{4x} + \frac{1}{8} \left( b^2 e^{\frac{1}{2}(-a-bx)} \sqrt{e^{a+bx}} \right) \int \frac{e^{\frac{1}{2}(a+bx)}}{x} dx \\
&= -\frac{\sqrt{e^{a+bx}}}{2x^2} - \frac{b\sqrt{e^{a+bx}}}{4x} + \frac{1}{8} b^2 e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \operatorname{Ei}\left(\frac{bx}{2}\right)
\end{aligned}$$

**Mathematica [A]**

time = 0.06, size = 56, normalized size = 0.79

$$\frac{e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \left( -2e^{\frac{bx}{2}} (2 + bx) + b^2 x^2 \operatorname{Ei}\left(\frac{bx}{2}\right) \right)}{8x^2}$$

Antiderivative was successfully verified.

`[In] Integrate[Sqrt[E^(a + b*x)]/x^3,x]``[Out] (Sqrt[E^(a + b*x)]*(-2*E^((b*x)/2)*(2 + b*x) + b^2*x^2*ExpIntegralEi[(b*x)/2]))/(8*E^((b*x)/2)*x^2)`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(53) = 106.

time = 0.02, size = 155, normalized size = 2.18

method	result
meijerg	$\frac{\sqrt{e^{bx+a}} e^{a-\frac{bx}{2}} b^2 \left( \frac{e^{-a} \left( \frac{9b^2 x^2 e^a}{4} + 6bx e^{\frac{a}{2}} + 6 \right)}{3b^2 x^2} - \frac{2e^{-a+\frac{bx}{2}} \left( 3 + 3bx e^{\frac{a}{2}} \right)}{3b^2 x^2} - \frac{\ln\left(-\frac{bx}{2} e^{\frac{a}{2}}\right)}{2} - \frac{\operatorname{expIntegral}\left(1, -\frac{bx}{2} e^{\frac{a}{2}}\right)}{2} - \frac{3}{4} + \frac{\ln(x)}{2} - \frac{\ln(2)}{2} \right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(exp(b*x+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)`

```
[Out] 1/4*exp(b*x+a)^(1/2)*exp(a-1/2*b*x*exp(1/2*a))*b^2*(1/3/b^2/x^2*exp(-a)*(9/4*b^2*x^2*exp(a)+6*b*x*exp(1/2*a)+6)-2/3/b^2/x^2*exp(-a+1/2*b*x*exp(1/2*a))*(3+3/2*b*x*exp(1/2*a))-1/2*ln(-1/2*b*x*exp(1/2*a))-1/2*Ei(1,-1/2*b*x*exp(1/2*a))-3/4+1/2*ln(x)-1/2*ln(2)+1/2*ln(-b*exp(1/2*a))-2/x^2/b^2*exp(-a)-2/x/b*exp(-1/2*a))
```

**Maxima [A]**

time = 0.31, size = 15, normalized size = 0.21

$$-\frac{1}{4} b^2 e^{\left(\frac{1}{2} a\right)} \Gamma\left(-2, -\frac{1}{2} b x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(b*x+a)^(1/2)/x^3,x, algorithm="maxima")``[Out] -1/4*b^2*e^(1/2*a)*gamma(-2, -1/2*b*x)`**Fricas [A]**

time = 0.36, size = 38, normalized size = 0.54

$$\frac{b^2 x^2 \operatorname{Ei}\left(\frac{1}{2} b x\right) e^{\left(\frac{1}{2} a\right)} - 2 (b x + 2) e^{\left(\frac{1}{2} b x + \frac{1}{2} a\right)}}{8 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(b*x+a)^(1/2)/x^3,x, algorithm="fricas")``[Out] 1/8*(b^2*x^2*Ei(1/2*b*x)*e^(1/2*a) - 2*(b*x + 2)*e^(1/2*b*x + 1/2*a))/x^2`**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e^a e^{bx}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(b*x+a)**(1/2)/x**3,x)``[Out] Integral(sqrt(exp(a)*exp(b*x))/x**3, x)`**Giac [A]**

time = 2.93, size = 46, normalized size = 0.65

$$\frac{b^2 x^2 \operatorname{Ei}\left(\frac{1}{2} b x\right) e^{\left(\frac{1}{2} a\right)} - 2 b x e^{\left(\frac{1}{2} b x + \frac{1}{2} a\right)} - 4 e^{\left(\frac{1}{2} b x + \frac{1}{2} a\right)}}{8 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(b*x+a)^(1/2)/x^3,x, algorithm="giac")``[Out] 1/8*(b^2*x^2*Ei(1/2*b*x)*e^(1/2*a) - 2*b*x*e^(1/2*b*x + 1/2*a) - 4*e^(1/2*b*x + 1/2*a))/x^2`

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e^{a+bx}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(a + b\*x)^(1/2)/x^3,x)

[Out] int(exp(a + b\*x)^(1/2)/x^3, x)

$$3.98 \quad \int \frac{\sqrt{e^{a+bx}}}{x^4} dx$$

**Optimal.** Leaf size=92

$$-\frac{\sqrt{e^{a+bx}}}{3x^3} - \frac{b\sqrt{e^{a+bx}}}{12x^2} - \frac{b^2\sqrt{e^{a+bx}}}{24x} + \frac{1}{48}b^3e^{-\frac{bx}{2}}\sqrt{e^{a+bx}}\text{Ei}\left(\frac{bx}{2}\right)$$

[Out]  $-1/3*\exp(b*x+a)^{(1/2)}/x^3-1/12*b*\exp(b*x+a)^{(1/2)}/x^2-1/24*b^2*\exp(b*x+a)^{(1/2)}/x+1/48*b^3*\text{Ei}(1/2*b*x)*\exp(b*x+a)^{(1/2)}/\exp(1/2*b*x)$

**Rubi [A]**

time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2208, 2213, 2209}

$$\frac{1}{48}b^3e^{-\frac{bx}{2}}\sqrt{e^{a+bx}}\text{Ei}\left(\frac{bx}{2}\right) - \frac{b^2\sqrt{e^{a+bx}}}{24x} - \frac{\sqrt{e^{a+bx}}}{3x^3} - \frac{b\sqrt{e^{a+bx}}}{12x^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[E^(a + b\*x)]/x^4,x]

[Out]  $-1/3*\text{Sqrt}[E^{(a + b*x)}]/x^3 - (b*\text{Sqrt}[E^{(a + b*x)}])/(12*x^2) - (b^2*\text{Sqrt}[E^{(a + b*x)}])/(24*x) + (b^3*\text{Sqrt}[E^{(a + b*x)}]*\text{ExpIntegralEi}[(b*x)/2])/(48*E^{((b*x)/2)})$

Rule 2208

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)\*((b\*F^(g\*(e + f\*x)))^n/(d\*(m + 1))), x] - Dist[f\*g\*n\*(Log[F]/(d\*(m + 1))), Int[(c + d\*x)^(m + 1)\*(b\*F^(g\*(e + f\*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2\*m] && !TrueQ[\$UseGamma]

Rule 2209

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - c\*(f/d)))/d)\*ExpIntegralEi[f\*g\*(c + d\*x)\*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

Rule 2213

Int[((b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(b\*F^(g\*(e + f\*x)))^n/F^(g\*n\*(e + f\*x)), Int[(c + d\*x)^m\*F^(g\*n\*(e + f\*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{e^{a+bx}}}{x^4} dx &= -\frac{\sqrt{e^{a+bx}}}{3x^3} + \frac{1}{6}b \int \frac{\sqrt{e^{a+bx}}}{x^3} dx \\
 &= -\frac{\sqrt{e^{a+bx}}}{3x^3} - \frac{b\sqrt{e^{a+bx}}}{12x^2} + \frac{1}{24}b^2 \int \frac{\sqrt{e^{a+bx}}}{x^2} dx \\
 &= -\frac{\sqrt{e^{a+bx}}}{3x^3} - \frac{b\sqrt{e^{a+bx}}}{12x^2} - \frac{b^2\sqrt{e^{a+bx}}}{24x} + \frac{1}{48}b^3 \int \frac{\sqrt{e^{a+bx}}}{x} dx \\
 &= -\frac{\sqrt{e^{a+bx}}}{3x^3} - \frac{b\sqrt{e^{a+bx}}}{12x^2} - \frac{b^2\sqrt{e^{a+bx}}}{24x} + \frac{1}{48} \left( b^3 e^{\frac{1}{2}(-a-bx)} \sqrt{e^{a+bx}} \right) \int \frac{e^{\frac{1}{2}(a+bx)}}{x} dx \\
 &= -\frac{\sqrt{e^{a+bx}}}{3x^3} - \frac{b\sqrt{e^{a+bx}}}{12x^2} - \frac{b^2\sqrt{e^{a+bx}}}{24x} + \frac{1}{48} b^3 e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \operatorname{Ei}\left(\frac{bx}{2}\right)
 \end{aligned}$$

### Mathematica [A]

time = 0.07, size = 64, normalized size = 0.70

$$\frac{e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \left( -2e^{\frac{bx}{2}} (8 + 2bx + b^2x^2) + b^3x^3 \operatorname{Ei}\left(\frac{bx}{2}\right) \right)}{48x^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[E^(a + b\*x)]/x^4,x]

[Out] (Sqrt[E^(a + b\*x)]\*(-2\*E^((b\*x)/2)\*(8 + 2\*b\*x + b^2\*x^2) + b^3\*x^3\*ExpIntegralEi[(b\*x)/2]))/(48\*E^((b\*x)/2)\*x^3)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(69) = 138.

time = 0.02, size = 189, normalized size = 2.05

method	result
meijerg	$  \frac{\sqrt{e^{bx+a}} e^{\frac{3a}{2} - \frac{bx}{2}} b^3 \left( -\frac{e^{-\frac{3a}{2}} \left( \frac{11b^3x^3e^{\frac{3a}{2}}}{4} + 9b^2x^2e^a + 18bxe^{\frac{a}{2}} + 24 \right)}{9b^3x^3} + \frac{e^{-\frac{3a}{2} + \frac{bx}{2}} \left( b^2x^2e^a + 2bxe^{\frac{a}{2}} + 8 \right)}{3b^3x^3} + \frac{\ln\left(-\frac{bx}{2}\right)}{6} + \frac{\operatorname{ExpIntegralEi}\left(\frac{bx}{2}\right)}{48} \right)}{48x^3}  $

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(b\*x+a)^(1/2)/x^4,x,method=\_RETURNVERBOSE)

[Out] -1/8\*exp(b\*x+a)^(1/2)\*exp(3/2\*a-1/2\*b\*x\*exp(1/2\*a))\*b^3\*(-1/9/b^3/x^3\*exp(-3/2\*a)\*(11/4\*b^3\*x^3\*exp(3/2\*a)+9\*b^2\*x^2\*exp(a)+18\*b\*x\*exp(1/2\*a)+24)+1/3/



$$b^3/x^3 \exp(-3/2*a + 1/2*b*x*\exp(1/2*a)) * (b^2*x^2*\exp(a) + 2*b*x*\exp(1/2*a) + 8) + 1/6*\ln(-1/2*b*x*\exp(1/2*a)) + 1/6*Ei(1, -1/2*b*x*\exp(1/2*a)) + 11/36 - 1/6*\ln(x) + 1/6*\ln(2) - 1/6*\ln(-b*\exp(1/2*a)) + 8/3/x^3/b^3*\exp(-3/2*a) + 2/x^2/b^2*\exp(-a) + 1/x/b*\exp(-1/2*a)$$

**Maxima** [A]

time = 0.32, size = 15, normalized size = 0.16

$$\frac{1}{8} b^3 e^{(\frac{1}{2} a)} \Gamma\left(-3, -\frac{1}{2} b x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)^(1/2)/x^4,x, algorithm="maxima")

[Out] 1/8\*b^3\*e^(1/2\*a)\*gamma(-3, -1/2\*b\*x)

**Fricas** [A]

time = 0.39, size = 46, normalized size = 0.50

$$\frac{b^3 x^3 Ei(\frac{1}{2} b x) e^{(\frac{1}{2} a)} - 2(b^2 x^2 + 2 b x + 8) e^{(\frac{1}{2} b x + \frac{1}{2} a)}}{48 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)^(1/2)/x^4,x, algorithm="fricas")

[Out] 1/48\*(b^3\*x^3\*Ei(1/2\*b\*x)\*e^(1/2\*a) - 2\*(b^2\*x^2 + 2\*b\*x + 8)\*e^(1/2\*b\*x + 1/2\*a))/x^3

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e^a e^{bx}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)\*\*(1/2)/x\*\*4,x)

[Out] Integral(sqrt(exp(a)\*exp(b\*x))/x\*\*4, x)

**Giac** [A]

time = 2.29, size = 63, normalized size = 0.68

$$\frac{b^3 x^3 Ei(\frac{1}{2} b x) e^{(\frac{1}{2} a)} - 2 b^2 x^2 e^{(\frac{1}{2} b x + \frac{1}{2} a)} - 4 b x e^{(\frac{1}{2} b x + \frac{1}{2} a)} - 16 e^{(\frac{1}{2} b x + \frac{1}{2} a)}}{48 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b\*x+a)^(1/2)/x^4,x, algorithm="giac")

[Out]  $\frac{1}{48}(b^3x^3\text{Ei}(1/2bx)e^{1/2a} - 2b^2x^2e^{1/2bx + 1/2a} - 4bx e^{1/2bx + 1/2a} - 16e^{1/2bx + 1/2a})/x^3$

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e^{a+bx}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(a + b*x)^(1/2)/x^4,x)`

[Out] `int(exp(a + b*x)^(1/2)/x^4, x)`

# Chapter 4

## Appendix

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## 4.1 Download section

The following zip files contain the raw integrals used in this test.

**Mathematica format** Mathematica\_syntax.zip

**Maple and Mupad format** Maple\_syntax.zip

**Sympy format** SYMPY\_syntax.zip

**Sage math format** SAGE\_syntax.zip

## 4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7, 2022. add second output which gives reason for the grade *)
(*           Small rewrite of logic in main function to make it*)
(*           match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*           is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*           antiderivative*)
(* "A" if result can be considered optimal*)
```

```

GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
  expnResult = ExpnType[result];
  expnOptimal = ExpnType[optimal];
  leafCountResult = LeafCount[result];
  leafCountOptimal = LeafCount[optimal];

  (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
  If[expnResult<=expnOptimal,
    If[Not[FreeQ[result,Complex]], (*result contains complex*)
      If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
          finalresult={"A","none"}
          ,(*ELSE*)
          finalresult={"B","Both result and optimal contain complex but leaf count
        ]
        ,(*ELSE*)
        finalresult={"C","Result contains complex when optimal does not."}
      ]
      ,(*ELSE*)(*result does not contains complex*)
      If[leafCountResult<=2*leafCountOptimal,
        finalresult={"A","none"}
        ,(*ELSE*)
        finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
      ]
    ]
    ,(*ELSE*)(*expnResult>expnOptimal*)
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      finalresult={"C","Result contains higher order function than in optimal. Order "<
    ,
    finalresult={"F","Contains unresolved integral."}
  ]
];

finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)

```

(\*9 = unknown function\*)

```

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType, expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]], 2]],
      Max[ExpnType[expn[[1]], ExpnType[expn[[2]], 3]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3, ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
  If[Head[expn]===RootSum,
    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
  9]]]]]]]]]]]

```

```

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

```

```

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,

```

```

ExpIntegralE, ExpIntegralEi, LogIntegral,
SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
Gamma, LogGamma, PolyGamma,
Zeta, PolyLog, ProductLog,
EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]

```

## 4.2.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems
#Nasser 4/07/2022 add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
      leaf_count_optimal,
      ExpnType_result,
      ExpnType_optimal,
      debug:=false;

  leaf_count_result:=leafcount(result);
  #do NOT call ExpnType() if leaf size is too large. Recursion problem
  if leaf_count_result > 500000 then
    return "B","result has leaf size over 500,000. Avoiding possible recursion issues";
  fi;

  leaf_count_optimal := leafcount(optimal);
  ExpnType_result := ExpnType(result);
  ExpnType_optimal := ExpnType(optimal);

```

```

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
#   is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
#   antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A","";
            else
                return "B",cat("Both result and optimal contain complex but leaf count of r
                    convert(leaf_count_result,string)," vs. $2 (" ,
                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co

        end if
    else #result contains complex but optimal is not
        if debug then
            print("result contains complex but optimal is not");
        fi;
        return "C","Result contains complex when optimal does not.";
    fi;
else # result do not contain complex

```



```

    # this assumes optimal do not as well. No check is needed here.
    if debug then
        print("result do not contain complex, this assumes optimal do not as well")
    fi;
    if leaf_count_result<=2*leaf_count_optimal then
        if debug then
            print("leaf_count_result<=2*leaf_count_optimal");
        fi;
        return "A","";
    else
        if debug then
            print("leaf_count_result>2*leaf_count_optimal");
        fi;
        return "B",cat("Leaf count of result is larger than twice the leaf count of o
                        convert(leaf_count_result,string)," $ vs. $2(",
                        convert(leaf_count_optimal,string),")=",convert(2*leaf_cou

    fi;
    fi;
else #ExpnType(result) > ExpnType(optimal)
    if debug then
        print("ExpnType(result) > ExpnType(optimal)");
    fi;
    return "C",cat("Result contains higher order function than in optimal. Order ",
                  convert(ExpnType_result,string)," vs. order ",
                  convert(ExpnType_optimal,string),".");
fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
    return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function

```

```

# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+`) or type(expn,'*`) then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,

```

```

    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func, [
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func, [Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func, [AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

### 4.2.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#      Port of original Maple grading function by
#      Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#      added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
    ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
    ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

```

```

except AttributeError as error:
    return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'`^`')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:

```

```

    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation = ""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation = ""
                    else:
                        grade = "B"
                        grade_annotation = "Both result and optimal contain complex but leaf count of result is larger"
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation = "Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation = ""
                else:
                    grade = "B"
                    grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_count_result)
            else:
                grade = "C"
                grade_annotation = "Result contains higher order function than in optimal. Order "+str(ExpnType_result)

```

```

# print("Before returning. grade=", grade, " grade_annotation=", grade_annotation)

return grade, grade_annotation

```

#### 4.2.4 SageMath grading function

```

# Dec 24, 2019. Nasser: Ported original Maple grading function by
#       Albert Rich to use with Sagemath. This is used to
#       grade Fracas, Giac and Maxima results.
# Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#       'arctan2', 'floor', 'abs', 'log_integral'
# June 4, 2022 Made default grade_annotation "none" instead of "" due
#       issue later when reading the file.
# July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    # print("Enter tree_size, expr is ", expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: # isinstance(expr, Pow):
        if expr.operands()[1] == 1/2: # expr.args[1] == Rational(1,2):
            if debug: print("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arcoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral',
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func , " is special_function")
        else:
            print ("func ", func , " is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric'] #[appellf1] can't find this in sagemath

```



```

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-try:
    if expn.parent() is SR:
        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().gens()

    return False

except AttributeError as error:
    print("Exception,AttributeError in is_atom")
    print ("caught exception" , type(error).__name__ )
    return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)

```

```

    return 1
  else:
    return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
  else:
    return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or instan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

```

```

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = "none"
            else:
                grade = "B"
                grade_annotation = "Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation = "Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation = "none"
        else:
            grade = "B"
            grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation = "Result contains higher order function than in optimal. Order "+str(expnType_resu

print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```